Computation of Neutrino Masses in $R-$parity Violating Supersymmetry in SOFTSUSY

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Abstract

The program SOFTSUSY can calculate tree–level neutrino masses in the $R-$parity violating minimal supersymmetric standard model (MSSM) with real couplings. At tree–level, only one neutrino acquires a mass, in contradiction with neutrino oscillation data. Here, we describe an extension to the SOFTSUSY program which includes one–loop $R-$parity violating effects' contributions to neutrino masses and mixing. Including the one–loop effects refines the radiative electroweak symmetry breaking calculation, and may result in up to three massive, mixed neutrinos. This paper serves as a manual to the neutrino mass prediction mode of the program, detailing the approximations and conventions used.

Keywords: sparticle, MSSM
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PACS: 14.80.Ly

1. Program Summary

Program title: SOFTSUSY
Program obtainable from: http://projects.hepforge.org/softsusy/
Distribution format: tar.gz
Programming language: C++, fortran
Computer: Personal computer
Operating system: Tested on Linux 4.x
Word size: 32 bits
External routines: None
Typical running time: A second per parameter point.
Nature of problem: Calculation of neutrino masses and the neutrino mixing matrix at one–loop level in the R–parity violating minimal supersymmetric standard model. The solution to the renormalisation group equations must be consistent with a high or weak–scale boundary condition on supersymmetry breaking parameters and R–parity violating parameters, as well as a weak–scale boundary condition on gauge couplings, Yukawa couplings and the Higgs potential parameters.
Solution method: Nested iterative algorithm.
Restrictions: SOFTSUSY will provide a solution only in the perturbative régime and it assumes that all couplings of the MSSM are real (i.e. $CP-$conserving).

2. Introduction

Supersymmetric (SUSY) models provide an attractive weak–scale extension to the Standard Model (SM). The R–parity conserving (R_p) minimal supersymmetric extension of the Standard Model (R_p MSSM) is often used as a reference model for phenomenological studies. There exist several publicly available spectrum generators for the R_p MSSM: `ISASUGRA` [1], `SOFTSUSY` [2], `SUSPECT` [3] and `SPHENO`[1] [4]. Spectrum information is typically transferred to decay packages and event generators via a file in the SUSY Les Houches Accord format [5, 6].

The most general renormalisable superpotential of the MSSM contains R−Parity violating ($\not{R}_p$) couplings which violate baryon and lepton number [7]. A symmetry can be imposed upon the model in order to maintain stability of the proton, for example baryon triality [8] or proton hexality [9]. It has been shown that $\not{R}_p$ models may have interesting features, such as the generation of non–zero neutrino masses and mixing without the addition of right–handed neutrino fields [10]. In fact, neutrino oscillation data indicates that at least two neutrinos must be massive, so a realistic extension to the SM should include mechanisms to generate these masses. Here, we describe an extension to `SOFTSUSY` which calculates neutrino masses and mixing to one–loop order in the presence of $\not{R}_p$ couplings. The latest version of `SOFTSUSY` including $\not{R}_p$ effects can be downloaded from the address

```
http://projects.hepforge.org/softsusy/
```

Installation instructions and more detailed technical documentation of the code may also be found there.

The R_p and $\not{R}_p$ aspects of the `SOFTSUSY` calculation leading to self–consistent spectra are already explained in detail in Refs. [2] and [11] respectively, with the technical differences between these two calculations detailed in the latter reference. They shall not be repeated here. Instead we shall concentrate on the calculation of neutrino masses and mixing, and differences in the $\not{R}_p$ calculations between the new and the previous release, which are called the neutrino mode and the $\not{R}_p$ mode respectively. Two new features of `SOFTSUSY3.2` are the $\not{R}_p$ one–loop tadpole corrections to the two Higgs vacuum expectation values (VEVs), and the complete one–loop tadpole corrections to the sneutrino VEVs. These improvements allow the computation of the neutrino spectrum and neutrino mixings, with minimal additional computational cost. They also slightly change some predictions in the non-neutrino sector.

We proceed with a definition of the `SOFTSUSY` convention for the $\not{R}_p$ parameters and mixings relevant for the neutrino mass calculation in section 3. Next, in section 4, we discuss the calculation of the neutrino masses, detailing the approximations made. Installation instructions can be found on the `SOFTSUSY` web–site, but instructions to run the program can be found in Appendix A. The output from a `SOFTSUSY` sample run in the neutrino mode is displayed and discussed in Appendix C, whereas a sample main program is shown and explained in Appendix B. Some more technical information on the structure of the program can be found in Appendix D. It is expected that the information in Appendix D will only be of use to users who wish to 'hack' `SOFTSUSY` in some fashion.

3. MSSM $\not{R}_p$ Parameters

In this section, we introduce the $\not{R}_p$ MSSM parameters in the `SOFTSUSY` conventions relevant for the neutrino mass and 1–loop tadpole calculations. A detailed description of the complete set of $\not{R}_p$ MSSM parameters is presented in Ref. [11]. The latter follows Ref. [12] and so the notation and conventions employed are similar.

3.1. $\not{R}_p$ supersymmetric and SUSY breaking parameters

The chiral superfield particle content of the MSSM has the following $SU(3)_c \times SU(2)_L \times U(1)_Y$ quantum numbers:

$$\begin{aligned} L: &\quad (1,2,-\tfrac{1}{2}), \quad \bar{E}: \quad (1,1,1), \qquad Q: (3,2,\frac{1}{6}), \quad \bar{U}: (\bar{3},1,-\frac{2}{3}), \\ \bar{D}: &\quad (\bar{3},1,\tfrac{1}{3}), \quad H_1: \quad (1,2,-\frac{1}{2}), \quad H_2: (1,2,\frac{1}{2}). \end{aligned} \tag{1}$$

[1] `SPHENO` includes a small subset of the $\not{R}_p$ interactions.

L, Q, H_1, and H_2 are the left handed doublet lepton and quark superfields and the two Higgs doublets. $\bar{E}$, $\bar{U}$, and $\bar{D}$ are the lepton, up–type quark and down–type quark right–handed superfield singlets, respectively. The $\not{R}_p$ part of the renormalisable MSSM superpotential that violates lepton number, written in the interaction eigenbasis, is

$$W_{\not{R}_p} \subset \epsilon_{ab}\left[\frac{1}{2}\lambda_{ijk}L_i^a L_j^b \bar{E}_k + \lambda'_{ijk}L_i^a Q_j^{xb}\bar{D}_{kx} - \kappa_i L_i^a H_2^b\right]. \tag{2}$$

Here, κ_i [λ_{ijk} and λ'_{ijk}] are the bi–linear [trilinear] couplings. We denote an $SU(3)$ colour index of the fundamental representation by $\{x, y, z\} \in \{1, 2, 3\}$. The $SU(2)_L$ fundamental representation indices are denoted by $\{a, b, c\} \in \{1, 2\}$ and the generation indices by $\{i, j, k\} \in \{1, 2, 3\}$. $\epsilon_{ab} = \epsilon^{ab}$ is a totally antisymmetric tensor, with $\epsilon_{12} = 1$. Currently, only real couplings in the superpotential and Lagrangian are included.

The corresponding soft $\not{R}_p$ breaking interaction potential, together with the bi–linear mixing term between the scalar component of the lepton doublets and the H_1 superfields, is involved in tadpole and 1–loop corrections to the neutrino–neutralino masses. It is given by

$$V_{\not{R}_p} \subset \epsilon_{ab}\left[\frac{1}{2}h_{ijk}\tilde{L}_i^a\tilde{L}_j^b\tilde{e}_k + h'_{ijk}\tilde{L}_i^a\tilde{Q}_j^{bx}\tilde{d}_{kx} - D_i\tilde{L}_i^a H_2^b \;+\; \text{H.c.}\right] + \left[m^2_{\tilde{L}_i H_1}\tilde{L}^\dagger_{ia}H_1^a + \;\text{H.c.}\right], \tag{3}$$

where fields with a tilde are the scalar components of the superfield with the same capital letter. The scalar components of the superfields H_1 and H_2 and the superfields themselves have the same notation. The electric charges of $\tilde{d}$ and $\tilde{e}$ are $\frac{1}{3}$, and 1, respectively. h_{ijk} and h'_{ijk} are trilinear soft SUSY breaking parameters that correspond to λ_{ijk} and λ'_{ijk}, and D_i and $m^2_{\tilde{L}_i H_1}$ are bi–linear SUSY breaking parameters. "H.c." denotes the Hermitian conjugate of the preceding terms.

3.2. Neutral and charged fermion masses

In the presence of $\not{R}_p$ interactions that violate lepton number, the neutrinos and neutralinos mix, and the charginos and leptons mix with each other. Sneutrino–anti-sneutrino mixing is also present in principle; in practice this has been shown to have negligible phenomenological consequences once experimental bounds have been applied [13], and is neglected in our calculation.

The (7×7) neutrino–neutralino Lagrangian mass term, containing three families of neutrinos is given in [12] and reads

$$\mathcal{L} = -\frac{1}{2}\,(\nu_i, -i\widetilde{\mathcal{B}}, -i\widetilde{W}^{(3)}, \tilde{h}_1^0, \tilde{h}_2^0)\;\mathcal{M}_{\rm N}\begin{pmatrix}\nu_j\\ -i\widetilde{\mathcal{B}}\\ -i\widetilde{W}^{(3)}\\ \tilde{h}_1^0\\ \tilde{h}_2^0\end{pmatrix}, \tag{4}$$

where at tree level,

$$\mathcal{M}_{\rm N} = \begin{pmatrix} 0_{ij} & -\frac{g'}{2}v_i & \frac{g_2}{2}v_i & 0 & -\kappa_i\\ -\frac{g'}{2}v_j & M_1 & 0 & -\frac{g'}{2}\langle H_1^0\rangle & \frac{g'}{2}\langle H_2^0\rangle\\ \frac{g_2}{2}v_j & 0 & M_2 & \frac{g_2}{2}\langle H_1^0\rangle & -\frac{g_2}{2}\langle H_2^0\rangle\\ 0 & -\frac{g'}{2}\langle H_1^0\rangle & \frac{g_2}{2}\langle H_1^0\rangle & 0 & -\mu\\ -\kappa_i & \frac{g'}{2}\langle H_2^0\rangle & -\frac{g_2}{2}\langle H_2^0\rangle & -\mu & 0\end{pmatrix}. \tag{5}$$

In Eq. (4), the flavour basis neutral fermions are the 3 neutrinos (ν_i), the gauginos ($\widetilde{\mathcal{B}}$,$\widetilde{W}$) and the higgsinos ($\tilde{h}_1^0$,$\tilde{h}_2^0$). In the mass matrix in Eq. (5), κ_i [μ] are the supersymmetric $\not{R}_p$ [R_p] bi–linear mixing parameters, v_i [$\langle H_2^0\rangle$ and $\langle H_1^0\rangle$]

are the sneutrino [H_2^0 and H_1^0] vacuum expectation values (VEVs), M_1, M_2 are the gaugino masses of hypercharge and weak isospin respectively, and g', g_2 are the corresponding gauge couplings. The mass eigenstates are obtained upon diagonalisation of $\mathcal{M}$: $\nu_{i=1,2,3}, \tilde{\chi}^0_{1,2,3,4}$ via a 7 by 7 orthogonal matrix O:

$$\mathcal{M}_{\rm N}^{diag} = O^T \mathcal{M}_{\rm N}\, O. \tag{6}$$

A simple multiplication of rows of O by factors of i can absorb any minus signs in $\mathcal{M}_{\rm N}^{diag}$. Mass eigenstates are ordered in increasing (absolute) mass eigenvalues.

To facilitate comparisons with neutrino oscillation data, it is useful to express $\mathcal{M}_{\rm N}$ as

$$\mathcal{M}_{\rm N} \;=\; \begin{pmatrix} m_\nu & m \\ m^T & \mathcal{M}_{\tilde{\chi}^0} \end{pmatrix}, \tag{7}$$

where m_ν is a 3×3 mass matrix, $\mathcal{M}_{\tilde{\chi}^0}$ is a 4×4 mass matrix, and m is a 3×4 matrix that mixes the neutrinos and the neutralinos. We define an effective 3×3 neutrino mass matrix $\mathcal{M}_\nu$ via the see–saw relation

$$\mathcal{M}_\nu \;\equiv\; m_\nu - m\, \mathcal{M}_{\tilde{\chi}^0}^{-1}\, m^T. \tag{8}$$

It can be diagonalised by a $O(3)$ matrix O_ν,

$$\mathcal{M}_\nu^{diag} = O_\nu^T \mathcal{M}_\nu\, O_\nu. \tag{9}$$

For realistic mass spectra, O_ν is practically the same as the 3×3 neutrino part of O (we do not assume this, however).

At tree level, the effective neutrino mass matrix is [12]

$$\mathcal{M}_\nu^{\rm tree} \;=\; \frac{\mu(M_1 g_2^2 + M_2 g'^2)}{2\langle H_1^0\rangle\langle H_1^0\rangle(M_1 g_2^2 + M_2 g'^2) - 2\mu M_1 M_2} \begin{pmatrix} \Delta_1\Delta_1 & \Delta_1\Delta_2 & \Delta_1\Delta_3 \\ \Delta_2\Delta_1 & \Delta_2\Delta_2 & \Delta_2\Delta_3 \\ \Delta_3\Delta_1 & \Delta_3\Delta_2 & \Delta_3\Delta_3 \end{pmatrix}, \tag{10}$$

where

$$\Delta_i \;\equiv\; v_i - \langle H_1^0\rangle \frac{\kappa_i}{\mu}, \qquad i = 1,2,3\,. \tag{11}$$

$\mathcal{M}_\nu^{\rm tree}$ is rank 1 and so contains two zero eigenvalues. Because of the presence of two massless neutrinos in the spectrum, the tree level values are not realistic. The numerical values of O_ν are also not meaningful in the tree-level approximation. The 1–loop corrections to $\mathcal{M}_\nu$ can lead to two or three non–zero neutrino masses, if more than one lepton flavour is violated.

By default, the neutrino masses are normal–ordered, i.e. $|m_{\nu_1}| < |m_{\nu_2}| < |m_{\nu_3}|$. A spectrum with inverted ordering can be obtained by

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \;\rightarrow\; \begin{pmatrix} \nu'_1 \\ \nu'_2 \\ \nu'_3 \end{pmatrix} = \begin{pmatrix} \nu_2 \\ \nu_3 \\ \nu_1 \end{pmatrix}, \tag{12}$$

together with the corresponding swaps (of the column vectors) in the mixing matrix O_ν. The mass ordering then becomes $|m_{\nu'_3}| < |m_{\nu'_1}| < |m_{\nu'_2}|$.

The presence of $\not{R}_p$ interactions also mix charged–leptons with the charginos. The Lagrangian contains the (5×5) chargino–lepton mass matrix

$$\mathcal{L} = -(-i\widetilde{W}^-, \tilde{h}_1^-, e_{L_j}^-)\, \mathcal{M}_{\rm C} \begin{pmatrix} -i\widetilde{W}^+ \\ \tilde{h}_2^+ \\ e_{R_k}^+ \end{pmatrix} + \text{H.c.}\,. \tag{13}$$

The mass eigenstates $\ell = (e,\mu,\tau), \tilde{\chi}^\pm_{1,2}$ are given upon the diagonalisation of the matrix $\mathcal{M}_{\rm C}$, where at tree level,

$$\mathcal{M}_{\rm C} = \begin{pmatrix} M_2 & \frac{g_2}{\sqrt{2}}\langle H_2^0\rangle & 0_j \\ \frac{g_2}{\sqrt{2}}\langle H_1^0\rangle & \mu & -\frac{1}{\sqrt{2}}(Y_E)_{ij} v_i \\ \frac{g_2}{\sqrt{2}} v_i & \kappa_i & \frac{1}{\sqrt{2}}\left((Y_E)_{ij}\langle H_1^0\rangle + \lambda_{kij} v_k\right) \end{pmatrix}, \tag{14}$$

Here, Y_E is the lepton Yukawa matrix from the R_p superpotential in Ref. [2]. We define the diagonalised mass matrix

$$\mathcal{M}_{\rm C}^{diag} = U \mathcal{M}_{\rm C} V^T, \tag{15}$$

U and V being orthogonal 5×5 matrices. The one–loop $\not{R}_p$ corrections to $\mathcal{M}_{\rm C}$ are related to neutrino masses. For $\not{R}_p$ effects giving small enough neutrino masses to pass empirical bounds, the one–loop $\not{R}_p$ corrections to $\mathcal{M}_{\rm C}$ are expected to be negligible, unless one assumes that the $\not{R}_p$ effects are large, but have a high degree of cancellation in the neutrino masses. At present, the one–loop $\not{R}_p$ corrections to $\mathcal{M}_{\rm C}$ are not implemented. The absolute mass eigenvalues are in increasing order along the diagonal of $\mathcal{M}_{\rm C}^{diag}$. An effective charged lepton mixing matrix, U_l, can be obtained from the 3×3, top right part of U. The PMNS matrix $U_{\rm PMNS}$ is then defined as

$$U_{\rm PMNS} = U_l^* O_\nu, \tag{16}$$

with O_ν including the (column) swapping when an inverted mass ordering is desired.

3.3. Neutral slepton masses

Lepton number violating interactions mix the sleptons and higgs'. In the limit of CP conservation, as assumed here, one obtains two mass squared matrices: one for the neutral CP even real scalars $\mathcal{M}^2_{\varphi^0_+}$, and one for neutral CP odd real scalars $\mathcal{M}^2_{\varphi^0_-}$. The Lagrangian containing these terms looks like

$$\mathcal{L} = -\frac{1}{2}(h^0_{2\pm}, h^0_{1\pm}, \tilde{\nu}_{j\pm})\, \mathcal{M}^2_{\varphi^0_\pm} \begin{pmatrix} h^0_{2\pm} \\ h^0_{1\pm} \\ \tilde{\nu}_{k\pm} \end{pmatrix}, \tag{17}$$

where the fields with subscript $+(-)$ are the real (imaginary) part of the complex fields in obvious notation. At tree level,

$$\begin{aligned} \mathcal{M}^2_{\varphi^0_\pm} \;=\; & \frac{(g'^2 + g_2^2)}{4} \left\{ \begin{pmatrix} \langle H_2^0\rangle^2 & -\langle H_2^0\rangle\langle H_1^0\rangle & -\langle H_2^0\rangle v_k \\ -\langle H_2^0\rangle\langle H_1^0\rangle & \langle H_1^0\rangle^2 & \langle H_2^0\rangle v_k \\ -\langle H_2^0\rangle v_j & \langle H_1^0\rangle v_j & v_j v_k \end{pmatrix} - \frac{\left(\langle H_2^0\rangle^2 - \langle H_1^0\rangle^2 - v_i^2\right)}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \delta_{jk} \end{pmatrix} \right\} \\ & + \begin{pmatrix} \frac{m_3^2\langle H_1^0\rangle + D_i v_i}{\langle H_2^0\rangle} & \mp m_3^2 & \mp D_k \\ \mp m_3^2 & \mu^2 + m^2_{H_1} & \mu\kappa_k + m^2_{\tilde{L}_k H_1} \\ \mp D_j & \mu\kappa_j + m^2_{\tilde{L}_j H_1} & \kappa_j\kappa_k + (m^2_{\tilde{L}})_{jk} \end{pmatrix}, \end{aligned} \tag{18}$$

where m_3^2 is the soft breaking term corresponding to the bilinear Higgs-mixing parameter μ.

Currently, $\mathcal{M}^2_{\varphi^0_\pm}$ is calculated at tree-level and is only used to calculate the one-loop corrections to neutrino masses and mixings. The mass eigenstates are ordered in mass $(m_{\varphi^0_\pm})_{i=1,\ldots,5}$

$$\text{diag}(m_{\varphi^0_\pm})_i^2 = O_\pm^T \mathcal{M}^2_{\varphi^0_\pm} O_\pm \tag{19}$$

with mixing matrix $O_\pm$.

4. Calculation Algorithm

The calculation to calculate the low energy $\not{R}_p$ mass spectrum proceeds mostly in the same way as the original $\not{R}_p$ mode, which is detailed in Ref. [11], but now also includes the full 1–loop $\not{R}_p$ tadpoles and additional methods to calculate the 1–loop 7x7 neutralino–neutrino mass matrix and to obtain the neutrino pole masses and the PMNS matrix via the seesaw–mechanism. The calculation algorithm is displayed in Fig. 1, where the changes with respect to the $\not{R}_p$ mode are displayed in bold print. For a given a set of inputs for $\not{R}_p$ couplings, assumptions about the measurable quark mixing make a physical difference to the results. It is well known that in the R_p MSSM, a $U(3)^5$ global family symmetry, where Yukawa couplings also transform as spurions, renders physical results invariant to where the CKM

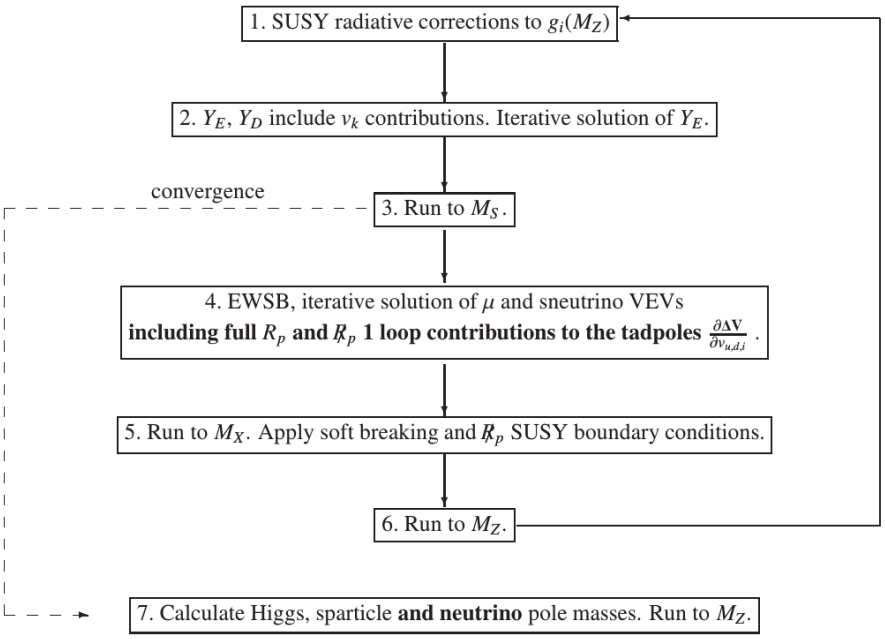

Figure 1: Iterative algorithm used to calculate the $\not{R}_p$ MSSM spectrum in the neutrino mode. Differences to the original $\not{R}_p$ mode are highlighted in bold print. The initial step is the uppermost one. M_S is the scale at which the EWSB conditions are imposed, as discussed in the text. M_X is the scale at which the high energy SUSY breaking boundary conditions are imposed.

quark mixing lies. For example, the physics is identical for the cases where CKM mixing lies solely in left handed up quarks and for the case where it lies solely in left handed down quarks, since the family symmetry relates the two cases [14]. However, in the $\not{R}_p$ MSSM, one would *also* have to transform the $\not{R}_p$ Yukawa couplings λ'_{ijk} and λ''_{ijk} to keep the theory invariant under the transformation. Since this is inconvenient, we wish to leave the $\not{R}_p$ couplings untransformed, and instead specify (in the basis in which the $\not{R}_p$ couplings are given), where the quark mixing lies. There is thus a physical difference depending on whether one assumes that CKM mixing lies e.g. solely in the down–type quarks or solely in the up–type quarks. We describe the parameter in the program which specifies this mixing in Appendix D. It can have a significant effect on the neutrino mass predictions.

4.1. $\not{R}_p$ tadpole contributions

The 1–loop $\not{R}_p$ contributions to the tadpoles $\frac{\partial \Delta \mathbf{V}}{\partial v_{u,d}}$, as well as R_p and $\not{R}_p$ contributions to $\frac{\partial \Delta \mathbf{V}}{\partial v_i}$, are implemented in the neutrino mode of `SOFTSUSY` . These calculations include all three SM fermion families in the loop. In the presence of $\not{R}_p$ interactions, the charged sleptons and down–type Higgs mix, and the neutral scalars and Higgs mix with each other, so out computation treats these contributions on an equal footing. The sneutrino VEVs are now also calculated at 1–loop level at the renormalisation scale $M_S \equiv \sqrt{m_{\tilde{l}_1}(M_S)\, m_{\tilde{l}_2}(M_S)}$. However,there are no renormalisation group equations (RGEs) implemented for the sneutrino VEVs, so they remain constant at any renormalisation scale.

4.2. Neutrino–neutralino mass corrections

The physical neutrino–neutralino masses are calculated to full one–loop order, including both R_p and $\not{R}_p$ contributions, following Refs. [15, 16] closely. The running parameters are evaluated at the renormalisation scale $\mu = M_{SUSY}$. However, note that the physical neutralino masses are displayed in the R_p limit in the `SOFTSUSY` output without including the $\not{R}_p$ contributions. However, in the absence of large cancellations, $\not{R}_p$ corrections to neutralino masses will be of order the neutrino mass, i.e. negligible. To calculate the physical neutrino masses, the external momentum for

the physical neutrino–neutralino mass matrix, $\mathcal{M}_{\rm N}^{phys}$, is set at $p = 0$. The matrix $\mathcal{M}_{\rm N}^{phys}$ is given by

$$\mathcal{M}_{\rm N}^{phys} = \mathcal{M}_{\rm N} + \frac{1}{2}\left(\Delta_{\rm N} + \Delta_{\rm N}^{T}\right), \tag{20}$$

where

$$\Delta_{\rm N} = \Sigma_D - \mathcal{M}_{\rm N}\Sigma_L. \tag{21}$$

In the above expression, Σ_D and Σ_L are mass corrections and wavefunction renormalisation respectively. The effective physical neutrino mass matrix $\mathcal{M}_\nu$ is obtained as defined in Eq. 8, from which the physical neutrino masses and mixings are extracted.

To avoid numerical instability due to large cancellations between the CP–even (CPE) and CP–odd (CPO) Σ_D contributions to m_ν in Eq. (7), the combined contribution is obtained using an analytic expansion in the 2×3 $\not{R}_p$ matrices that mix the Higgs with the sneutrinos in the 5×5 CPE and CPO neutral scalar mass matrices. Both R_p and $\not{R}_p$ corrections to the physical neutrino masses are computed simultaneously, treating the charged sleptons and down–type Higgs as well as the neutral scalars and Higgs particles on an equal footing, as in the tadpole calculation.

When calculating realistic inverted or quasi–degenerate neutrino mass spectra, the mass eigenvalues are much larger than their differences, so some fine tuning is expected. For example, in Ref. [17], the level of fine–tuning is estimated to be of $O(10^{-3})$ and $O(10^{-4})$ for the inverted and quasi–degenerate cases respectively. In order to obtain numerically stable results for neutrino masses and mixings for degenerate or inverted hierarchies, we recommend setting the `TOLERANCE` parameter to 10^{-5} and 10^{-6}, respectively.

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Appendix A. Running `SOFTSUSY`

`SOFTSUSY` produces an executable called `softpoint.x`. Instructions on how to run `softpoint.x` in the presence of $\not{R}_p$ couplings are detailed in Appendices A and B of Ref. [11]. Starting from version `SOFTSUSY-3.2`, output for both input options is compliant to the SLHA2 conventions, which are fully explained in Ref. [6]. A sample SLHA2 file, `rpvHouchesInput`, may be used as input, and `SOFTSUSY` run, by the command

```
./softpoint.x leshouches < rpvHouchesInput
```

When any non–zero $\not{R}_p$ couplings are set in either the command line or the $\not{R}_p$ Les Houches input file, the program automatically calls the neutrino mode instead of the original $\not{R}_p$ mode. There is a new optional SLHA2 input parameter in the block SOFTSUSY associated with the neutrino mode, which can be set in the following fashion:

```
Block SOFTSUSY
    9   0.000000000e+00      # output uses normal hierarchy (=0.0) or inverted (=1.0)
```

We also remind the reader that the quark mixing assumption can have a physical effect given an assumed set of $\not{R}_p$ couplings, and that the CKM mixing may be set solely in the up or down quark sector via

```
Block SOFTSUSY
    2   1.000000000e+00      # quark mixing: none (=0), up (=1) or down (=2)
```

Appendix B. Sample Program

In this section we present a sample main program, that illustrates a calculation of neutrino masses and mixings given a set of $\not{R}_p$ parameters at M_{GUT}. This main program is included as the `rpvneutmain.cpp` file with the standard `SOFTSUSY` distribution and performs the calculation assuming mSUGRA parameters $m_0 = 100$ GeV, $M_{1/2} = 500$ GeV, $A_0 = 932.9$ GeV, $\tan\beta = 20$ and $\mu > 0$, with non–zero $\not{R}_p$ parameters $\lambda'_{111}(M_{GUT}) = 0.039$, $\lambda'_{211}(M_{GUT}) = -0.016$, $\lambda'_{311}(M_{GUT}) = 0.018$, $\lambda'_{133}(M_{GUT}) = 3.0 \times 10^{-5}$, $\lambda'_{233}(M_{GUT}) = 3.0 \times 10^{-5}$, $\lambda'_{333}(M_{GUT}) = -3.5 \times 10^{-5}$.

The sample program has the following form:

```
/**
   Project:     SOFTSUSY
   File:        rpvmain.cpp
   Authors:     B.C. Allanach, Steve Kom and Marja Hanussek
   Manual:      B.C. Allanach, M. Hanussek and S. Kom, Comput. Phys. Commun.
                183 (2012) 785, arXiv:1109.3735 and
                B.C. Allanach,hep-ph/0104145, Comp. Phys. Comm. 143 (2002) 305
   Webpage:     http://projects.hepforge.org/softsusy/
   Description: main calling program example:
calculates neutrino masses for a certain RPV point
*/


#include "rpvmain.h"

int main() {
  /// Sets up exception handling
  signal(SIGFPE, FPE_ExceptionHandler);

  /// Apply SUSY breaking conditions at GUT scale, where g_1=g_2
  bool gaugeUnification = true;

  /// Sets format of output: 3 decimal places
  outputCharacteristics(3);

  /// "try" catches errors in main program and prints them out
  try {
    QedQcd oneset;      ///< See "lowe.h" for default parameter definitions
    oneset.toMz();      ///< Runs SM fermion masses to MZ
    oneset.runto(oneset.displayPoleMt());      ///< Runs SM fermion masses to MZ

    /// Guess at GUT scale
    double mxGuess = 2.e16;

    /// Close to scenario IH S2 from arXiv:1106.4338
    int sgnMu = 1;
    double tanb = 20., a0 = 924., m12 = 500., m0 = 100.;

    /// Define RpvNeutrino object
    RpvNeutrino kw;
    /// Quark mixing is defined to be in the up sector (1) at MZ
    kw.setMixing(1);

    /// Set the CKM angles in Wolfenstein parameterisation
    double lambda = 0.2272, aCkm = 0.818, rhobar = 0.221, etabar = 0.34;
    kw.setAngles(lambda, aCkm, rhobar, etabar);

    /// Set the GUT scale RPV SUSY couplings
```

```
    kw.setLamPrime(1, 1, 1,  0.0395);
    kw.setLamPrime(2, 1, 1, -0.018);
    kw.setLamPrime(3, 1, 1,  0.019);
    kw.setLamPrime(1, 3, 3,  3.0e-5);
    kw.setLamPrime(2, 3, 3,  3.2e-5);
    kw.setLamPrime(3, 3, 3, -3.5e-5);

    /// Store inputs into one vector
    DoubleVector pars(3); pars(1) = m0; pars(2) = m12; pars(3) = a0;

    /// Outputs the RPV couplings required into the vector pars used by lowOrg
    kw.rpvDisplay(pars);
    /// Makes sure the neutrino mass ordering will be as expected in inverted
    /// hierarchy output. If required, must be set before lowOrg is called
    kw.setInvertedOutput();

    /// Main driver routine: do the calculation
    kw.lowOrg(rpvSugraBcs, mxGuess, pars, sgnMu, tanb, oneset,
      gaugeUnification);

    /// Output the results in SLHA2 format
    double qMax = 0.;  char * modelIdent = (char *)"sugra";
    int numPoints = 1; bool ewsbBCscale = false;

    kw.lesHouchesAccordOutput(cout, modelIdent, pars, sgnMu, tanb, qMax,
      numPoints, ewsbBCscale);
  }
  catch(const string & a) {
    cerr << a; return -1;
  }
  catch(const char *a) {
    cerr << a; return -1;
  }
  return 0;
}
```

The structure of the main file above is as follows. After including a header file, an exception handler is set up, followed by specifying a (sub–) set of global variables, all of which are described in the R_p manual [2]. These include the `mixing` switch, which determines how any quark mixing is implemented, the `gaugeUnification` switch, which determines M_X as the scale of M_{GUT} of electroweak gauge unification. The switch `outputCharacteristics`, which specifies the output accuracy, is then set.

The running masses of the SM fermions and the QED and QCD gauge couplings are determined at M_Z from data with the method `toMz`. In order for `SOFTSUSY` to determine M_{GUT}, it requires an initial guess, which must be supplied as the initial value of the variable `mxGuess` (in GeV), and is later over–written by the program with a more accurate calculated value[2].

Next, the mSUGRA parameters sgnμ =`sgnMu`, tanβ =`tanb`, A_0/GeV=`a0`, $M_{1/2}$/GeV=`m12` and m_0/GeV=`m0` are defined. This is then followed by the instantiation of the `RpvNeutrino` object `kw` and the assignment of the $\not{R}_p$ parameters `kw.setLamPrime(i,j,k,val)` for λ'_{ijk} =`val`. In the iterative `SOFTSUSY` algorithm the parameters in the `RpvNeutrino` object change due to the RGE running. The `pars` vector is needed to keep track of the boundary conditions set at M_{GUT}. These boundary conditions are re–set in every iteration at M_{GUT} from the unchanged `DoubleVector pars` parameters.

We do not fill the other 102 $\not{R}_p$ entries of `pars` explicitly. This would be tedious and an additional source of potential bugs. Instead, we fill the `RpvSoftsusy` object itself using the `setLamPrime` method in this example. We use

[2]If the user wishes to provide this, 2×10^{16} GeV is a good initial guess for M_{GUT}.

the rpvDisplay method: this fills the pars vector automatically with what was set already inside the RpvNeutrino object, while leaving the first nine entries in the vector unchanged. The rpvDisplay method automatically changes the length of pars appropriately.

The neutrino masses may be presented in normal (default) and inverted mass orderings. This is controlled by the method setInvertedOutput which must be invoked before the actual SOFTSUSY main driving method lowOrg is called. When lowOrg is called, the first argument specifies the type of boundary condition (currently rpvSugraBcs), which assumes that pars has already been prepared by using the rpvDisplay object. This is followed by a print to standard output of the model parameters at M_Z and the physical parameters SLHA2 compliant format. Finally, the catch commands print any errors produced by the code.

Appendix C. Sample Output

The output is in standard SLHA2 format, including neutrino masses in GeV units, neutrino mass ordering (inverted hierarchy in this example) and the PMNS mixing matrix:

```
[ ... ]
Block MASS                      # Mass spectrum
        12     4.93408789e-11   # Mnu1 inverted hierarchy output
        14     1.24343048e-10   # Mnu2 inverted hierarchy output
        16     8.30886600e-14   # Mnu3 inverted hierarchy output
[ ... ]
Block RVSNVEV Q= 8.95402562e+02 # sneutrino VEVs D
     1   -5.85864597e-04   # SneutrinoVev_{1}
     2   -5.78522008e-05   # SneutrinoVev_{2}
     3    6.97534248e-05   # SneutrinoVev_{3}
[ ... ]
Block UPMNS Q= 9.11876000e+01 # neutrino mixing matrix (inverted  hierarchy)
  1  1     1.40526098e-01   # UPMNS_{11} matrix element
  1  2     9.90041013e-01   # UPMNS_{12} matrix element
  1  3     8.43849607e-03   # UPMNS_{13} matrix element
  2  1    -6.74893108e-01   # UPMNS_{21} matrix element
  2  2     8.95510591e-02   # UPMNS_{22} matrix element
  2  3     7.32461535e-01   # UPMNS_{23} matrix element
  3  1     7.24411284e-01   # UPMNS_{31} matrix element
  3  2    -1.08625044e-01   # UPMNS_{32} matrix element
  3  3     6.80756117e-01   # UPMNS_{33} matrix element
[ ... ]
```

The block MASS includes the three pole neutrino masses calculated in units of GeV, given the $R\!\!\!/_p$ model parameters. Note that the corresponding particle data group (PDG) codes [19] are for the neutrino flavour eigenstates. However, since we are dealing with massive neutrinos, we slightly adapt this definition such that the mass ordering of the neutrinos is specified to be either normal or inverted hierarchy according to the value of the invertedOutput parameter. In block RVSNVEV, the 1–loop sneutrino VEVs are displayed at the energy scale Q at which the EWSB conditions are imposed (M_S). Finally, the block UPMNS displays the matrix elements of the matrix $U_{\rm PMNS}$ in Eq. 16.

Appendix D. The RpvNeutrino class

We now go on to sketch the class RpvNeutrino. This class publicly inherits from the class RpvSoftsusy, *cf.* Fig. D.2. The RpvSoftsusy methods physical, calculateSneutrinoVevs and doCalcTadpole1oneLoop, doCalcTadpole2oneLoop are overloaded in RpvNeutrino in order to include 1–loop neutrino masses, sneutrino VEVs and tadpoles. The data and methods in the RpvNeutrino class deemed of possible importance for prospective users are presented in Table D.1. Note that different quark mixing assumptions can significantly affect the predictions of the neutrino masses. The mixing parameter is implemented in the neutrino mode in the same fashion as in Ref. [2], and can take values 0, 1, 2 for no mixing, up quark mixing and down quark mixing, respectively.

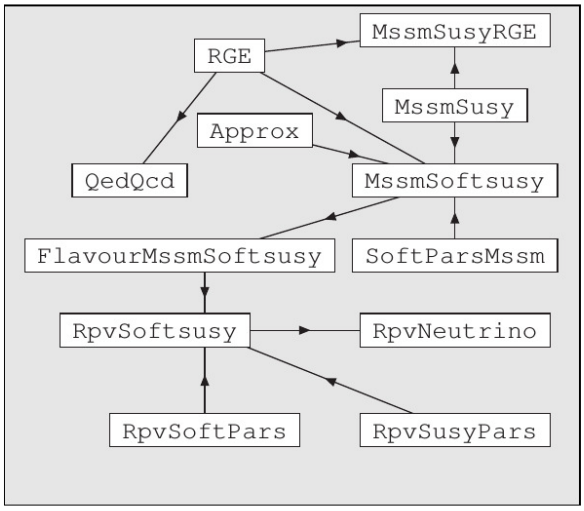

Figure D.2: Heuristic high-level object structure of SOFTSUSY-3.2. Inheritance is displayed by the direction of the arrows.

data variable		methods
`DoubleVector physNuMasses` $m_{\nu_i=1,2,3}$/GeV	physical neutrino masses	`displayPhysNuMasses` `setPhysNuMasses`
`DoubleMatrix uPmns` $U_{\rm PMNS}$ (3×3)	PMNS mixing matrix	`displayUpmns` `setUpmns`
`DoubleMatrix physNeutMix` O (7×7)	physical neutral fermion mixing	`displayPhysNeutMix` `setPhysNeutMix`

`bool invertedOutput`	neutrino mass ordering	`displayInvertedOutput`
		`setInvertedOutput`
		`setNormalOutput`

`double theta12, theta23`
`double theta13, deltaCkm`
$\theta_{12},\theta_{23},\theta_{13},\delta$/radians
	PDG parameterisation of CKM angles	`displayThetaCkm12, displayThetaCkm23`
		`displayThetaCkm13, displayDeltaCkm`
		`setThetaCkm12, setThetaCkm23`
		`setThetaCkm13, setDeltaCkm`

`DoubleVector CPEmasses` $(m_{\varphi^0_+})_{i=1\ldots5}$	tree-level CP even neutral scalar masses	`displayCPEMasses` `setCPEMasses`
`DoubleVector CPOmasses` $(m_{\varphi^0_-})_{i=1\ldots5}$	tree-level CP odd neutral scalar masses	`displayCPOMasses` `setCPOMasses`
`DoubleMatrix CPEscalars` $\mathcal{M}^2_{\varphi^0_+}$ (5×5)	tree-level CP even neutral scalar mass^2 matrix	`displayCPEscalars` `setCPEscalars`
`DoubleMatrix CPOscalars` $\mathcal{M}^2_{\varphi^0_-}(5\times 5)$	tree-level CP odd neutral scalar mass^2 matrix	`displayCPOscalars` `setCPOscalars`
`DoubleMatrix CPEscalarMixing` O_+ (5×5)	tree-level CP even neutral scalar mixing matrix	`displayCPEscalarMixing` `setCPEscalarMixing`
`DoubleMatrix CPOscalarMixing` O_- (5×5)	tree-level CP odd neutral scalar mixing matrix	`displayCPOscalarMixing` `setCPOscalarMixing`

Table D.1: `RpvNeutrino` class. We display the important data contained within the object, along with ways of accessing and setting their values. For the definitions of the methods, see the file `rpvneut.h` in the SOFTSUSY distribution.

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