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Higher Order Corrections and Unification in the Minimal Supersymmetric Standard Model: SOFTSUSY3.5

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Abstract

We explore the effects of three-loop minimal supersymmetric standard model renormalisation group equation terms and some leading two-loop threshold corrections on gauge and Yukawa unification: each being one loop higher order than current public spectrum calculators. We also explore the effect of the higher order terms (often 2-3 GeV) on the lightest CP even Higgs mass prediction. We illustrate our results in the constrained minimal supersymmetric standard model. Neglecting threshold corrections at the grand unified scale, the discrepancy between the unification scale α_s and the other two unified gauge couplings changes by 0.1% due to the higher order corrections and the difference between unification scale bottom-tau Yukawa couplings neglecting unification scale threshold corrections changes by up to 1%. The difference between unification scale bottom and top Yukawa couplings changes by a few percent. Differences due to the higher order corrections also give an estimate of the size of theoretical uncertainties in the minimal supersymmetric standard model spectrum. We use these to provide estimates of theoretical uncertainties in predictions of the dark matter relic density (which can be of order one due to its strong dependence on sparticle masses) and the LHC sparticle production cross-section (often around 30%). The additional higher order corrections have been incorporated into SOFTSUSY, and we provide details on how to compile and use the program. We also provide a summary of the approximations used in the higher order corrections.

Keywords: sparticle, MSSM
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1. Program Summary

Program title: SOFTSUSY
Program obtainable from: `http://softsusy.hepforge.org/`
Distribution format: tar.gz
Programming language: C++, `fortran`
Computer: Personal computer.
Operating system: Tested on Linux 3.4.6
Word size: 64 bits.
External routines: At least `GiNaC1.3.5` [1] and `CLN1.3.1` (both freely obtainable from `www.ginac.de`).
Typical running time: A minute per parameter point.

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Nature of problem: Calculating supersymmetric particle spectrum and mixing parameters in the minimal supersymmetric standard model. The solution to the renormalisation group equations must be consistent with boundary conditions on supersymmetry breaking parameters, as well as the weak-scale boundary condition on gauge couplings, Yukawa couplings and the Higgs potential parameters.
Solution method: Nested iterative algorithm.
Restrictions: SOFTSUSY will provide a solution only in the perturbative regime and it assumes that all couplings of the model are real (i.e. $CP-$conserving). If the parameter point under investigation is non-physical for some reason (for example because the electroweak potential does not have an acceptable minimum), SOFTSUSY returns an error message. The higher order corrections included are for the real $R-$parity conserving minimal supersymmetric standard model (MSSM) only.
CPC Classification: 11.1 and 11.6.
Does the new version supersede the previous version?: Yes.
Reasons for the new version: Extension to include additional two and three-loop terms.
Summary of revisions: All quantities in the minimal supersymmetric standard model are extended to have three-loop renormalisation group equations (including 3-family mixing) in the limit of real parameters and some leading two-loop threshold corrections are incorporated to the third family Yukawa couplings and the strong gauge coupling.

2. Introduction

The recent discovery of the Higgs boson [2, 3] and the measurement of its mass at around 125-126 GeV [4] solidify the important and well-known question of how its mass is stabilised with respect to quantum corrections, which are expected to be of order the largest fundamental mass scale divided by the $16\pi^2$ loop factor. In particular, the Planck mass at $\sim 10^{19}$ GeV is expected to be the largest such relevant mass scale. However, since a quantum field theoretic description of gravity does not exist it is possible, if not expected, that our effective field theory description breaks down and such huge corrections are absent for some reason. In any case, mass scales associated with the string scale $\sim 10^{17}$ GeV or the grand unified theory (GUT) scale $M_{GUT} \sim 10^{16}$ GeV reintroduce the question of stability of the Higgs mass. Imposing softly-broken supersymmetry upon the Standard Model provides a well-known answer to this question, and this approach has been pursued with vigour in the literature and at various high energy colliders (see, for example, Refs. [5, 6]), where the predicted Standard Model particles' supersymmetric partners are being searched for. To date, no unambiguous direct collider signals of supersymmetric particles have been found, and a significant portion of the most interesting parameter space has been ruled out. In order to rule a parameter point out, one predicts sparticle masses using a supersymmetric spectrum generator and then simulates various collisions, comparing to data to see if the predicted signals are significantly excluded or not (or conversely, to see if there is statistically significant evidence for a signal). The accurate measurement of a Higgs boson now has become an important constraint upon any supersymmetric model. In order for this constraint to be as useful and as accurate as possible, the prediction of the MSSM Higgs masses needs to be as accurate as possible. With a current estimated theoretical uncertainty in its prediction of around 3 GeV for 'normal' supersymmetric spectra (i.e. sparticles in the TeV range), a reduction in the theoretical uncertainty in the lightest CP even Higgs mass[1] prediction is welcome.

There are currently several available sparticle generators: `ISAJET` [7], `SOFTSUSY` [8, 9, 10, 11], `SPheno` [12], `SUSEFLAV` [13], `SUSPECT` [14] as well as tailor made generators `FlexibleSUSY` [15] and `SphenoMSSM` [16] based on `SARAH` [17, 18]. Even specialising to the minimal supersymmetric standard model (MSSM) with real parameters, these programs have slightly different approximations, resulting in numerical predictions that are not identical [19, 20, 21]. Even when calculations have at the same headline order of approximation (for example, two-loop renormalisation group equations (RGEs) and one-loop threshold corrections at M_Z), legitimate differences can result from the fact that higher order corrections contribute to the calculation implicitly in different ways. If we take the example of a one-loop threshold correction to, for example, the prediction of the stop mass, there are various contributions from Standard Model and supersymmetric particles. If we consider an internal loop with a gluino propagator, which mass do we use for the gluino? One achieves numerically distinct results if one uses two of the obvious choices: the pole mass or the modified dimensional reduction ($\overline{DR}$) running mass. The difference between the two prescriptions is a two-loop threshold effect, and so either choice is allowed if one is working only at one loop threshold effect order (numerically this is equivalent to working to two-loop order in the RGEs, which are enhanced by a large logarithm). Such choices occur hundreds of times within the calculation, multiplying the possibilities for numerical differences. Thus, the numerical differences between the spectrum calculators gives a very rough estimate of the size of theoretical uncertainties associated with the calculation.

One obvious way to reduce such a theoretical uncertainty is to incorporate higher order effects, pushing the associated theoretical uncertainty to yet higher orders. That is what we have done for the present paper: we have picked some available higher order terms that are expected to affect the predictions of the spectrum mass calculation, and included them in `SOFTSUSY3.5.1`. The previous version of the program, `SOFTSUSY3.4.1`, contained two-loop RGEs and one-loop threshold corrections. The higher order terms that we have included in the present paper are:

1. Three-loop RGEs [22] to all soft and supersymmetry preserving MSSM parameters, assuming that such parameters are real. Both the supersymmetric and soft-breaking MSSM parameters contain the possibility of full three-family mixing.
2. The following two-loop threshold corrections calculated in the (electroweak) gaugeless limit [23] of the MSSM[2]

[1] In a large part of parameter space the lightest CP even Higgs boson behaves approximately like the Standard Model Higgs boson.

[2] With the only exception being the top-quark mass, for which only strong interactions are taken into account.

Name	Description
Δm_t	$O(\alpha_s^2)$ [24, 25] corrections to m_t.
$\Delta\alpha_s$	$O(\alpha_s^2)$ [26, 27], $O(\alpha_s\alpha_{t,b})$ [28] corrections to α_s.
$\Delta m_b, m_\tau$	$O(\alpha_s^2)$ [27, 29], $O(\alpha_s\alpha_{t,b})$, $O(\alpha_{t,b}^2)$, $O(\alpha_t\alpha_b)$, $O(\alpha_{t,b}\alpha_\tau)$ [30] corrections to m_b. m_τ includes $O(\alpha_\tau^2)$ and $O(\alpha_\tau\alpha_{t,b})$ [30] corrections.

For our phenomenological analysis, we take the superpotential of the MSSM to be:

$$W = \mu H_2 H_1 + Y_t Q_3 H_2 u_3 + Y_b Q_3 H_1 d_3 + Y_\tau L_3 H_1 e_3, \tag{1}$$

where the chiral superfields of the MSSM have the following $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ quantum numbers

$$\begin{aligned} &L_i: \quad (1,2,-\tfrac{1}{2}), \quad e_i: \quad (1,1,1), \qquad Q_i: (3,2,\frac{1}{6}), \quad u_i: (\bar{3},1,-\frac{2}{3}), \\ &d_i: \quad (\bar{3},1,\tfrac{1}{3}), \quad H_1: \quad (1,2,-\frac{1}{2}), \quad H_2: (1,2,\frac{1}{2}), \end{aligned} \tag{2}$$

$i \in \{1,2,3\}$ is a family index and we have neglected all Yukawa couplings except those of the third family. In the table above, α_s denotes the strong coupling constant, m_t the top mass and $\alpha_{t,b,\tau} = Y_{t,b,\tau}^2/(4\pi)$. None of Δm_t, $\Delta\alpha_s$ or $\Delta m_b, m_\tau$ have been, to the best of our knowledge, made available to the public in a supported computer program before.

We shall illustrate our results with two different assumptions about supersymmetry breaking soft terms. The first is the constrained minimal supersymmetric standard model (CMSSM), which makes a simplifying assumption about the supersymmetry breaking terms: each soft supersymmetry (SUSY) breaking scalar mass is set to a common value m_0 at a high scale $M_{GUT} \sim 10^{16}$ GeV (defined here to be the scale at which the electroweak gauge couplings unify), the gaugino masses are set to a common value $M_{1/2}$ at M_{GUT} and the SUSY breaking trilinear scalar couplings are all fixed to a value A_0 at M_{GUT}. The other relevant input parameters are $\tan\beta$, the ratio of the two Higgs vacuum expectation values, and the sign of a parameter μ that appears in the Higgs potential (its magnitude is fixed by the empirically measured central value of the Z^0 boson mass via the minimisation of the Higgs potential).

GUTs make the gauge unification prediction

$$\alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = \alpha_3(M_{GUT}), \tag{3}$$

where α_1 is the hypercharge gauge coupling in the GUT normalisation and α_2 is the $SU(2)_L$ gauge coupling. If one uses gauge couplings inferred from measurements near the electroweak scale and evolves them with the Standard Model RGEs, Eq. 3 is not satisfied: the gauge couplings α_1 and α_3 meet at a very different renormalisation scale than α_1 and α_2. However, if we instead assume the MSSM and calculate the evolution of gauge couplings at one loop order, the prediction Eq. 3 agrees with data well [31]. Two-loop predictions spoil this good agreement [32], but discrepancies between the equalities are small and easily explained by heavy particles present in realistic GUTs which are not far below the GUT scale, for example $\sim O(M_{GUT}/10)$ [33]. These particles (for example heavy coloured triplets that come from spontaneous breaking of the GUT group) affect the running of the gauge couplings between their mass and M_{GUT}. Since we do not know of their existence in our effective MSSM field theory, and we do not know their mass, these effects are not taken into account in a general MSSM gauge unification calculation, allowing for some small apparent 'GUT threshold corrections' instead. In practice, we define M_{GUT} to be the renormalisation scale Q where $\alpha_1(Q) = \alpha_2(Q)$, allowing $\alpha_3(M_{GUT})$ to differ by a small amount due to the unknown heavy GUT-scale threshold corrections. Some GUTs such as $SU(5)$ [34, 35] predict bottom-tau Yukawa unification

$$Y_b(M_{GUT}) = Y_\tau(M_{GUT}), \tag{4}$$

because both particles reside in the same multiplet. In larger GUTs such as SO(10) [36], the top Yukawa coupling is unified with the other two:

$$Y_t(M_{GUT}) = Y_b(M_{GUT}) = Y_\tau(M_{GUT}). \tag{5}$$

In a similar way to gauge unification, small GUT threshold corrections may slightly spoil apparent Yukawa unification. We shall therefore bear in mind that there may be small corrections to Eqs. 4, 5.

The effect of the three-loop RGEs upon the relative mass shifts of Snowmass (SPS) benchmark points [37] were presented and studied in Ref. [22] without the inclusion of the two-loop threshold effects. We shall demonstrate that three-loop RGEs typically provide smaller effects than the two-loop threshold effects. Thus the additional higher order terms that were included in Ref. [22] were of smaller size as compared to other terms that were missing in the calculation. Effects upon sparticle mass predictions of around 1-2% were typically found, although one point studied did have an 8% difference in the light stop mass at the SPS5 point. We go beyond this calculation by including the two-loop thresholds.

Subsequently, one of us performed [28] a preliminary study of the SPS 4 (CMSSM high $\tan\beta$) benchmark point [37], with a modified version of SOFTSUSY that included both the three-loop RGEs and the two-loop thresholds that we consider here. 1-2% mass shifts in the strongly interacting sparticles, a 3% correction to the higgsino mass and a 1% decrease in the lightest CP even higgs boson mass was observed.

Our purpose here is to provide a more extensive study of the higher order effects as well as to present a public version of SOFTSUSY that incorporates them, along with instructions on how to use it. In particular, we shall study the effects on Yukawa and gauge unification, whose accuracy is improved by the inclusion of the higher orders. In the prediction of the sparticle mass spectrum, there are other, two-loop direct mass threshold contributions of the same order as the ones that we have included. The mass spectrum does not therefore increase in precision, but the shift in masses observed is a good estimator for the size of the theoretical uncertainty induced by such two-loop direct mass threshold contributions. We shall study these uncertainties and the induced uncertainties on other observables. For the CP-even Higgs masses we have the leading two-loop corrections implemented in SOFTSUSY. However, SOFTSUSY still misses higher powers of logarithms of the form $\log(M_{SUSY}/m_t)$ coming either from the diagrams with massive SM particles, e.g., top quarks (if the renormalisation scale Q is chosen to be the stop mass scale $Q = M_{SUSY} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$) or from those with sparticles (if one uses Q chosen at the top mass m_t instead)[3], and this fact induces an uncertainty on the prediction of the lightest CP-even Higgs mass that is larger than its experimental uncertainty.

We go beyond the pioneering study Ref. [28] in the following ways: in section 3, we show case the effect of the higher order terms in a CMSSM focus-point [41, 42, 43], where a high sensitivity to the precise value of the top Yukawa coupling leads to large order one uncertainties in the mass spectrum. We examine induced uncertainties in the predicted LHC production cross-sections and the dark matter relic density. We then present a detailed breakdown in the case of a phenomenological MSSM point, where the effects of the RGEs are small, allowing us to focus, to a reasonable approximation, purely on the size of the threshold corrections. We then quantify the effects of the higher order corrections in a CMSSM plane that is used by ATLAS to interpret their searches for supersymmetric particles in order to get an idea of the size of the corrections for generic points in parameter space that are not excluded by the current experimental limits. We scan a high $\tan\beta$ CMSSM plane to illustrate the large effect that the higher order terms can have upon the dark matter relic density prediction, across parameter space. In the appendix, we give details on how to install and use the increased accuracy mode in a publicly available version of SOFTSUSY.

3. Effects of Higher Order Terms

Here, we shall examine how the higher order terms change unification predictions and the Higgs and sparticle spectrum. The accuracy of the unification calculation is improved with the additional terms, and we shall investigate how much they affect the accuracy with which gauge and Yukawa unification is (or is not) achieved.

Two-loop threshold effects are not included in the calculation of sparticle masses in any of the public programs, indeed most of them have not been calculated yet (with the notable exceptions of the squark and gluino masses [44, 45]). This means that we are missing some terms *of the same order* as those that we include in our higher order corrections for these quantities (for example, two-loop threshold corrections to squark and gluino masses). Thus we cannot claim to have increased the accuracy of the sparticle mass predictions. Differences in sparticle masses due to the higher order corrections do give an estimate of the size of the missing terms, however, and are therefore instructive. Their inclusion is also a necessary step for the future when the two and three-loop sparticle mass threshold corrections are included.

[3]These logarithms could be re-summed if we were to integrate in the particles one-by-one with rising renormalisation scale, matching the effective field theories at each sparticle mass scale. Such a scheme would be very complicated, which is why is hasn't been attempted so far, although steps in this direction have started [38, 39, 40].

Name	RGEs	Quantity
Q	2	Standard `SOFTSUSY3.5.1` calculation without higher orders
Q_3	3	Only 3-loop RGEs are included, not the 2-loop threshold corrections
Q_{α_s}	2	Included 2-loop threshold corrections to α_s
Q_{m_t}	2	Only the 2-loop threshold corrections to m_t are included
Q_{m_b,m_τ}	2	Only the 2-loop threshold corrections to m_b and m_τ are included
Q_{All}	3	All higher order corrections

Table 1: Different approximations for the calculation of a quantity Q. The column headed 'RGEs' labels the number of loops used in the MSSM RGEs.

The tree-level lightest CP even Higgs mass is suppressed $m_{h^0} < M_Z$ and the one-loop corrections (dominantly due to Yukawa interactions of the top squarks) are larger than one would naively expect from formal perturbation theory, being numerically of the same order of magnitude as the tree-level contributions. A two-loop $O(\alpha_s^2)$ correction to m_t can be seen to affect m_{h^0} through a two-loop correction to a one-loop effect, and is thus in total a three-loop effect. Its numerical effect on the prediction of m_{h^0} is significant compared to the experimental uncertainties on the 125 GeV Higgs mass measurement. However, we are still missing other Feynman diagrammatic $O(\alpha_t\alpha_s^2)$ corrections to m_{h^0} and so again, its inclusion does not improve the accuracy of the prediction, but instead it can be used to estimate the size of the missing higher order pieces.

In the next subsection, we shall examine two parameter points in detail before performing parameter scans to characterise more generally the size of the higher order effects. Throughout this paper, we fix the important Standard Model parameters as follows at or near their central empirical values [46]: the top quark pole mass $m_t = 173.2$ GeV, the running bottom quark mass in the $\overline{MS}$ scheme $m_b(m_b) = 4.18$ GeV, the strong coupling in the $\overline{MS}$ scheme $\alpha_s(M_Z) = 0.1187$ where the Z^0 boson pole mass is fixed to $M_Z = 91.1876$ GeV, the Fermi decay constant of the muon $G_\mu = 1.16637 \times 10^{-5}$ GeV^{-2}, the fine structure constant in the $\overline{MS}$ scheme $\alpha(M_Z) = 1/127.934$ and the pole mass of the tau lepton $m_\tau = 1.7770$ GeV.

3.1. Dissection of the higher order effects at benchmark points

In order to dissect the various higher order points we first define different approximations to the prediction as in Table 1. In Table 2, we show the effects of the higher order terms on a CMSSM parameter point that is in the high $\tan\beta$ focus point region: ($m_0 = 7240$ GeV, $M_{1/2} = 800$ GeV, $A_0 = -6000$ GeV, $\tan\beta = 50$, $\mu > 0$) with some rather attractive phenomenological properties: it has a high lightest CP even Higgs mass of 124.6 GeV, agreeing with the experimental central value [4], once theoretical uncertainties (estimated to be around ± 3 GeV [20]) have been taken into account. It also has attractive dark matter properties: $\Omega_{CDM}h^2 = 0.122$ is close to the central value inferred from cosmological observations. In addition, the gluino and squark masses are heavy enough so as to not be ruled out by the LHC7/8 TeV data. Apart from these phenomenologically advantageous properties, the point has a high value of $\tan\beta = 50$, which may give the bottom and tau Yukawa corrections a higher impact than if $\tan\beta$ were smaller. At higher values of $\tan\beta$, the bottom and tau Yukawa couplings are roughly proportional to $\tan\beta \approx 1/\cos\beta$:

$$Y_b(M_Z) = \frac{\sqrt{2}m_b(M_Z)}{v\cos\beta}, \qquad Y_\tau(M_Z) = \frac{\sqrt{2}m_\tau(M_Z)}{v\cos\beta}. \tag{6}$$

We split the various higher order corrections up in the table: the 'base' calculation is taken to be `SOFTSUSY3.5.1` without the higher order corrections. By comparing the entries for Δ_3 with entries in other rows, we see that three-loop RGE corrections on their own tend to induce smaller changes to the quantities listed in the table than the threshold corrections. We can see that the two-loop threshold corrections to the strong coupling α_s and top-quark mass m_t are the most important ones. It is worth mentioning that both two-loop contributions to α_s and m_t (see., e.g., [47, 28]) tend to decrease the corresponding running MSSM parameters at the matching scale. We see that the row Δm_t, that includes two-loop threshold corrections to m_t coming from strong SUSY QCD interactions only contains 'N/A' entries, indicating that the calculation failed in this approximation because electroweak symmetry was not broken successfully, as we now explain.

	m_h	m_A	$m_{\tilde{g}}$	$m_{\chi_1^0}$	$m_{\chi_2^0}$	$m_{\chi_3^0}$	$m_{\chi_4^0}$
Q	124.6	2416	2015	363	702	1134	1140
Δ_3	+0.1	+18.9	-1.5	-0.1	+0.3	+37.2	+36.6
$\Delta\alpha_s$	+1.1	+411.2	-49.5	+2.1	+7.6	+729.4	+724.7
Δm_t	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$\Delta m_b, m_\tau$	+0.5	+490.5	-0.7	+0.3	+2.3	+188.7	+186.4
Δ All	-2.2	-347.5	-49.7	-18.4	-291.7	-718.8	-421.6
	$m_{\tilde{q}}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$	$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$
Q	7322	4951	4245	4946	5542	6265	5095
Δ_3	-4.2	-11.3	-18.7	-11.2	-7.2	-0.1	-0.5
$\Delta\alpha_s$	-2.2	-71.3	-167.9	-71.1	-0.3	+0.6	+1.6
Δm_t	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$\Delta m_b, m_\tau$	-0.5	+55.1	-34.2	+55.5	+125.9	+3.8	+10.2
Δ All	-6.6	+3.7	+65.3	+3.9	-46.3	+6.6	+15.9
	$m^{\pm}_{\chi_1}$	$m^{\pm}_{\chi_2}$	$g_3(M_{SUSY})$	$Y_t(M_{SUSY})$	$Y_b(M_{SUSY})$	$Y_\tau(M_{SUSY})$	$\mu(M_{SUSY})$
Q	703	1141	1.001	0.811	0.639	0.512	1114
Δ_3	+0	+37	+0.000	+0.001	-0.000	+0.000	+37
$\Delta\alpha_s$	+8	+725	-0.019	+0.014	-0.005	+0.000	+727
Δm_t	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$\Delta m_b, m_\tau$	+2	+186	-0.000	+0.004	-0.023	-0.001	+187
Δ All	-303	-422	-0.020	-0.016	+0.001	-0.001	-718
	$\Omega_{CDM}h^2$	σ^{TOT}_{SUSY}	$M_{GUT}/10^{16}$	$1/\alpha_{GUT}$			
Q	53.2	1.17	1.678	25.686			
Δ_3	+7.2	+0.01	+0.007	+0.003			
$\Delta\alpha_s$	+292.8	+0.36	-0.046	+0.058			
Δm_t	N/A	N/A	N/A	N/A			
$\Delta m_b, m_\tau$	+48.8	+0.00	-0.009	+0.011			
Δ All	-53.1	+0.36	+0.087	-0.067			
	$10\Delta(\alpha)$	$\Delta Y_{b\tau}$	ΔY_{tb}				
Q	-0.005	-0.130	0.158				
Q_3	-0.005	-0.129	+0.160				
Q_{α_s}	-0.005	-0.130	+0.158				
Q_{m_t}	N/A	N/A	N/A				
Q_{m_b,m_τ}	-0.005	-0.143	+0.184				
$Q_{\rm All}$	-0.013	-0.120	+0.140				

Table 2: Differences due to the highest order terms (three-loop RGEs for gauge and Yukawa couplings and two-loop threshold corrections to third family fermion masses and g_3) on various predicted quantities in the focus point of the CMSSM for m_0 = 7240 GeV, $M_{1/2}$ = 800 GeV, $A_0 = -6000$ GeV, $\tan\beta$ = 50, $\mu > 0$. We display massive quantities in units of GeV. The first column details which higher order threshold corrections are included. For some quantity, we have defined $\Delta_3 = Q_3 - Q$, $\Delta\alpha_s = Q_{\alpha_s} - Q$, $\Delta m_t = Q_{m_t} - Q$, $\Delta m_b, m_\tau = Q_{m_b m_\tau} - Q$ and Δ All= $Q_{\rm All} - Q$ (see Table 1). 'N/A' means that electroweak symmetry was not broken, and so reliable results cannot be reported. $m_{\tilde{q}}$ refers to the average mass of the squarks of the first two families. The column headed 'σ^{TOT}_{SUSY}' shows the total cross-section in fb for the production of gluinos and squarks at a 14 TeV LHC.

Minimising the MSSM Higgs potential with respect to the electrically neutral components of the Higgs vacuum expectation values, one obtains the well-known tree-level result for the Higgs mass parameter μ in the modified dimensional reduction scheme ($\overline{DR}$)

$$\mu^2 = \frac{\tan 2\beta}{2}\left[m_{H_2}^2 \tan\beta - m_{H_1}^2 \cot\beta\right] - \frac{M_Z^2}{2}. \tag{7}$$

In order to reduce[4] missing higher order corrections, all quantities in Eq. 7 are understood to be evaluated at a $\overline{DR}$ renormalisation scale $Q = M_{SUSY}$, where M_{SUSY} is the geometric mean of the two stop masses. $\tan\beta = \langle H_2^0\rangle/\langle H_1^0\rangle$ is the ratio of the two MSSM Higgs vacuum expectation values and $m_{H_{1,2}}$ are the soft SUSY breaking $\overline{DR}$ mass terms of the Higgs doublets. If $m_{H_1}^2$ and $m_{H_2}^2$ and $\tan 2\beta$ are such that $\mu^2 > 0$ results from Eq. 7, the model point may break electroweak symmetry successfully. On the other hand, if $\mu^2 \leq 0$, electroweak symmetry is *not* broken successfully and the model point is ruled out. At the focus point, the predicted value of μ derived from electroweak symmetry breaking is known to depend extremely sensitively upon the precise value of the top Yukawa coupling [48]. The parameter point in Table 2 appears to agree with the experimental result on the Higgs mass (which, in the CMSSM at high masses, acts to a good approximation with identical couplings to the Standard Model Higgs) according to the standard `SOFTSUSY3.5.1` calculation, bearing the ± 3 GeV theoretical uncertainty in mind. However, one would discard the point based on the predicted value of 53.2 for $\Omega_{CDM}h^2$, which disagrees with the cosmologically inferred value by hundreds of sigma. On the other hand, including all of the high order corrections ('Δ All'), we see that the Higgs mass prediction lowers somewhat, and the dark matter relic density is predicted to be the cosmologically acceptable value of 0.122 once all of the higher order corrections are included. Here, we use `micrOMEGAs3.3.13` [49, 50, 51] to predict the relic density of lightest neutralinos, identified to be our dark matter candidate. Fits to cosmological data constrain the relic density of dark matter to be $\Omega_{CDM}h^2 = 0.1198 \pm 0.0026$ from Planck data [52] (We allow a ± 0.02 error in the prediction coming from higher order annihilation diagrams [53]). Most of the CMSSM parameter space that is allowed by current sparticle searches predicts a relic density that is far too high compared with observations. However, there are isolated regions of parameter space that, for one reason or another, have an enhanced annihilation mechanism where the dark matter annihilates efficiently. In the focus point, the enhanced annihilation comes from the fact that the dark matter candidate (the lightest neutralino) has a significant higgsino component: small but real values of $\mu(M_{SUSY})$ lead to a higgsino-dominated lightest neutralino dark matter candidate, which annihilates efficiently into WW, ZZ, Zh or $t\bar{t}$ [54], reducing the dark matter relic density to an acceptable value. It also co-annihilates with the lightest chargino and the second lightest neutralino. The MSSM Lagrangian contains the neutralino mass matrix as $-\frac{1}{2}\tilde{\psi}^{0T}\mathcal{M}_{\tilde{\psi}^0}\tilde{\psi}^0$ + h.c., where $\tilde{\psi}^0 = (-i\tilde{b}, -i\tilde{w}^3, \tilde{h}_1, \tilde{h}_2)^T$ and, at tree level,

$$\mathcal{M}_{\tilde{\psi}^0} = \begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \end{pmatrix}, \tag{8}$$

where M_1 and M_2 are the bino and wino SUSY breaking soft mass parameters, respectively. We use s and c for sine and cosine, so that $s_\beta \equiv \sin\beta$, $c_\beta \equiv \cos\beta$ and $s_W(c_W)$ is the sine (cosine) of the weak mixing angle. When $\mu \sim O(\min(M_1,\ M_2))$, the lightest mass eigenstate thus picks up a significant higgsino component, which has enhanced annihilation into the channels mentioned above. At high m_0, other annihilation channels involving a $t-$channel scalar, are suppressed due to the high scalar mass. As each higher order correction is added, the value of Y_t changes slightly, changing the value of μ eventually predicted by Eq. 7. Moving down the rows in Table 2 through each successive approximation, we move from an approximation where $\mu(M_{SUSY}) > M_1$ and $\mu(M_{SUSY}) > M_2$ to a situation where $\mu^2 < 0$ (the Δm_t row, where 'N/A' is listed) to the approximation where all of our higher order corrections are included, and $\mu(M_{SUSY})$ is of a similar magnitude to M_1 and we have mixed higgsino-bino dark matter with an observationally acceptable predicted value. In the literature, a value of 2-3 GeV is often quoted as the spectrum calculators' theoretical uncertainty on the prediction of m_h. We see that this is borne out in our CMSSM model, where the higher order corrections give a 2.2 GeV shift in the prediction.

[4]This prescription at least ensures that the dominant terms do not involve large logarithms.

Two-loop threshold corrections to the strong gauge coupling have a significant effect upon some of the sparticle masses: particularly $m_{\chi_3^0}$ and $m_{\chi_4^0}$. $m_{\chi_3^0}$ and $m_{\chi_4^0}$ are controlled to leading order by μ (when $\mu > M_1, M_2$), which in turn is affected sensitively by the value of $m^2_{H_2}(M_{SUSY})$, as Eq. 7 shows. $m^2_{H_2}(Q)$ runs very quickly with renormalisation scale Q [55]:

$$\begin{aligned}16\pi^2 \frac{\partial m^2_{H_2}}{d\ln Q} &= 6\left[(m^2_{H_2} + m^2_{\tilde{Q}_3} + m^2_{\tilde{u}_3} + A_t^2)Y_t^2\right] - 6g_2^2 M_2^2 - \frac{6}{5}g_1^2 M_1^2 + \frac{3}{5}g_1^2\Big(m^2_{H_2} - m^2_{H_1} + \\ & \mathrm{Tr}[m^2_{\tilde{Q}} - m^2_{\tilde{L}} - 2m^2_{\tilde{u}} + m^2_{\tilde{d}} + m^2_{\tilde{e}}]\Big),\end{aligned} \tag{9}$$

to one-loop order, and it is strongly affected by the value of the top Yukawa coupling Y_t. The precise value of the top Yukawa coupling is affected by the strong threshold corrections to the top quark mass through [47]

$$Y_t(M_Z) = \frac{\sqrt{2}m_t(M_Z)}{v\sin\beta}, \qquad m_t = m_t(M_Z) + \Sigma_t(M_Z), \tag{10}$$

where $\Sigma_t(M_Z)$ represents the MSSM top quark mass threshold corrections (which include the strong threshold corrections). The two-loop threshold corrections to the bottom and tau Yukawa couplings have an effect, particularly on $m_{\chi_3^0}$ and $m_{\chi_4^0}$ through μ in a similar way to the effect due to Δm_t, as explained above. In addition, the masses of heavy higgs bosons are shifted by about ten percent. It is due to the fact that the running mass of CP-odd neutral higgs m_A is given at tree-level by the well-known relation

$$m_A^2 = \frac{1}{\cos 2\beta}\left(m^2_{H_2} - m^2_{H_1}\right) - M_Z^2, \tag{11}$$

into which both $m^2_{H_1}$ and $m^2_{H_2}$ enter on equal footing. Since the running of $m^2_{H_1}$ depends on the bottom Yukawa coupling [55], the reduction of the corresponding boundary condition at M_Z leads via RGE to comparatively larger values of $m^2_{H_1}$ at $Q = M_{SUSY}$, which, in turn, increase[5] the value of m_A^2. Although one cannot discern it from our table, the effect due to Δm_t is apparently in the opposite direction and tends to compensate that of Δm_b [28]. The lightest sbottom mass undergoes a 1% relative change, whereas the other masses are less affected by two loop contribution to m_b.

The smaller $\Delta\alpha_s$ changes in the gluino and average squark masses lead to associated changes in the squark and gluino production cross-sections. For the point in Table 2, SUSY particle production is dominated by the production of two gluinos and their subsequent decay, the squarks being too heavy to be produced with any appreciable cross-section. We calculate the next-to-leading order total QCD cross section for production at a 14 TeV LHC with `PROSPINO` [56, 57]. The relative statistical accuracy of this cross-section is ±0.001. We see that there is a large 30% increase due to the higher order effects modifying α_s, which in turn changes the gluino mass. We emphasise again that the changes in the spectrum that we see as a result of the higher order corrections are indicative of the size of theoretical uncertainties in each mass prediction, but that the results with the higher order corrections are only as accurate as those without them. However, our results on Yukawa and gauge couplings and their unification *are* more accurate.

For brevity, we have defined

$$\Delta(\alpha) = \alpha_3(M_{GUT}) - \alpha_1(M_{GUT}), \qquad \Delta Y_{b\tau} = Y_b(M_{GUT}) - Y_\tau(M_{GUT}), \qquad \Delta Y_{tb} = Y_t(M_{GUT}) - Y_b(M_{GUT}), \tag{12}$$

where α_{GUT} in the table refers to $\alpha_1(M_{GUT}) = \alpha_2(M_{GUT})$. Table 2 shows that the threshold corrections to α_s and three-loop RGEs change M_{GUT} and the unified gauge coupling α_{GUT} slightly. The discrepancy between α_3 and α_1 is generally small, and is not affected much by the higher order corrections. The discrepancy between the third family Yukawa couplings relatively decreases by some 20-30% once all of our higher order corrections are taken into account.

We now wish to decouple the "pure" three-loop RGE effects from the two-loop threshold effects as far as possible while still giving a valid prediction for a point in MSSM parameter space. This can be achieved by studying the spectrum at a point in pMSSM parameter space, where supersymmetry breaking boundary conditions are imposed already at the SUSY breaking scale, defined to be $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$. We use the point pMSSM1.6 from Ref. [58], which is

[5]Both $\cos 2\beta$ in (11) and $\tan 2\beta$ in (7) are negative for $\tan\beta > 1$.

	m_h	$m_{\tilde{g}}$	$m_{\tilde{q}}$	$m_{\chi_1^0}$	$m_{\chi_2^0}$	$m_{\chi_3^0}$	$m^0_{\chi_4}$
Q	125.4	1088	1042	790	2428	2499	2550
Δ_3	+0.1	+0.0	+0.0	-0.0	-0.0	-0.0	-0.0
$\Delta\alpha_s$	+1.5	-3.4	-1.7	+0.0	-0.2	-0.4	-0.3
Δm_t	-2.6	+0.0	+0.0	-0.0	+0.3	+0.6	+0.4
$\Delta m_b, m_\tau$	+0.7	-0.0	-0.0	+0.0	-0.1	-0.1	-0.1
Δ All	-1.5	-3.4	-1.7	-0.0	+0.1	+0.3	+0.2
	$m_{\tilde{t}_2}$	$m_{\tilde{t}_1}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$	$m_{\tilde{\tau}_2}$	$m_{\tilde{\tau}_1}$	$m^\pm_{\chi_1}$
Q	2616	2323	2498	2535	2523	2499	2429
Δ_3	+0.0	-0.3	-0.1	+0.0	+0.0	+0.0	-0.0
$\Delta\alpha_s$	-1.1	-4.4	-2.0	-0.9	-0.0	+0.0	-0.2
Δm_t	+0.2	+5.6	+1.8	+0.2	+0.0	-0.0	+0.3
$\Delta m_b, m_\tau$	-0.1	-1.4	-0.3	-0.1	+0.0	-0.0	-0.1
Δ All	-0.7	+1.9	+0.2	-0.8	+0.0	-0.0	+0.1
	$g_3(M_{SUSY})$	$Y_t(M_{SUSY})$	$Y_b(M_{SUSY})$	$Y_\tau(M_{SUSY})$	$\mu(M_{SUSY})$	$\Omega_{CDM}h^2$	σ^{TOT}_{SUSY}
Q	1.041	0.823	0.129	0.100	2500	0.12	1684
Δ_3	+0.000	+0.001	+0.000	+0.000	+0	+0.00	-0
$\Delta\alpha_s$	-0.014	+0.011	+0.001	-0.000	+0	+0.00	+25
Δm_t	+0.000	-0.017	-0.000	+0.000	+0	+0.00	-0
$\Delta m_b, m_\tau$	-0.000	+0.004	-0.003	+0.000	-0	+0.00	+0
Δ All	-0.014	-0.008	-0.002	+0.000	+0	+0.00	+25

Table 3: Differences due to the highest order terms (three-loop RGEs for gauge and Yukawa couplings and two-loop threshold corrections to third family fermion masses and α_s) in a modified phenomenological MSSM point 1.6 [58]. $\tan\beta = 10$, $A_t = -5$ TeV and most other mass parameters are set to 2.5 TeV except for $M_3 = 960$ GeV, $M_1 = 800$ GeV and the first two generation squark masses being set to 960 GeV (see text for more details). The first column details which higher order threshold corrections are included. We display massive quantities in units of GeV. For some quantity Q, $\Delta_3 = Q_3 - Q$, $\Delta\alpha_s = Q_{\alpha_s} - Q$, $\Delta m_t = Q_{m_t} - Q$, $\Delta m_b, m_\tau = Q_{m_b m_\tau} - Q$ and Δ All= $Q_{\mathrm{All}} - Q$ (see Table 1). $m_{\tilde{q}}$ refers to the average mass of the squarks of the first two families. The rows marked with a Δ show the *change* with respect to the `SOFTSUSY3.5.1` prediction. The column headed 'σ^{TOT}_{SUSY}' shows the total cross-section in fb for the production of gluinos and squarks at a 14 TeV LHC.

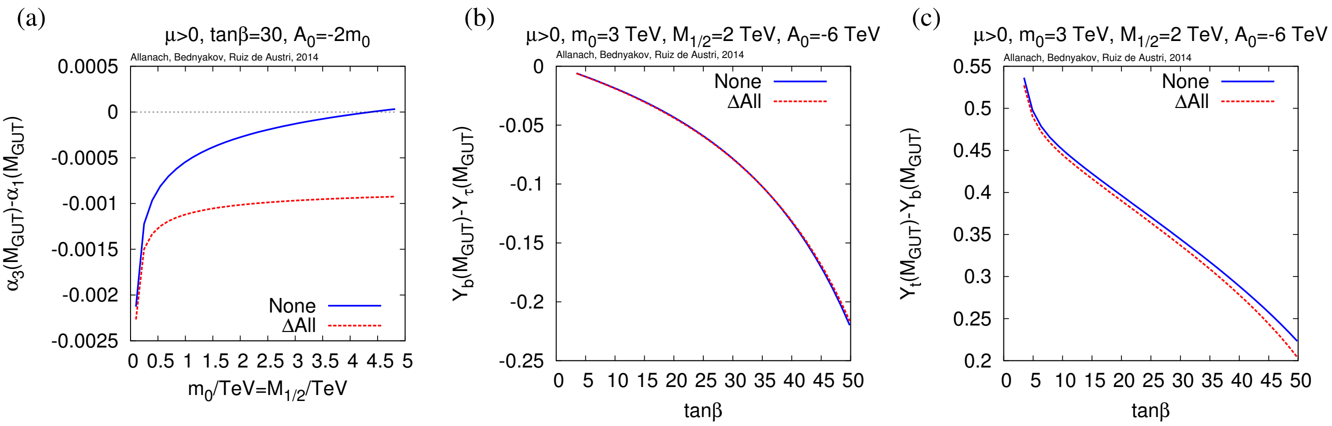

Figure 1: Effects of higher order terms on unification in the CMSSM. The legend defines whether the higher order terms are included ('ΔAll') or not ('None').

defined to have $\tan\beta = 10$, all scalar trilinear couplings set to 0, the tree-level first two generation squark masses, and tree-level gluino masses all set to 960 GeV and a tree-level bino mass of 800 GeV. The tree-level wino mass μ and all other tree-level squark and slepton masses are fixed to 2500 GeV. We change the point however, the pseudo-scalar Higgs pole mass is fixed to $M_{A^0} = 1580$ GeV: this provides efficient dark matter annihilation of two neutralinos (whose mass is approximately $M_{A^0}/2$). We also change the stop mixing parameter, setting it to be $A_t = -5$ TeV, which puts the prediction of the lightest CP even higgs mass to be near the experimental central value. The aim of using the pMSSM is to reduce the effects of the RGEs in order to study the threshold contributions more cleanly. However, we cannot eliminate RGE effects completely because there is still running between M_Z and $M_{SUSY} \sim 2.5$ TeV. However, these running effects are relatively small, being of order $1/(16\pi^2) \log M_{SUSY}/M_Z$, unlike the CMSSM case above with its large logarithms $1/(16\pi^2) \log M_{GUT}/M_Z \sim O(1)$. We see that the lightest CP-even Higgs mass prediction decreases by 1.5 GeV, mainly because of the corrections to the top mass. Three-loop RGEs appear unnecessary here, only inducing relative changes of order 10^{-4} in the predictions. The high order strong corrections to α_s reduce squark and gluino masses at the per-mille level, which will only have a very small effect on collider signatures. We see that the RGE corrections do induce a small additional change: but it is at the per-mille or smaller level for this point. The sparticle mass predictions only change by a very small amount, which is contrary to the case of the focus-point CMSSM (which is admittedly very sensitive to small changes in the top Yukawa coupling) that is shown in Table 2. The effect on the total 14 TeV LHC gluino/squark production cross-section is only around the 1% level or so. We conclude that in this point of the pMSSM, the higher order effects are not needed for collider studies except for those involving the lightest CP-even Higgs. We find it likely, where no input mass parameters are lighter than 700 GeV that this conclusion will hold more generally for the pMSSM because of sparticle decoupling in the corrections. However, to be sure of this conclusion, one should calculate the spectrum at any point in question including the higher order effects in order check. We shall show next that even for more generic CMSSM points, there are typically relative changes in the spectra of 2-3%.

3.2. *CMSSM parameter scans*

We now perform scans over CMSSM parameter space in order to examine the effects of the higher order terms on unification. We shall often consider scans through CMSSM parameter space around a $m_0 - M_{1/2}$ parameter plane with $\mu > 0$, $\tan\beta = 30$ and $A_0 = -2m_0$, which was recently used by ATLAS to place bounds upon the CMSSM from various 8 TeV, LHC 'jets plus missing energy searches' [59] with 20 fb^{-1} of integrated luminosity. We see in Fig. 1a that exact gauge unification occurs, for $A = -2m_0$, $\mu > 0$ and $\tan\beta = 30$, at around $m_0 = M_{1/2} \approx 4.5$ TeV in the CMSSM at the usual SOFTSUSY accuracy. However, including the higher order terms, $\alpha_3(M_{GUT})$ as predicted by data is around 0.001 times smaller than the other gauge couplings. Such a discrepancy may be explained within a more detailed GUT model via GUT threshold effects, but the precise value of $\alpha_3(M_{GUT}) - \alpha_1(M_{GUT})$ is important for constraining these. We see from Fig. 1b that bottom-tau Yukawa unification is only possible at low values of

$\tan\beta$. There, however, the Higgs mass prediction is too low compared with recent measurements. The higher order corrections change the prediction of the $Y_b - Y_\tau$ Yukawa coupling difference (to be acquired through GUT threshold effects) very little. Top-bottom Yukawa GUT unification is not possible for $\tan\beta < 40$, as is evident in Fig. 1c where $Y_t(M_{GUT}) - Y_b(M_{GUT})$ is too large to be explained by small loop effects. If the high-scale thresholds are instead well below the GUT scale, perhaps a large enough correction may be possible at high $\tan\beta > 40$. The higher order effects make a large difference of several percent at high $\tan\beta$, and would significantly change the constraints upon these thresholds.

We shall now display some of our results in the parameter plane recently defined by ATLAS. We have combined the two most restrictive exclusion limits from their direct LHC 7/8 TeV searches [59] into one exclusion limit: if a point is excluded at 95% confidence level by either or both of them, we count it as excluded. Comparing Figs. 2a,2b, we see that the higher order corrections introduce a constant term which makes $\alpha_3(M_{GUT})$ approximately 0.001 below $\alpha_1(M_{GUT}) = \alpha_2(M_{GUT})$. When studying Yukawa unification, we diverge from the ATLAS plane and instead vary two parameters that control Yukawa unification more directly: $\tan\beta$ and A_0. We fix $m_0 = 2$ TeV, $M_{1/2} = 0.6$ TeV so that, at $\tan\beta = 30$, the point is allowed (as shown by reference to Fig. 2a,b). It is expected [60] that the LHC limits should only be weakly dependent upon $\tan\beta$ and so we expect this $\tan\beta$, A_0 plane to not be excluded by them. Figs 2c,2d show that the difference in bottom and tau Yukawa couplings doesn't change much in the region closest to Yukawa unification at around $\tan\beta \approx 2$: less than half a percent (in the less unified direction) is accounted for by the higher order corrections there. However, bottom-top Yukawa unification is made slightly better, by a few percent or so (seen from Figs 2e,2f), but even at high values of $\tan\beta$, there is a 10% discrepancy between the two couplings. One would need quite large GUT scale threshold corrections to explain this sizable discrepancy.

In Fig. 3, we show some contours of important MSSM particle masses as well as their relative change due to the higher order corrections. The region below the dashed line is excluded by either one or both of the most restrictive ATLAS SUSY searches [59]. We see that the gluino, the lightest CP even Higgs mass, the first two generation average squark mass and the CP odd Higgs mass m_A typically become reduced by 1-3% by the higher order corrections in the region allowed by the search. In the CMSSM, the dominant production of SUSY particles is via gluino and squark production. The mass of the lightest neutralino changes less: typically at the one per-mille level, whereas the lightest stop mass has larger contributions from the higher order corrections: up to about $\pm 8\%$. CMSSM signatures involving the lightest stops are therefore more sensitive to the higher order contributions. The reduction of gluino and squark masses makes the SUSY production cross-section larger. As Fig. 4 shows, this results in an increase of 10-26% in the cross-section within the region not excluded by current searches. This is therefore our estimate for the theoretical uncertainty upon the next-to-leading order cross-section induced from spectrum uncertainties (note however that it does not include theoretical uncertainties coming from the next-to-next-to leading order cross-section, however that can easily be obtained from scale dependence in the next-to-leading order result).

We next perform a scan at high $\tan\beta$, displaying a region where the dark matter relic abundance appears to have the correct value compared to the value inferred from cosmological observationals. In Fig. 5, we show this as the region between the two green contours. On the other hand, the background colour shows the apparent induced theoretical uncertainty in the prediction of the dark matter relic density from our higher order terms. We have defined $\Delta\Omega_{CDM}h^2$ to be the 'Δ All'$-$'None' value. We see that, for $m_0 > 1$ TeV, it is swamped by theoretical uncertainties and the prediction is completely unreliable. This is not unexpected at the focus point, because of huge sensitivities to Y_t [61]. While the uncertainties for fixed CMSSM parameters are huge, it is true that the region of dark matter relic abundance that agrees with observations will be present somewhere. However, it may move significantly with m_0. The contours shown track to be close to the boundary of successful electroweak symmetry breaking, shown by the white region. As we move across the plot from left to right, the value of $\mu^2(M_{SUSY})$ as predicted by minimisation of the Higgs potential decreases, and finally becomes less than zero in the white region, signalling incorrect electroweak symmetry breaking. If we omit the higher order corrections, we obtain instead the grey region, which would have its own contours of dark matter relic density predicted to be compatible with observations.

4. Summary and Conclusions

We have incorporated full three-loop MSSM $R-$parity and CP-conserving conserving RGEs as well as some leading two-loop threshold corrections to the QCD gauge coupling and third family fermion masses into the `SOFTSUSY` spectrum calculator. The corrections included are: $O(\alpha_s^2)$ corrections to m_t, $O(\alpha_s^2)$, and $O(\alpha_s\alpha_{t,b})$ corrections to α_s,

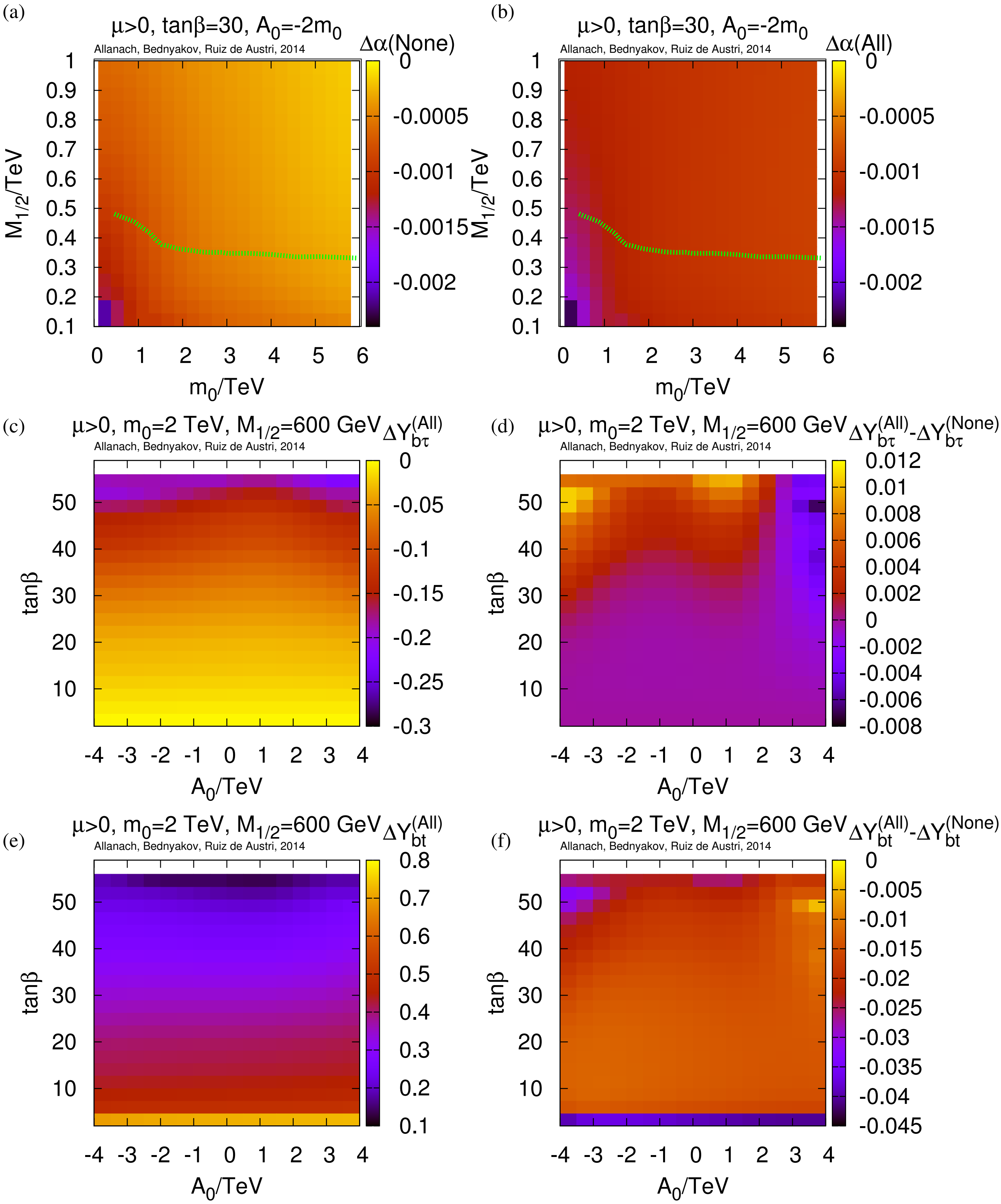

Figure 2: Relative effect of highest order terms on unification in the CMSSM. The CMSSM parameters in (a) and (b) coincide with the latest ATLAS searches for jets and missing energy interpreted in the CMSSM [59]. On the colour legend we have labeled the default SOFTSUSY calculation by (None) and the one including the higher order corrections by (All). The regions in (a) and (b) below the dashed line are excluded at the 95% confidence level by at least one of the most restrictive ATLAS jets plus missing energy searches.

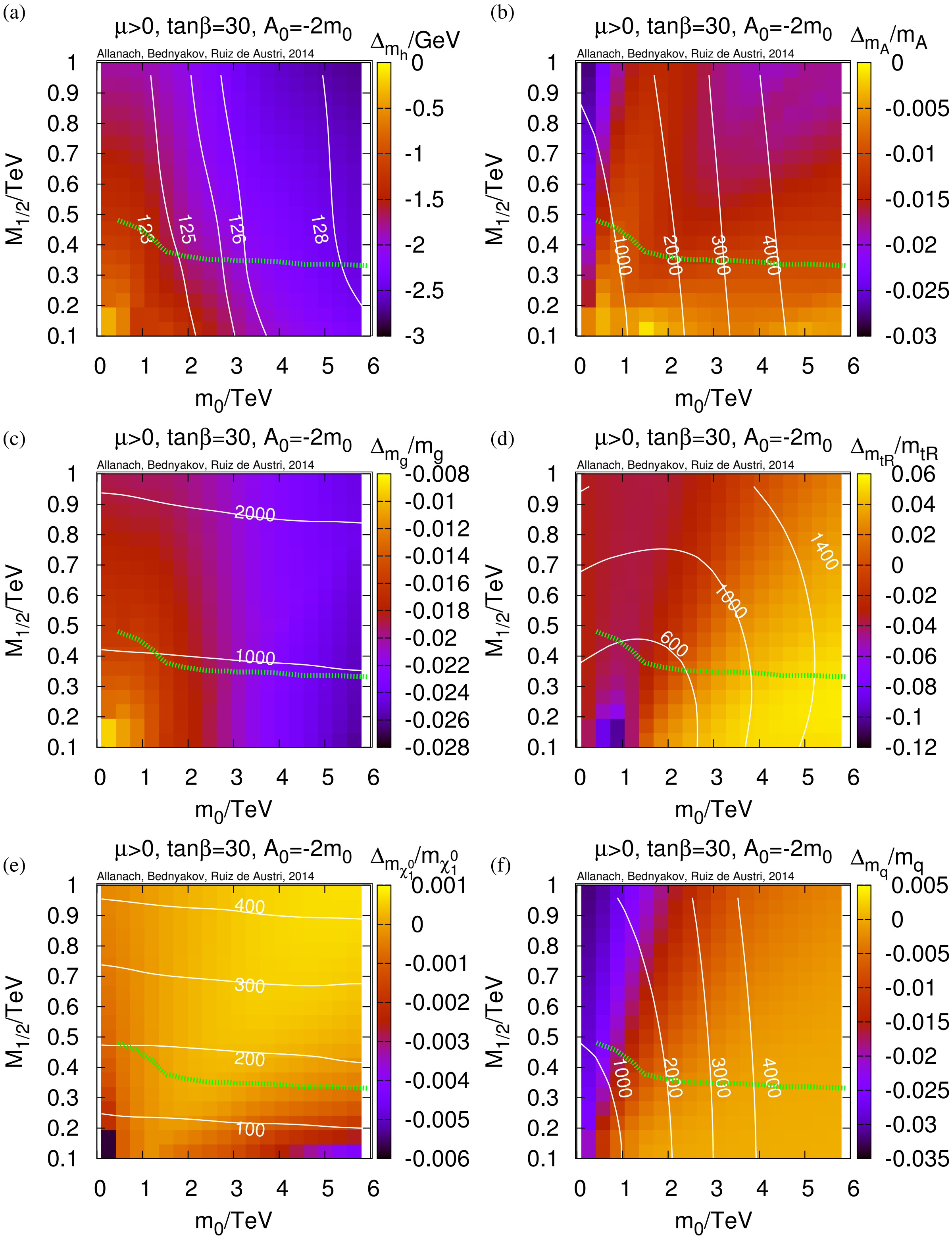

Figure 3: Relative effect of highest order terms (three-loop RGEs for gauge and Yukawa couplings and two-loop threshold corrections to third family fermion masses and g_3) on various particle pole masses in the CMSSM. The CMSSM parameters coincide with a parameter plane where limits from the latest ATLAS searches for jets and missing energy were presented in the CMSSM [59]. Solid contours of iso-mass calculated including all of our higher order corrections are overlayed on each figure, with each contour labelling the mass in GeV. $\Delta m/m$ denotes the change that was induced by the higher order corrections and is shown as the background colour in each plot. Here, m_g denotes the gluino mass and m_q the average squark mass from the first two generations. The region below the dashed line is excluded at the 95% confidence level by at least one of the most restrictive ATLAS jets plus missing energy searches.

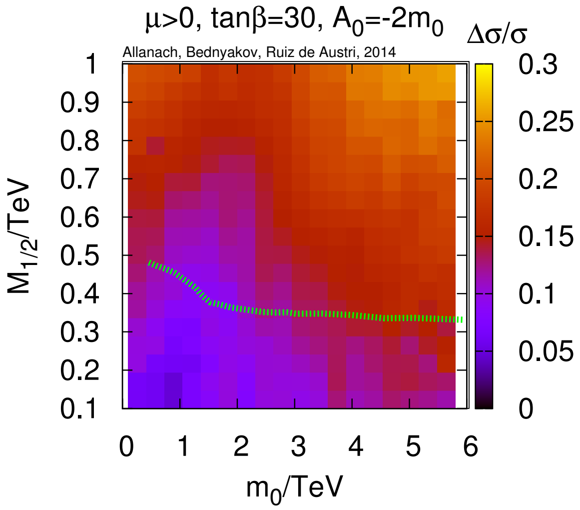

Figure 4: Relative effect of highest order terms (three-loop RGEs for gauge and Yukawa couplings and two-loop threshold corrections to third family fermion masses and g_3) on the predicted LHC SUSY production cross-section. The CMSSM parameters coincide with the latest ATLAS searches for jets and missing energy interpreted in the CMSSM [59]. The region below the dashed line is excluded at the 95% confidence level by at least one of the most restrictive ATLAS jets plus missing energy searches.

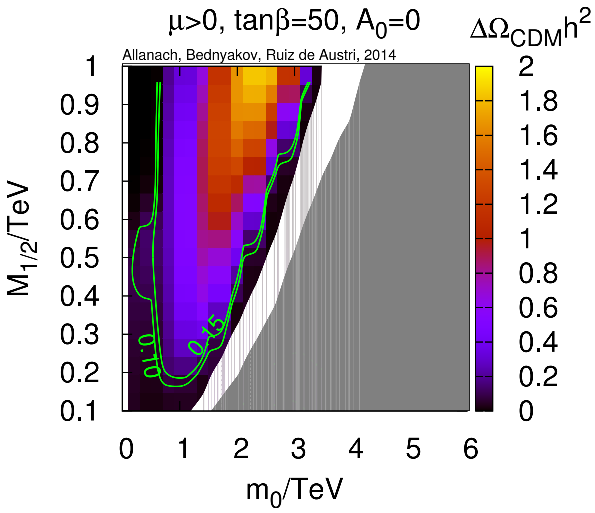

Figure 5: Effect of highest order terms (three-loop RGEs for gauge and Yukawa couplings and two-loop threshold corrections to third family fermion masses and g_3) on the predicted dark matter relic density in the CMSSM in a high $\tan\beta$ scenario. Contours of iso-relic density for the highest order prediction are overlayed. Also shown is the change in the position of the border of successful electroweak symmetry breaking: the white region (and to the right) is ruled out for higher loop corrections, whereas the lighter one is ruled out according to the standard `SOFTSUSY` calculation.

and $O(\alpha_s^2)$, $O(\alpha_s\alpha_{t,b})$, $O(\alpha_{t,b}^2)$, $O(\alpha_t\alpha_b)$, and $O(\alpha_{t,b}\alpha_\tau)$ corrections to m_b. $O(\alpha_\tau^2), O(\alpha_{t,b}\alpha_\tau)$ corrections to m_τ are also included. These corrections make gauge and Yukawa unification predictions more accurate. We report up to a 3 GeV (usually *negative*) change in the prediction of the lightest CP even Higgs mass, mostly due to the reduction of the running top Yukawa coupling originating from the two-loop threshold correction to the top quark mass. It is interesting to mention that our result looks complementary to that obtained by means of the H3M package [38, 62, 63] – three-loop diagrammatic corrections $O(\alpha_t\alpha_s^2)$ to the lightest Higgs mass are estimated to be of the same order but *positive*. The authors of H3M took the $O(\alpha_s^2)$ contribution to the running top mass into account but used a very different setup[6]. It would be instructive in the future to implement the leading three-loop contribution [62, 66] to the lightest CP-even Higgs boson mass in SOFTSUSY and cross-check results available in the literature.

The inclusion of the higher order terms also gives a good estimate for the size of theoretical uncertainties in the sparticle mass predictions from higher order corrections. Some sparticle masses have 10% uncertainties when running to and from the GUT scale, as in the CMSSM, where small threshold effects become amplified by sensitive renormalisation group running. On the other hand, in the pMSSM, where there is only running between M_{SUSY} and M_Z, the theoretical uncertainties in sparticle masses are smaller: typically at the one percent level. The uncertainties in the spectrum have a knock-on effect on derived observables: for example, the predicted relic density of dark matter, since it depends so sensitively on sparticle masses in some parts of parameter space, can have order 1 relative theoretical uncertainties. The total LHC sparticle production cross-section can have a 30% error in the CMSSM (this decreases to a percent or so in the pMSSM). The change in the running value of the top quark mass induces a particularly large change in the higgsino mass parameter μ at the focus point of the CMSSM at large m_0, resulting in huge theoretical uncertainties in some neutralino and chargino masses. This is thus an important input for global fits of the CMSSM (see, for example Refs. [67, 68, 69, 70]). It is probable that regions of parameter space at high m_0 are weighted incorrectly. Ideally, the fit would be performed including a calculated theoretical error (particularly that coming from the dark matter relic density constraint [21]). This could come from estimating the corrections using our higher order corrections in order to quantify the uncertainty, or from renormalisation scale dependence of observables (for instance, how much would the dark matter relic density prediction change if M_{SUSY} were varied by a factor of 2?)

Neither $O(\alpha_s\alpha_t)$ nor $O(\alpha_t^2)$ corrections are included in the calculation of the running value of m_t. Since parts of the phenomenology are so sensitively dependent upon the precise value of of m_t in parts of parameter space (especially the focus point of the CMSSM), an important future work will be to include these. We estimate that current uncertainties on the extreme focus point region are huge, and need to be decreased by the calculation and addition of these terms. We note that currently, no other publicly supported spectrum calculator contains our higher order terms. There has been a tendency in the recent literature for some authors to increase the SUSY breaking mass scales to several tens of TeV, or even higher. In this case, to get a m_h prediction that is very accurate, the fixed order calculations employed in SOFTSUSY could be subject to corrections of several GeV [39, 40]. For a more accurate prediction, additional log re-summation should be implemented: another important possible future direction for research.

There has been attention in the literature on the question of whether full top-bottom-tau Yukawa unification is possible in supersymmetric minimal SO(10) GUT models while respecting current data [71, 72, 73]. It will be an interesting future project to examine to what extent this is possible or not while including the important effects of the higher order corrections, although this should only be done after the inclusion of $O(\alpha_s\alpha_t)$ and $O(\alpha_t^2)$ corrections to m_t. A more formidable future enterprise would be to include direct two-loop threshold corrections to sparticle masses (e.g., with the help of TSIL [74] package [44, 45]). The corrections that we have included are necessary if such corrections are to increase the accuracy of sparticle mass predictions.

We have examined the effects of the higher order terms that we include upon apparent discrepancies in various predictions of unification at the GUT scale. We fix gauge and Yukawa couplings to data at M_Z, assuming some value of $\tan\beta$. Then, by evolving to M_{GUT}, where the electroweak gauge couplings meet, we obtain GUT scale gauge and Yukawa couplings. The discrepancy between $\alpha_3(M_{GUT}) - \alpha_1(M_{GUT})$ is typically larger once higher order corrections are included (particularly the two-loop threshold corrections to $\alpha_3(M_Z)$). However, it is in any case only at the per mille level and can easily be explained by small GUT threshold corrections. Yukawa unification has larger apparent GUT-scale discrepancies in generic parts of parameter space. It is affected mostly by higher order top mass and $\alpha_3(M_Z)$ threshold corrections. We have studied examples where these change the GUT-scale discrepancies by 4%.

[6]One- and two-loop corrections to the CP-even Higgs mass matrix are calculated by means of the FeynHiggs [64, 65] program.

This would certainly have an impact on detailed GUT model building, in order to explain the discrepancy with, for example, GUT-scale threshold corrections.

We have provided details in the Appendix of how to compile and run a new publicly available version of SOFTSUSY that incorporates the higher order terms discussed above[7]. We hope that this provision will aid other studies of unification and quantification of theoretical uncertainties in the sparticle spectrum. In addition, if SUSY is discovered at the LHC, the inclusion of higher order corrections will be important for testing various SUSY breaking hypotheses and measuring the SUSY breaking parameters.

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Appendix A. Installation of the Increased Accuracy Mode

The two freely available programs CLN (at least version 1.3.1) and GiNaC (at least version 1.3.5) should be installed before the user attempts to install SOFTSUSY with the higher order threshold corrections. However, SOFTSUSY should compile without problems without these libraries if our higher order corrections are not required. Two compilation arguments to the `./configure` command are provided:

`--enable-three-loop-rge-compilation` - compile three-loop RGEs in the MSSM[8]

`--enable-two-loop-gauge-yukawa-compilation` - compile additional two-loop threshold corrections to the third generation Yukawa couplings and the strong coupling constant.

Thus, if all higher order corrections are desired, CLN and GiNaC should be first installed, then the program should be compiled, via:

```
> ./configure --enable-three-loop-rge-compilation --enable-two-loop-gauge-yukawa-compilation
> make
```

We have included two global boolean variables that control the higher order corrections at run time, provided the program has already been compiled with the higher order corrections included:

- `USE_THREE_LOOP_RGE` - add three-loop contribution to MSSM RGE (corresponds to the `SOFTSUSY Block` parameter 19).
- `USE_TWO_LOOP_GAUGE_YUKAWA` - add two-loop threshold corrections to the third generation Yukawa couplings and the strong coupling constant (corresponds to the `SOFTSUSY Block` parameter 20). If this variable is switched on, `MssmSoftsusy` object constructors will automatically include all higher order threshold corrections.

[7]There may be minimal changes (from a user's perspective) to this procedure in future versions where additional corrections are added.
[8]GiNaC and CLN are not required for three-loop RGEs.

By default, both of these sets of higher order corrections are switched off (the boolean values are set to `false`), unless the user sets them in their main program, or in the input parameters (see Appendix B).

We also add the variable `double TWOLOOP_NUM_THRESH = 0.1` for finer control. It is used in the iterative algorithm to prevent lengthy re-evaluation of two-loop thresholds. If the relative difference between the two-loop thresholds obtained in the current iteration and the value calculated in the previous iteration is less than `TWOLOOP_NUM_THRESH`, the thresholds are not re-evaluated for the next iteration. See Ref. [8] for details of the standard `SOFTSUSY` fixed point iteration algorithm employed.

Appendix B. Running `SOFTSUSY` in the Increased Accuracy Mode

`SOFTSUSY` produces an executable called `softpoint.x`. One can run this executable from command line arguments, but the higher order corrections will be, by default, switched off. One may switch all of the higher order corrections on with the arguments `--three-loop-rges --two-loop-gauge-yukawa` (provided they have been compiled as specified above). Thus, in order to produce the spectrum detailed in the Δ(all) row of Table 2, one may use the command with `SOFTSUSY3.5.1`:

```
./softpoint.x sugra --tol=1.0e-5 --m0=7240 --m12=800 --a0=-6000 --tanBeta=50 --sgnMu=1 --mt=173.2
--alpha_s=0.1187 --mbmb=4.18 --two-loop-gauge-yukawa --three-loop-rges
```

For the calculation of the spectrum of single points in parameter space, one could alternatively use the SUSY Les Houches Accord (SLHA) [75] input/output option. The user must provide a file (e.g. the example file included in the `SOFTSUSY` distribution `inOutFiles/lesHouchesInput`), that specifies the model dependent input parameters. The program may then be run with

```
./softpoint.x leshouches < inOutFiles/lesHouchesInput
```

One can change whether the 3-loop RGE corrections are switched on with `SOFTSUSY Block` parameter 19, whereas the 2-loop third family and g_3 threshold corrections are switched on with `SOFTSUSY Block` parameter 20 in the SLHA input file:

```
Block SOFTSUSY               # Optional SOFTSUSY-specific parameters
   19   1.000000000e+00      # Include 3-loop RGE terms (default of 0 to disable)
   20  31.000000000e+00      # Include all 2-loop thresholds (default of 0 to disable)
```

A comment in the SLHA output file states which of the higher order terms is included in the calculation, provided `SOFTSUSY` has been compiled to include them. If only some of the additional two loop threshold corrections are required, they can be switched with a finer control by changing the value of the `SOFTSUSY Block 20` parameter, as specified below.

The considered two-loop threshold corrections in a `MssmSoftsusy` object are controlled by an `integer` parameter `included_thresholds`. Depending upon the value of this integer, different approximations of the various thresholds are included. For SUSY Les Houches Accord input, `included_thresholds` is fixed to the `SOFTSUSY Block 20` parameter input. The various options are presented in Table B.4. For convenience, we have included three `MssmSoftsusy` methods that can be used from within main programs to switch on and off some sub-classes of threshold corrections. Each takes a `bool` argument, which will switch the correction on if it is `true` and switch it off if `false`. Table B.5 displays these.

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	Two-loop threshold correction				
Value	m_t	α_s	Strong m_b	Yukawa m_b	Yukawa m_τ
0					
1	✓				
2		✓			
3	✓	✓			
4			✓		
5	✓		✓		
6		✓	✓		
7	✓	✓	✓		
8				✓	
9	✓			✓	
10		✓		✓	
11	✓	✓		✓	
12			✓	✓	
13	✓		✓	✓	
14		✓	✓	✓	
15	✓	✓	✓	✓	
16					✓
17	✓				✓
18		✓			✓
19	✓	✓			✓
20			✓		✓
21	✓		✓		✓
22		✓	✓		✓
23	✓	✓	✓		✓
24				✓	✓
25	✓			✓	✓
26		✓		✓	✓
27	✓	✓		✓	✓
28			✓	✓	✓
29	✓		✓	✓	✓
30		✓	✓	✓	✓
31	✓	✓	✓	✓	✓

Table B.4: Options for finer control of the threshold loop corrections. A ✓ indicates that the correction is present, whereas the absence of a ✓ indicates that the correction is absent. The column labelled 'm_t' refers to the $O(\alpha_s^2)$ corrections to m_t, 'α_s' refers to the $O(\alpha_s^2)$, $O(\alpha_s\alpha_{t,b})$ corrections to α_s, 'Strong m_b' refers to the $O(\alpha_s^2)$ correction to m_b, 'Yukawa m_b' refers to the $O(\alpha_s\alpha_{t,b})$,$O(\alpha_{b,t}^2)$, $O(\alpha_b\alpha_t)$, and $O(\alpha_{t,b}\alpha_\tau)$ corrections to m_b and 'Yukawa m_τ' refers to the $O(\alpha_\tau^2)$, and $O(\alpha_\tau\alpha_{t,b})$ corrections to m_τ.

Method	Corrections
`setAllTwoLoopThresholds`	Δm_t, $\Delta\alpha_s$, $\Delta m_b, m_\tau$
`setTwoLoopAlphasThresholds`	$\Delta\alpha_s$
`setTwoLoopMtThresholds`	Δm_t
`setTwoLoopMbMtauThresholds`	$\Delta m_b, m_\tau$

Table B.5: Methods for switching on and off sub-classes of higher order threshold corrections.

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