

# The Calculation of Sparticle and Higgs Decays in the Minimal and Next-to-Minimal Supersymmetric Standard Models: SOFTSUSY4.0

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## Abstract

We describe a major extension of the SOFTSUSY spectrum calculator to include the calculation of the decays, branching ratios and lifetimes of sparticles into lighter sparticles, covering the next-to-minimal supersymmetric standard model (NMSSM) as well as the minimal supersymmetric standard model (MSSM). This document acts as a manual for the new version of SOFTSUSY, which includes the calculation of sparticle decays. We present a comprehensive collection of explicit expressions used by the program for the various partial widths of the different decay modes in the appendix.

*Keywords:* MSSM, NMSSM, branching ratio, lifetime

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## 1. Program Summary

*Program title:* SOFTSUSY

*Program obtainable from:* <http://softsusy.hepforge.org/>

*Distribution format:* tar.gz

*Programming language:* C++, fortran

*Computer:* Personal computer.

*Operating system:* Tested on Linux 4.4.0-36-generic, Linux 3.13.0-93-generic

*Word size:* 64 bits.

*External routines:* None

*Typical running time:* 0.1-1 seconds per parameter point.

*Nature of problem:* Calculating supersymmetric particle partial decay widths in the MSSM or the NMSSM, given the parameters and spectrum which has already been calculated by SOFTSUSY.

*Solution method:* Analytic expressions for tree-level 2 body decays and loop-level decays and one-dimensional numerical integration for 3 body decays.

*Restrictions:* Decays are only calculated in the real  $R$ -parity conserving MSSM or the real  $R$ -parity conserving NMSSM only. No additional charge-parity violation (CPV) relative to the Standard Model (SM). Sfermion mixing has only been accounted for in the third generation of sfermions in the decay calculation. Decays in the MSSM are 2-body and 3-body, whereas decays in the NMSSM are 2-body only.

*CPC Classification:* 11.1 and 11.6.

*Does the new version supersede the previous version?:* Yes.

*Reasons for the new version:* Significantly extended functionality. The decay rates and branching ratios of sparticles are particularly useful for collider searches. Decays calculated in the NMSSM will be a particularly useful check of

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the other programs in the literature, of which there are few.

*Summary of revisions:* Addition of the calculation of sparticle and Higgs decays. All 2-body and important 3-body tree-level decays, including phenomenologically important loop-level decays (notably, Higgs decays to  $gg$ ,  $\gamma\gamma$  and  $Z\gamma$ ). Next-to-leading order corrections are added to neutral Higgs decays to  $q\bar{q}$  for quarks  $q$  of any flavour and to the neutral Higgs decays to  $gg$ .

## 2. Introduction

The phenomenology of simple supersymmetric extensions of the Standard Model has become something of an industry in particle physics of late. The potential of such models to explain the technical hierarchy problem (i.e. the relative smallness of the Higgs mass as compared to the Planck or gauge unification scale) has motivated its study. The models have potential implications for cosmology as well as for collider physics. Several different computational tools are necessary for the study of the models' phenomenology [1]. The initial step is a calculation of the supersymmetric spectrum, as well as particle couplings. This is the step that the computer program SOFTSUSY has previously performed, in the  $R$ -parity conserving MSSM [2], the  $R$ -parity conserving NMSSM [3] and the  $R$ -parity violating MSSM [4]. By adding one-loop corrections to neutrino masses, the program has been extended [5] to calculate neutrino masses and mixings in the presence of lepton number violating  $R$ -parity violation. Further non-trivial additions of higher order corrections result in increased precision for the gauge and Yukawa couplings [6] and the squark and gluino pole masses [7]. Other publicly available computer programs exist which perform the task of computing the supersymmetric spectrum: for the MSSM, there is FLEXIBLESUSY [8], ISASUSY [9], SUSEFLAV, SUSPECT [10] and sPHENO [11]. In the NMSSM, there is only one stand-alone dedicated tool for spectrum calculation: NMSSMTools [12, 13], whereas the SARAH [14] framework can be combined with FLEXIBLESUSY or sPHENO in order to calculate the spectrum. The plethora of computer programs is useful: some of them use different approximations and extend to different models. Even the programs having the same apparent approximations differ in their numerical output because the higher order corrections not included are implicitly different: both for the sparticle spectrum [15] and the Higgs masses [16, 17, 18]. Thus, the size of differences between the programs for observables calculated at the same order or approximation serve as an rough estimate of the size of higher order corrections. In some cases, different approximations are used and these can help investigate different régimes of parameter space. For example, one can deal with sparticle threshold effects differently: either at fixed order at say  $M_Z$  using the MSSM or NMSSM as an effective field theory above  $M_Z$  (this is the approach taken by SOFTSUSY, sPHENO and SUSPECT for instance), or one could integrate the particles out in the renormalisation group equations at some higher scale (the effective field theory approach taken by ISAJET and NMSSMTools). The former approach includes finite terms of order  $M_Z^2/(16\pi^2 M_p^2)$ , where  $p$  is the mass of some sparticle, but takes the sparticle mass splittings  $\Delta M_p$  into account at some level approximation, typically  $\sim O(\log[\Delta M_p^2/(16\pi^2)])$ , whereas the latter approach often misses the finite terms but re-sums the mass splitting terms. One generically expects the former approach to be more accurate when sparticles are not too heavy, and the latter when the sparticles are very heavy and when the splittings between them are large. Which one is more accurate given current lower bounds upon sparticle masses is a quantitative question that is observable dependent.

In order to provide predictions for future sparticle and Higgs searches at colliders, or indeed in order to interpret searches at them in terms of the MSSM and NMSSM, cross-section estimates as well as simulations of collisions are required, in order to estimate acceptances and efficiencies. For the collision simulations, estimates of the various decay partial widths for the sparticles and Higgs particles are required. Some Monte-Carlo event generators perform this task in the MSSM, for example PYTHIA [19] or HERWIG7 [20], but there also exist dedicated tools like SUSYHIT [21] or HDECAY [22] and FeynHiggs [23] (the latter two specialising in the Higgs boson decays). sPHENO also contains a decay calculation for the MSSM. In the NMSSM however, the options for calculating sparticle and Higgs decays are rather slim: NMSSMTools is the only stand-alone option, whereas SARAH can be combined with sPHENO. Previous versions of SOFTSUSY contained an interface to NMSSMTools so that the NMSSM spectrum and couplings could then be fed into the program in order to predict sparticle and Higgs boson partial decay widths.

The present paper describes a significant extension in functionality in SOFTSUSY: to calculate and output the various partial widths for the decays of sparticles and Higgs bosons in the MSSM and in the NMSSM. Emphasis has been placed on speed of execution, preferring to perform as much of the calculation analytically as is practicable. We

hope that this addition of functionality to SOFTSUSY will facilitate collider studies of sparticle and supersymmetric Higgs searches: both through the study of differences with the other programs as an estimate of the size of theoretical uncertainty in the prediction, and through a fast and unified computation.

The rest of the paper is organised as follows: in Section 3, we specify the conventions used in this paper and the assumptions made in the decay calculator and we provide some details about the method of computation. Following this, in section 4, we list the decay modes included in the program. Next, in section 5, we provide a few examples of comparison tests with a couple of other publicly available tools for the MSSM and the NMSSM, before summarising in Section 6. We show how to run the program and provide explicit flags for controlling its behaviour in Appendix A, providing some sample output in Appendix B. We provide explicit formulae for the partial widths in Appendix D to Appendix J; these have not been collected together in one reference before.

### 3. Conventions, Assumptions and Method

Throughout  $\tilde{Z}_i$  and  $\tilde{W}_j$  are used for neutralinos ( $i = 1, 2, 3, 4$  in the MSSM or  $i = 1, 2, 3, 4, 5$  in the NMSSM) and charginos ( $j = 1, 2$ ), respectively. This is different to the commonly used  $\tilde{\chi}_i^0$  and  $\tilde{\chi}_j^\pm$  notation for ease of reading, particularly when they appear in subscripts. The notation for the mass-ordered CP even and CP odd neutral Higgs bosons is that  $(h, H, H3)$  are the CP even neutral Higgs bosons in order of increasing mass, whilst  $(A, A2)$  are the CP odd neutral Higgs bosons again in order of increasing mass, remembering that  $H3$  and  $A2$  occur only in the NMSSM.

The partial width formulae for all of the decay modes included in the SOFTSUSY decay calculation<sup>1</sup> are listed in Appendix D onwards, many of these were rederived and have been written in one consistent set of conventions.

#### 3.1. MSSM

While the conventions used in the decays code are largely those used in SOFTSUSY [2], there are differences in a few places in order to allow easier comparison with partial width (PW) formulae provided elsewhere. The few differences with respect to Ref. [2] are as listed below<sup>2</sup>:

- In our calculations, it is convenient to work in a basis where the third generation sfermions are mass ordered with  $m_{\tilde{f}_1} < m_{\tilde{f}_2}$ . In order to ensure this, the mixing angle  $\theta_f$  is transformed accordingly ( $\theta_f \rightarrow \theta_f + \pi/2$ ) in the case where the SOFTSUSY spectrum generator has  $m_{\tilde{f}_1} > m_{\tilde{f}_2}$ .
- The mixing angles for the charginos are transformed with respect to the SOFTSUSY spectrum generator in order to match conventions used elsewhere (e.g. [10]). Therefore  $\theta_{L/R}$  as indicated below is given by  $\theta_{L/R}^{decays} = -\theta_{L/R}^{spectrum} + \pi/2$ .

#### 3.2. NMSSM

The conventions used in the decay code are predominantly those described previously in the SOFTSUSY NMSSM manual [3], but there are differences in a few places. As well as those listed above, there are a few changes specific to the NMSSM, to allow straightforward comparison with NMSSMTools [12, 13, 24, 25]:

- The Charge Parity (CP) even neutral Higgs mixing matrix is altered relative to the matrix  $R$  provided by SOFTSUSY [3]. The matrix  $S$  used in the decay formulae is given by :

$$S = \begin{pmatrix} R(1,2) & R(1,1) & R(1,3) \\ R(2,2) & R(2,1) & R(2,3) \\ R(1,3) & R(2,3) & R(3,3) \end{pmatrix} \quad (1)$$

i.e. the first two columns of the first two rows are interchanged.

<sup>1</sup>The source code for the calculations is in the file `src/decays.cpp`, which is in the C++ programming language.

<sup>2</sup>In the decay code itself, the neutralino mixing matrix used ( $N$  in SOFTSUSY notation) is transposed.

- The CP odd neutral Higgs mixing matrix is altered relative to the matrix provided by SOFTSUSY [3], the matrix  $P$  detailed in the decay formulae (different to the  $P$  in Ref. [3] which we write here as  $P^{prov}$ ) is given below. The differences are that the first row of  $P^{prov}$  is dropped (as this refers to the goldstone boson) and the first and second columns are interchanged. The ratio of the Higgs vacuum expectation values (vevs)  $\beta$  and the mixing angle  $\theta_A$  are as used elsewhere in SOFTSUSY [2, 3]:

$$P = \begin{pmatrix} P^{prov}(2,2) & P^{prov}(2,1) & P^{prov}(2,3) \\ P^{prov}(3,2) & P^{prov}(3,1) & P^{prov}(3,3) \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \cos\beta \cos\theta_A & \sin\beta \cos\theta_A & \sin\theta_A \\ \cos\beta \sin\theta_A & \sin\beta \sin\theta_A & -\cos\theta_A \\ 0 & 0 & 0 \end{pmatrix} \quad (2)$$

### 3.3. Mass Choices and Scales Used

The input parameters from the SOFTSUSY spectrum generator [2, 3] can be evaluated at a variety of different scales. The choice of the renormalisation scale used is an important consideration and can result in differences between decay calculator predictions. Different scales effectively correspond to including different higher order terms in the calculation. For example, consider the decay of a gluino into a top and a stop. One must choose a renormalisation scale for the coupling. The masses of the particles involved could be running masses evaluated at different scales, or pole masses. Each choice affects the numerical value of the PW, but are all equivalent at tree-level. In SOFTSUSY the following choices are made:

- In general, unless explicitly stated otherwise, the masses of the supersymmetric (SUSY) and Higgs particles and other parameters, such as mixing angles and gauge couplings, are evaluated at the scale  $M_{SUSY} = x \sqrt{m_{\tilde{t}_1}(M_{SUSY})m_{\tilde{t}_2}(M_{SUSY})}$ , where  $x$  can be set by the user but by default is taken to be 1. Here,  $m_{\tilde{t}_i}(M_{SUSY})$  is the running  $i^{th}$  stop mass evaluated at a modified dimensional reduction [26] ( $\overline{DR}$ ) renormalisation scale  $M_{SUSY}$ .
- For Higgs loop decays the gauge coupling strengths  $\alpha_s$  and  $\alpha$  are evaluated at the mass of the decaying Higgs.
- For Higgs loop decays to  $\gamma\gamma$  or  $Z\gamma$  the masses of the important quarks (i.e.  $m_t, m_b, m_c$ ) are evaluated at the mass of the decaying Higgs. Below  $M_Z$ , these are run in 3 loop QCD and 1 loop in QED. In the calculation of decays of  $H, H3, A$  and  $A2$  quark masses are run to  $m_H, m_{H3}, m_A, m_{A2}$  in the (N)MSSM as appropriate, whilst for the lightest CP even Higgs  $h$  the  $m_t, m_b$  and  $m_c$  are run to  $m_h$  in 1 loop QED and 3 loop QCD.
- Throughout the program, unless otherwise stated here, we generally use two different quark masses; “kinematic masses” for the kinematics (i.e. for masses of particles in the initial or final states) and “running masses” for the evaluation of couplings. This hopefully allows a large part of some higher order corrections to be incorporated into the quark legs via the mass running. The way in which these masses are evaluated is listed in Table 1 below.
- In addition to the above quark masses, there are extra masses `mcpole` and `mcpole` defined in `decays.h` which are set to particular values used in the QCD corrections to neutral Higgs boson decays to  $q\bar{q}$  or  $g\bar{g}$ .
- If the QCD corrections to these decays are turned off in the program then the running masses for the quarks are used in order to attempt to hopefully incorporate some of the Next-to-Leading-Order (NLO) corrections to the quark legs.

As detailed in Table 1, for the third generation sfermions the “kinematic” masses are pole masses obtained from the propagators whilst the “running (coupling)” masses are in the  $\overline{DR}$  scheme. For the “kinematic” masses of the first two generation fermions, the  $\overline{MS}$  mass at  $M_Z$  is used, whilst the “running” masses are extracted from the running Yukawa couplings. For the electron and muon, the running is again small (only QED). The “kinematic” masses for the vector bosons are pole masses, whilst for the “running(coupling)” masses they are running  $\overline{DR}$  masses evaluated at  $M_{SUSY}$ .

- $\overline{MS}$  masses include only SM corrections within SOFTSUSY; 3-loop QCD and 1-loop QED corrections are included.
- Yukawa-extracted masses are in the  $\overline{DR}$  scheme and include SM and SUSY corrections.

kinematic masses		running (coupling) masses	
mtPole	pole mass from propagator	runmt	$\overline{DR}$ mass at $M_Z$
mbPole	pole mass from propagator	runmb	$\overline{DR}$ mass at $M_Z$
mtauPole	pole mass from propagator	runmtau	$\overline{DR}$ mass at $M_Z$
mc	$\overline{MS}$ mass at $M_Z$	runmc	Yukawa-extracted mass at $M_Z$
ms	$\overline{MS}$ mass at $M_Z$	runms	Yukawa-extracted mass at $M_Z$
mup	$\overline{MS}$ mass at $M_Z$	runmu	Yukawa-extracted mass at $M_Z$
mdu	$\overline{MS}$ mass at $M_Z$	runmd	Yukawa-extracted mass at $M_Z$
mel	$\overline{MS}$ mass at $M_Z$	runmel	Yukawa-extracted mass at $M_Z$
mmu	$\overline{MS}$ mass at $M_Z$	runmmu	Yukawa-extracted mass at $M_Z$
polemw	pole mass from propagator	runmw	running $W$ mass at $M_{SUSY}$
polemz	pole mass from propagator	runmz	running $Z$ mass at $M_{SUSY}$

Table 1: The two different types of masses used for the fermions and gauge bosons. The names given are those used in the code (`src/decays.cpp`). “kinematic” masses are used for the masses of initial and final state particles in the decay formulae whilst “running (coupling)” masses are used where masses appear in expressions involving couplings in the partial width formulae. Note that within `SOFTSUSY`, the  $\overline{MS}$  masses include only SM corrections whilst the Yukawa-extracted masses ( $\overline{DR}$  masses) include SM and SUSY corrections.

- Quark input masses can be reset by the user within the `SMINPUTS` block of the `SLHA/SLHA2` input file. The kinematic and running masses used will then change accordingly.

The different choices of scales for the input parameters is one of the key sources of differences between different decay calculator programmes. It is worth noting that an experimental value of Fermi’s constant,  $G_F$ , is also used, this is inconsistent with the tree-level expression  $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$  as it is an empirical quantity and so incorporates higher order terms.

### 3.4. Assumptions Made

The following assumptions are made in the decay calculator:

- $R$ -parity conservation in the MSSM and in the NMSSM.
- No additional CP violation relative to the SM.
- No additional flavour violation relative to the SM.
- Sfermion mixing has only been accounted for in the third generation of sfermions as it is proportional to the Yukawas, which are negligible for the first two generations.

### 3.5. Method

For 2 body decay modes, partial widths are explicitly used in order to provide fast evaluation. Similarly, for the loop decays the integrals were performed analytically and the resulting formulae used. For 3 body decay modes, the phase space integral has been analytically reduced to a one-dimensional integral, which is then performed using adaptive Gaussian numerical integration [27]. The exception to this is the neutral CP even lightest Higgs decays to a vector boson and an off-shell vector boson, which then decays into a fermion antifermion pair, this calculation was performed explicitly and all integrals were evaluated analytically.

The 3 body decay modes were therefore where most complications arose. In general for an  $N$ -body tree-level decay there are  $N$  integrals to perform, one over the three-momenta of each of the final state particles. One of these integrals is always trivial to perform using the momentum-conserving delta function. For the 2 body decay widths this leaves one remaining integral with the energy delta function, this can then be performed using the standard result for the integral of a function of a variable multiplied by a delta function containing another function of the same variable. For 3 body decay widths however, one has two integrals to perform and in general they are non-trivial to determine analytically. In certain cases the symmetry of the integrands, along with certain assumptions, may allow them to be

performed. For example, in the case of  $h \rightarrow VV^* \rightarrow V f \bar{f}$  (where  $V$  refers to  $W^\pm$  or  $Z^0$  bosons) the mass of the Higgs boson ensures that the outgoing fermions may not be top quarks, therefore one can neglect the masses of the outgoing fermions and greatly simplify the calculation. Passarino-Veltman reduction [32] can then allow reduction of the integrals to a one-dimensional integral, which in this case may be determined explicitly analytically; the result given in both Appendix F.6 and in the “Higgs Hunter’s Guide” [28]. For a general 3 body decay mode the calculations are however considerably more involved. There are two approaches that can be taken once the first trivial integral using the momentum-conserving delta function is performed; at this stage the partial width can be written as a double differential decay rate in two (reduced) Mandelstam variables as is the case in SUSYHIT-1.4, following the work performed in reference [29], these two dimensional integrals can then be performed numerically. Alternatively, often one of the two integrals (remaining after the first trivial integral is performed) may be evaluated analytically, leaving a single one-dimensional integral to be performed numerically. This is approach used in the work in reference [30] and is the method adopted in sPHENO [11, 31], from which the expressions we use for the 3 body decays originate. The Feynman diagrams involved, effects included and any assumptions made for each of the 3 body decays are given in detail in Appendix G with the corresponding partial width formulae.

The situation is of course more complicated for loop decays than at tree-level, each loop provides an additional loop integral to be performed, in the case of the 1-loop decays included in SOFTSUSY, they were performed explicitly with the help of Passarino-Veltman reduction [32].

#### 4. Decay Modes Included

The following section provides a list of all the decay modes included in the decay part of the SOFTSUSY package; they are split into MSSM SUSY tree-level 2 body decays, MSSM Higgs tree-level 2 body decays, MSSM Higgs 1-loop 2 body decays, MSSM tree-level 3 body decays, Next-to-Lightest Supersymmetric Particle (NLSP) decays to the gravitino LSP (Lightest Susy Particle), NMSSM SUSY and Higgs tree-level 2 body decays, NMSSM 1-loop 2 body decays and decays for which QCD corrections have been included. A comprehensive list of the formulae for all of the decays included is given explicitly in Appendix D to Appendix J for ease of reference. To summarise, we include:

- All MSSM 2 body decays at tree-level, both sparticle and Higgs boson decays.
- The phenomenologically most relevant three-body decays of gluinos, charginos and neutralinos.
- All Higgs 2 body decays at tree-level.
- Higgs decays to  $\gamma\gamma$ ,  $Z\gamma$  and  $g\bar{g}$  at leading order (i.e. one-loop).
- Next-to-Lightest SUSY Particle (NLSP) 2 body decays to gravitinos in the MSSM.
- All NMSSM 2 body decays at tree-level, including both the extended neutralino and extended Higgs sectors.
- QCD corrections to neutral Higgs decays to quarks and to gluons in the MSSM and NMSSM.

Whilst the majority of the decay modes are therefore calculated at tree-level, some effects of higher order corrections are approximated via the use of running masses and couplings, as calculated using the SOFTSUSY spectrum generator [2], the details of the mass choices were given in Section 3.3. In Appendix D there are a series of tables indicating all the modes included, along with appendix references for their partial width formulae as used in SOFTSUSY.

The branching ratios for each mode are grouped into decay tables for each parent SUSY or Higgs particle and are all printed to standard output in the SLHA/SLHA2 convention [33, 34] in order to allow it to be passed straightforwardly to other programs (such as PYTHIA [19], HERWIG7 [20], MadGraph [35], for instance).

#### 4.1. MSSM SUSY Tree-Level Two Body Decays

The detailed formulae for these modes are in Appendix F. The gluino  $\tilde{g}$  decays included are:

$$\tilde{g} \rightarrow q\tilde{q}_{L/R}^*, \tilde{q}\tilde{q}_{L/R}^*, t\tilde{t}_{1/2}^*, \tilde{t}\tilde{t}_{1/2}, b\tilde{b}_{1/2}^*, \tilde{b}\tilde{b}_{1/2},$$

where 1/2 means 1 or 2, and L/R means L or R. The sfermion  $\tilde{f}$  decays included are, for the first two generations where there is no sfermion mixing:

( $f'$  indicates a fermion in the same generation as the  $f$  fermion but with opposite third component of weak isospin, i.e.  $f$  and  $f'$  could be  $u$  and  $d$  or  $\nu_e$  and  $e^-$ ):

$$\begin{aligned}\tilde{q}_{L/R} &\rightarrow \tilde{g}q, \\ \tilde{f}_L &\rightarrow \tilde{W}_j f', \\ \tilde{f}_{L/R} &\rightarrow \tilde{Z}_i f.\end{aligned}$$

For the third generation sfermions:

$$\begin{aligned}\tilde{b}_{1/2} &\rightarrow \tilde{g}b, \tilde{W}_j t, \tilde{Z}_i b, \tilde{t}_{1/2} W^-, \tilde{t}_{1/2} H^-, \\ \tilde{t}_{1/2} &\rightarrow \tilde{g}t, \tilde{W}_j b, \tilde{Z}_i t, \tilde{b}_{1/2} W^+, \tilde{b}_{1/2} H^+, \\ \tilde{b}_2 &\rightarrow \tilde{b}_1 Z, \tilde{b}_1 h/H/A, \\ \tilde{t}_2 &\rightarrow \tilde{t}_1 Z, \tilde{t}_1 h/H/A, \\ \tilde{\tau}_{1/2} &\rightarrow \tilde{W}_j \nu_\tau, \tilde{Z}_i \tau, \tilde{\nu}_\tau W^-, \tilde{\nu}_\tau H^-, \\ \tilde{\nu}_\tau &\rightarrow \tilde{W}_j \tau, \tilde{Z}_i \nu_\tau, \tilde{\tau}_{1/2} W^+, \tilde{\tau}_{1/2} H^+, \\ \tilde{\tau}_2 &\rightarrow \tilde{\tau}_1 Z, \tilde{\tau}_1 h/H/A.\end{aligned}$$

For charginos, the two-body decay modes included are:

$$\begin{aligned}\tilde{W}_j &\rightarrow \tilde{q}_L \tilde{q}'^*, \tilde{q}_{1/2} \tilde{q}'^*, \tilde{l}_L \tilde{\nu}_l, \tilde{\nu}_l \tilde{l}, \tilde{\tau}_{1/2} \tilde{\nu}_\tau, \tilde{\nu}_\tau \tilde{\tau}, \tilde{Z}_i W^+, \tilde{Z}_i H^+, \\ \tilde{W}_2 &\rightarrow \tilde{W}_1 Z, \tilde{W}_1 h/H/A.\end{aligned}$$

For neutralinos the two-body decay modes are ( $k > i$  as the neutralinos are mass ordered):

$$\begin{aligned}\tilde{Z}_i &\rightarrow \tilde{f}_{L/R} \tilde{f}^*, \tilde{f}_{1/2} \tilde{f}^*, \tilde{W}_j W^+, \tilde{W}_j H^+, \\ \tilde{Z}_k &\rightarrow \tilde{Z}_i Z, \tilde{Z}_i h/H/A.\end{aligned}$$

#### 4.2. MSSM Higgs Tree-Level Two Body Decays

The tree-level 2 body decay modes included for the Higgs particles in the MSSM are as follows (see later for the 1-loop Higgs decays included), the formulae for the partial widths are explicitly given in Appendix F.6. Note that for the decays to sfermions any combination of handedness is allowed,  $LL$ ,  $LR$ ,  $RL$ ,  $RR$ . Similarly, for the third generation where there is squark mixing, and for decays to charginos, all combinations 11, 12, 21, 22 are possible:

$$\begin{aligned}h/H/A &\rightarrow \tilde{f}_{L/R} \tilde{f}_{L/R}^*, \tilde{f}_{1/2} \tilde{f}_{1/2}^*, \tilde{W}_{1/2} \tilde{W}_{1/2}, \tilde{Z}_i \tilde{Z}_i, q\bar{q}, l^+ l^-, \\ H^+ &\rightarrow \tilde{Z}_i \tilde{W}_j, q\bar{q}', \nu_l \tilde{l}, \tilde{f}_{L/R} \tilde{f}_{L/R}^*, \tilde{t}_{1/2} \tilde{b}_{1/2}^*, \nu_\tau \tilde{\tau}_{1/2}^*, hW^+, \\ h/H &\rightarrow AA, AZ, \\ H &\rightarrow H^+ H^-, hh, \\ A &\rightarrow h/HZ.\end{aligned}$$

Note that the quarks which are considered for neutral Higgs decays are only  $c$ ,  $s$ ,  $b$  for  $h$  but also  $t$  for  $H/A$ . Decays to  $u$  and  $d$  are negligible. For  $H^+$ , decays to CKM suppressed combinations of  $q$  and  $q'$  are also considered, for example  $H^+ \rightarrow u\bar{s}$ . Note also that the decays  $H^+ \rightarrow H/AW^+$  are not included as they are kinematically forbidden in the MSSM (assuming tree-level mass formulae), these modes are included in the NMSSM.

#### 4.3. MSSM Higgs 1-loop Two body decays

The key Higgs 1-loop decays are also included as these are very important channels for LHC Higgs discovery and measurement. The explicit expressions for their partial widths are in Appendix F.6:

$$h/H/A \rightarrow \gamma\gamma, Z\gamma, gg.$$

#### 4.4. MSSM Tree-Level Three Body Decays

The phenomenologically most important three-body decays in the MSSM are included, for the neutralino decays to another neutralino and a fermion antifermion pair then  $i > j$  as the neutralinos are mass-ordered.:

$$\begin{aligned} h &\rightarrow V f \bar{f}. \\ \tilde{g} &\rightarrow \tilde{Z}_i q \bar{q}, \tilde{W}_i q \bar{q}'. \\ \tilde{Z}_i &\rightarrow \tilde{Z}_j f \bar{f}, \tilde{W}_j f \bar{f}'. \\ \tilde{W}_j &\rightarrow \tilde{Z}_i f \bar{f}'. \end{aligned}$$

As of yet, there are no three-body decays of sfermions included; this may be resolved in future versions. Similarly, there are no radiative corrections included for sfermions; however, whilst these affect the partial widths, in general the branching ratios are not significantly affected [36]. The explicit formulae used, for which sPHENO [11] provided a useful reference, are given in Appendix G. In our decay calculator the 3 body decay modes are only calculated where no 2 body modes are available for the particular decaying SUSY or Higgs particle to the specific daughter(s) considered. Often 3 body modes will also not be output, even if they have been calculated, as in scenarios where other 2 body decay modes are kinematically available to the decaying parent particle these 2 body modes will dominate the total decay width. In such cases the 3 body modes often have branching ratios smaller than BRTo1, which is the minimum BR output in the decay tables and may be set by the user in the input file.

#### 4.5. MSSM Decays to Gravitinos

The following NLSP  $\rightarrow \tilde{G} + SM$  decays are included, for when the gravitino  $\tilde{G}$  is the lightest supersymmetric particle (LSP):

$$\tilde{g} \rightarrow g\tilde{G}, \quad \tilde{q}_i \rightarrow q\tilde{G}, \quad \tilde{l} \rightarrow l\tilde{G}, \quad \tilde{Z}_i \rightarrow \gamma\tilde{G}, \quad \tilde{Z}_i \rightarrow Z\tilde{G}, \quad \tilde{Z}_i \rightarrow \phi\tilde{G},$$

where  $\phi$  denotes one of the neutral Higgs bosons  $h, H$  or  $A$ . The formulae for the partial widths are in Appendix H.

#### 4.6. NMSSM SUSY and Higgs Tree-Level Two Body decays

In the NMSSM, decays not involving the extended Higgs or neutralino sectors are the same as in the MSSM. For the extended neutralino and Higgs sectors the allowed decays are largely as before with the exception that now the neutralino index  $i$  runs from 1 to 5, meanwhile any decays involving the heaviest neutral CP even Higgs,  $H$ , in the MSSM can now also occur for the heaviest Higgs  $H3$  in the NMSSM. Similarly, decays involving the neutral CP odd Higgs,  $A$ , in the MSSM, can now also occur for the heavier  $A2$ . Additional decay modes (other than simply extending decays into neutralinos to include  $\tilde{Z}_5$  or decays into  $H3, A2$ ) are the decays of the  $\tilde{Z}_5, H3$  and  $A2$ . Here,  $n = 1, 2, 3, 4$  since the  $\tilde{Z}_5$  decays into lighter neutralinos:

$$\begin{aligned} \tilde{Z}_5 &\rightarrow W\tilde{W}_{1/2}, Z\tilde{Z}_n, H^\pm\tilde{W}_{1/2}, \tilde{Z}_n h/H/H3/A/A2, \tilde{f}_{L/R}\tilde{f}, \tilde{f}_{1/2}\tilde{f}, \\ H3 &\rightarrow \tilde{f}_{L/R}\tilde{f}_{L/R}^*, \tilde{f}_{1/2}\tilde{f}_{1/2}^*, \tilde{W}_{1/2}\tilde{W}_{1/2}, \tilde{Z}_i\tilde{Z}_l, q\bar{q}, l^+l^-, AA, AA2, \\ H3 &\rightarrow A2A2, ZA/A2, H^+H^-, hh, hH, HH, W^-H^+, VV, \\ A2 &\rightarrow \tilde{f}_L\tilde{f}_R^*, \tilde{f}_1\tilde{f}_2^*, \tilde{W}_{1/2}\tilde{W}_{1/2}, \tilde{Z}_i\tilde{Z}_l, q\bar{q}, l^+l^-, Zh/H/H3, Ah/H/H3, W^-H^+, \end{aligned}$$

where  $VV \in \{W^+W^-, ZZ\}$ . As before, for the decays to two sfermions, any combination of handedness is permitted  $LL, LR, RL, RR$ ; similarly for the decays to mixed sfermions or to charginos 11, 12, 21, 22 are all allowed. For the decays to quarks only  $c, s, t, b$  are considered. For the  $A2$  there are fewer decays than the  $H3$  as many decays are ruled out by CP conservation.

The explicit partial width expressions used within the decay calculator SOFTSUSY are given in Appendix I, the expressions in NMSSMTools [12, 24, 25] were used, with appropriate changes.



#### 4.7. NMSSM 1-loop Two Body Decays

Just as in the MSSM, in the NMSSM the phenomenologically important 1-loop decays of Higgs bosons are included:

$$h/H/H3/A/A2 \rightarrow \gamma\gamma, Z\gamma, gg.$$

See Appendix I for the detailed formulae used within the code for the partial widths of these modes.

#### 4.8. QCD Corrected Decays

NLO QCD corrections have been incorporated for the decays in which such effects are most important, these are the neutral Higgs decays to quarks and decays to gluons,  $h/H/H3/A/A2 \rightarrow q\bar{q}, gg$ . The expressions used are given in Appendix J and are based on those provided in the calculations in [37, 38].

### 5. Validation and Comparison with other programs

We implemented a number of different approaches to validate the program and the formulae used.

1. The partial width of every channel was compared against corresponding channels in other programs where possible; for example the 2 body MSSM decays were compared against results from SUSYHIT, the 3 body MSSM decays against sPHENO and the 2 body NMSSM decays against NMSSMTools. This ensured that the formulae were correct and any differences were down to different input parameters resulting from different schemes of running, different mass choices and other similar effects. These checks were performed at several parameter space points for each partial width.
2. Most of the formulae used in the SOFTSUSY program were rederived in order to determine any issues, the exception being the 3 body decays for which there are very complicated expressions for the integrals to be performed, for these the formulae are as in sPHENO-3.3.8 [11]. Meanwhile, NMSSM decay expressions were generalised from MSSM expressions and checked against NMSSMTools.
3. MSSM 2 body partial widths were also checked against both Baer and Tata “Weak Scale Supersymmetry” [39] and The Higgs Hunter’s Guide [28]. Some small differences were found, in particular relative to Baer and Tata, these differences were largely simple typographical errors although there were several other differences in the 3 body decay formulae, where additional contributions are included. The full formulae for the partial widths used in SOFTSUSY-4.0 are given in Appendix D to Appendix J.
4. Once all the decay modes of a given initial state were implemented the branching ratios were compared against those of a relevant program, i.e. SUSYHIT for the MSSM 2 body decays, sPHENO for MSSM 3 body decays, and NMSSMTools (as linked with the SOFTSUSY spectrum generator) for NMSSM decays.
5. Many of the modes have been analysed in detail by scanning over the mass of the decaying particle and comparing the branching ratio structure with known results and with corresponding results in other decay calculator programs.

As described above we therefore performed specific and fairly extensive tests for particular benchmark points with sPHENO and SUSYHIT. Comparisons for some of these benchmark points are provided here for a selection of decaying SUSY and Higgs particles, in addition scans over the mass of the decaying particle are given for the decays of the lightest SM-like Higgs and for the decays of a gluino  $\tilde{g}$ . This allowed both a qualitative check of the behaviour of the decays in the program and a quantitative comparison of the level of agreement with other programs. In particular the level of agreement with the same input parameters and with our set of input parameters is detailed in some specific cases. This method uses the results the SOFTSUSY program produces to validate it.

#### 5.1. Supersymmetric Decays - $\tilde{t}_1$

First of all consider the decays of the lightest stop,  $\tilde{t}_1$ , the verbatim output of the SOFTSUSY decay calculator is given in Table 2 and was generated with the lesHouchesInput file. The comparison of the results for this benchmark point between the new SOFTSUSY decay calculator and those of SUSYHIT-1.4 is given in Table 3.

#	PDG	Width	# Stop1 decays			
DECA	1000006	5.72622764e+00				
#	BR	NDA	PDG1	PDG2	Comments	PW
	3.20176449e-01	2	5	1000024	# $\tilde{t}_1 \rightarrow b \tilde{\chi}_{1+}$	1.83340323e+00
	2.21769170e-01	2	5	1000037	# $\tilde{t}_1 \rightarrow b \tilde{\chi}_{2+}$	1.26990075e+00
	2.27726880e-01	2	6	1000022	# $\tilde{t}_1 \rightarrow t \tilde{\chi}_{10}$	1.30401596e+00
	1.25396681e-01	2	6	1000023	# $\tilde{t}_1 \rightarrow t \tilde{\chi}_{20}$	7.18049943e-01
	1.04930819e-01	2	6	1000025	# $\tilde{t}_1 \rightarrow t \tilde{\chi}_{30}$	6.00857758e-01

Table 2: The decays of a  $\tilde{t}_1$  at the parameter point given by the `lesHouchesInput` file provided with `SOFTSUSY`. For reference this has  $m_{\tilde{t}_1} = 808.7$  GeV,  $m_{\tilde{W}_1} = 385.0$  GeV,  $m_{\tilde{W}_2} = 637.5$  GeV,  $m_{Z_1} = 204.0$  GeV,  $m_{Z_2} = 385.0$  GeV,  $m_{Z_3} = -622.7$  GeV,  $m_{Z_4} = 637.2$  GeV. The partial widths are given in GeV units.

SOFTSUSY standard inputs		SOFTSUSY SUSYHIT's inputs		SUSYHIT		mode
PW/GeV	BR	PW/GeV	BR	PW/GeV	BR	
1.8334e+00	3.2018e-01	1.7080e+00	3.2111e-01	1.7080e+00	3.2181e-01	$\tilde{t}_1 \rightarrow b \tilde{W}_1$
1.2699e+00	2.2177e-01	1.1027e+00	2.0730e-01	1.1027e+00	2.0775e-01	$\tilde{t}_1 \rightarrow b \tilde{W}_2$
1.3040e+00	2.2773e-01	1.2986e+00	2.4414e-01	1.2992e+00	2.4478e-01	$\tilde{t}_1 \rightarrow t \tilde{Z}_1$
7.1805e-01	1.2540e-01	6.8479e-01	1.2874e-01	6.7286e-01	1.2677e-01	$\tilde{t}_1 \rightarrow t \tilde{Z}_2$
6.0086e-01	1.0493e-01	5.2503e-01	9.8706e-02	5.2485e-01	9.8887e-02	$\tilde{t}_1 \rightarrow t \tilde{Z}_3$

Table 3: The  $\tilde{t}_1$  decay partial widths and branching ratios as output by `SOFTSUSY` with our mass choices (and corresponding Yukawa couplings) and with the masses and Yukawa couplings in `SUSYHIT`, compared with the results of `SUSYHIT-1.4`. This illustrates the differences of order 10% or more that may arise depending upon mass (“kinematic” and “running”) choices, the differences here reduce to order 1% once the same masses are taken. The `lesHouchesInput` file provided with `SOFTSUSY` gives the parameter point here, it has a common scalar mass  $m_0 = 125$  GeV, a common gaugino mass  $m_{1/2} = 500$  GeV, ratio of Higgs vacuum expectation values  $\tan\beta = 10$ , sign of the superpotential  $\mu$  parameter  $\text{sign}(\mu) = +1$  and common soft SUSY breaking trilinear parameter  $A_0 = 0$  in the constrained MSSM (CMSSM). Here, as elsewhere in this paper, we present numerical results in mantissa-exponent notation (i.e.  $e-0a = \dots \times 10^{-a}$  and  $e+0b = \dots \times 10^b$  for  $a, b \in \{0, 1, 2, 3, \dots\}$ ).

In this the input values used for the various masses are: top pole mass  $m_{\text{tPole}} = 174.3$  GeV, bottom pole mass  $m_{\text{bPole}} = 4.985$  GeV, running top mass  $\text{runmt} = 145.555$  GeV and running bottom mass  $\text{runmb} = 2.576$  GeV. These differ from the default values used for these quantities in `SUSYHIT`, Table 3 illustrates the differences observed between `SOFTSUSY` and `SUSYHIT-1.4`, as well as the differences when `SOFTSUSY` has the `SUSYHIT` mass inputs inserted by hand. This demonstrates that the level of agreement between the programs is around 10%, dropping down to 1% when the same input masses and coupling constants are used in both programs. These differences result from the different mass and scheme choices, as outlined in Section 3.3.

## 5.2. Higgs Decays - $h$

Now, in Table 4, we can perform similar comparisons between `SOFTSUSY` and `HDECAY-3.4` of `SUSYHIT-1.4` for Higgs decays. Here we have taken a SM-like Higgs, in the decoupling limit so all the SUSY decays are kinematically forbidden, with Higgs mass set by hand to the observed 125.09 GeV [40]. The results of our decay calculator without QCD corrections included, with QCD corrections included, and with the same input quark and gauge boson masses and same input gauge couplings as `SUSYHIT`, again with QCD corrections, are compared with `HDECAY-3.4`.

In Table 4 the comparison of the PWs with QCD corrections switched on and switched off clearly demonstrates the significant difference these corrections make to neutral Higgs decays to quarks and to gluons, as is widely known in the literature [38, 41, 42, 43]. Furthermore, it is clear that the main source of differences in partial widths between the decay calculator of `SOFTSUSY-4.0` and `HDECAY` is in the choice of masses used. Remaining differences tend to be small and are due largely to differences in other inputs, the exception being the decays to two vector bosons,

SOFTSUSY no QCD corrections		SOFTSUSY with QCD corrections		SOFTSUSY with SUSYHIT's masses and QCD corrections		HDECAY-3.4 with QCD corrections		mode
PW/GeV	BR	PW/GeV	BR	PW/GeV	BR	PW/GeV	BR	
1.03e-04	3.36e-02	2.23e-04	3.99e-02	2.23e-04	4.56e-02	2.23e-04	4.35e-02	$h \rightarrow cc$
8.05e-07	2.62e-04	1.56e-06	2.80e-04	1.56e-06	3.20e-04	1.57e-06	3.06e-04	$h \rightarrow ss$
1.70e-03	5.54e-01	3.98e-03	7.13e-01	3.29e-03	6.74e-01	3.46e-03	6.76e-01	$h \rightarrow bb$
8.65e-07	2.82e-04	8.65e-07	1.55e-04	9.27e-07	1.90e-04	9.31e-07	1.82e-04	$h \rightarrow \mu\mu$
2.71e-04	8.83e-02	2.71e-04	4.86e-02	2.62e-04	5.36e-02	2.63e-04	5.14e-02	$h \rightarrow \tau\tau$
1.06e-05	3.46e-03	1.06e-05	1.90e-03	9.29e-06	1.90e-03	9.29e-06	1.81e-03	$h \rightarrow \gamma\gamma$
1.71e-04	5.57e-02	2.82e-04	5.04e-02	2.84e-04	5.82e-02	2.84e-04	5.55e-02	$h \rightarrow gg$
6.95e-06	2.26e-03	6.95e-06	1.25e-03	6.06e-06	1.24e-03	6.29e-06	1.23e-03	$h \rightarrow Z\gamma$
7.23e-04	2.36e-01	7.23e-04	1.30e-01	7.23e-04	1.48e-01	7.87e-04	1.54e-01	$h \rightarrow WW^*$
8.13e-05	2.65e-02	8.13e-05	1.46e-02	8.13e-05	1.66e-02	8.56e-05	1.67e-02	$h \rightarrow ZZ^*$

Table 4: The  $h$  decay partial widths and branching ratios as output by SOFTSUSY without QCD corrections, with QCD corrections, with HDECAY's quark and gauge boson masses and gauge couplings and with QCD corrections, and the results of HDECAY-3.4 from SUSYHIT-1.4. This illustrates the necessity of including QCD corrections for decays to quarks and decays to gluons, as well as the fact that differences in mass choices are the primary source of differences between SOFTSUSY and HDECAY-3.4. This is for a CMSSM point with  $m_0 = 1000$  GeV,  $m_{1/2} = 1000$  GeV,  $\tan\beta = 10$ ,  $\text{sign}(\mu) = +1$ ,  $A_0 = 0$  and with  $m_h = 125.09$  GeV set artificially in the code.

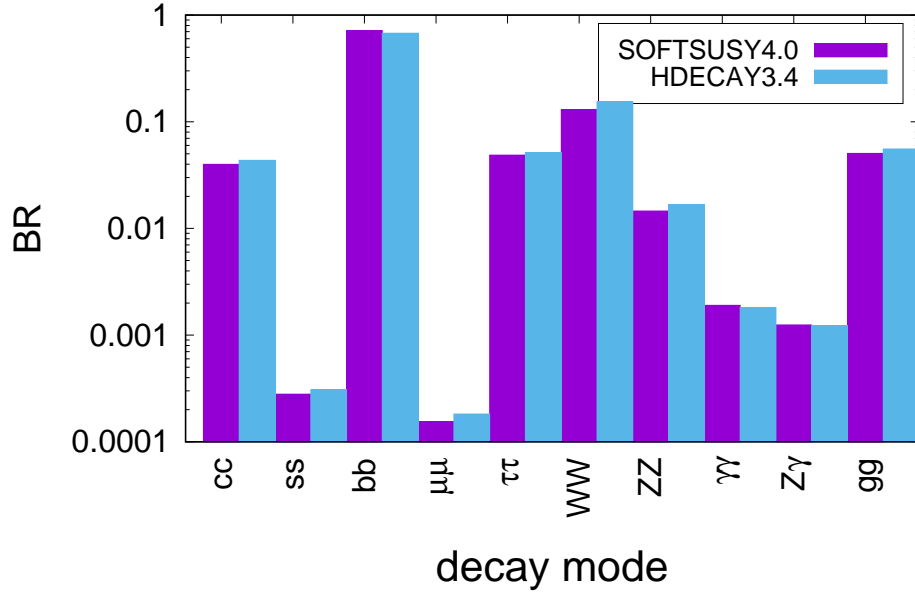


Figure 1: Branching ratios for a SM-like Higgs predicted by SOFTSUSY and by HDECAY-3.4 in SUSYHIT-1.4 for  $m_h = 125.09$  GeV. This is for a CMSSM point with  $m_0 = 1000$  GeV,  $m_{1/2} = 1000$  GeV,  $\tan\beta = 10$ ,  $\text{sign}(\mu) = +1$ ,  $A_0 = 0$  and with  $m_h = 125.09$  GeV set artificially in the code (i.e. identical to Table 4).

where order 10% differences are observed. This is due to HDECAY incorporating additional effects such as the width of the resonance and NLO corrections which are not included in SOFTSUSY. It should also be noted here that HDECAY performs a numerical integration whilst SOFTSUSY has an explicit expression with no integration required so the calculation methods are different. A comparison of the branching ratios output for this SM-like Higgs are given in Figure 1.

In order to provide a qualitative demonstration that the decay calculator is functioning correctly one may also scan the mass of the decaying particle and investigate how the partial widths and branching ratios change. Figure 2 shows how the BRs of an SM-like Higgs change as its mass is scanned from the  $Z^0$  boson mass up to 200 GeV as calculated in (a) SOFTSUSY and in (b) a well-known plot produced by the LHC Higgs Cross Section Working Group [44] in 2011. This shows good level of agreement, with small differences due to effects detailed previously in the quantitative comparison at  $m_h = 125.09$  GeV.

So far we have demonstrated the validation of SUSY 2 body and Higgs MSSM decays, including the loop-decays, QCD corrections and Higgs 3 body decays. Similar validation and comparison was also performed for the MSSM 3 body decays and the NMSSM decays.

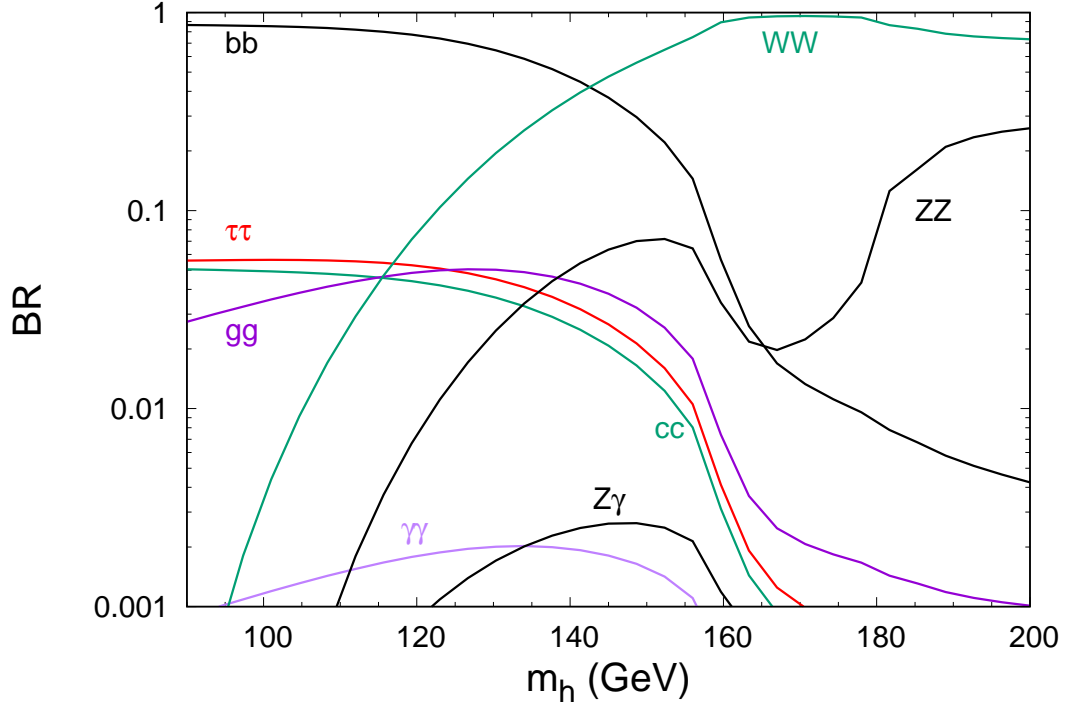
### 5.3. Three-Body Decays - $\tilde{g}$

First, consider the MSSM 3 body decays: an explicit comparison can be performed for the gluino 3 body decays with the spectrum given in Figure 3, the gluino three-body decays to neutralinos and quark anti-quark pairs are indicated. A comparison of the partial widths and branching ratios given by SOFTSUSY, sPHENO-3.3.8 and SUSYHIT-1.4 for this spectrum is presented in Table 5. This was performed taking the mass, coupling and other input decay parameters from sPHENO, as and inputting these directly by hand into the SOFTSUSY decay calculator in order to evaluate only differences due to the decay calculation, not any differences which might arise as a result of differing parameters from the spectrum generators. The agreement between the three programs is generally very good, in particular the agreement between SOFTSUSY and sPHENO-3.3.8, upon which the calculations of the 3 body decays is based, is usually between 1 and 5% with the larger differences often occurring where there are larger differences between SUSYHIT-1.4 and sPHENO-3.3.8. The exceptions to this are the decays to third generation quark-antiquark pairs and the third and fourth heaviest neutralinos; i.e.  $\tilde{g} \rightarrow t\bar{t}\tilde{Z}_3$ ,  $\tilde{g} \rightarrow t\bar{t}\tilde{Z}_4$ ,  $\tilde{g} \rightarrow b\bar{b}\tilde{Z}_3$  and  $\tilde{g} \rightarrow b\bar{b}\tilde{Z}_4$ . Here the differences observed are 10 – 20% and they arise because of differences in the Yukawa couplings taken, for example for the  $b$  quark here the Yukawa coupling used in SOFTSUSY is determined by a running bottom mass of  $\text{runmb}=2.63$  GeV, whereas sPHENO has a Yukawa coupling corresponding to a mass of  $\text{runmb}=2.37$  GeV. In order to show this results in the differences observed, the running  $b$  mass in SOFTSUSY was temporarily set to that of sPHENO and the comparison for  $\tilde{g} \rightarrow b\bar{b}\tilde{Z}_i$  is provided in Table 6. This demonstrates that the decays to  $\tilde{Z}_1$  and  $\tilde{Z}_2$  are not significantly altered by the new Yukawa coupling whereas the decays to  $\tilde{Z}_3$  and  $\tilde{Z}_4$  (i.e. those which showed differences with respect to sPHENO) now have significantly altered partial widths which are in much closer agreement with sPHENO, back down to the few percent level agreement seen in the other 3 body decays.

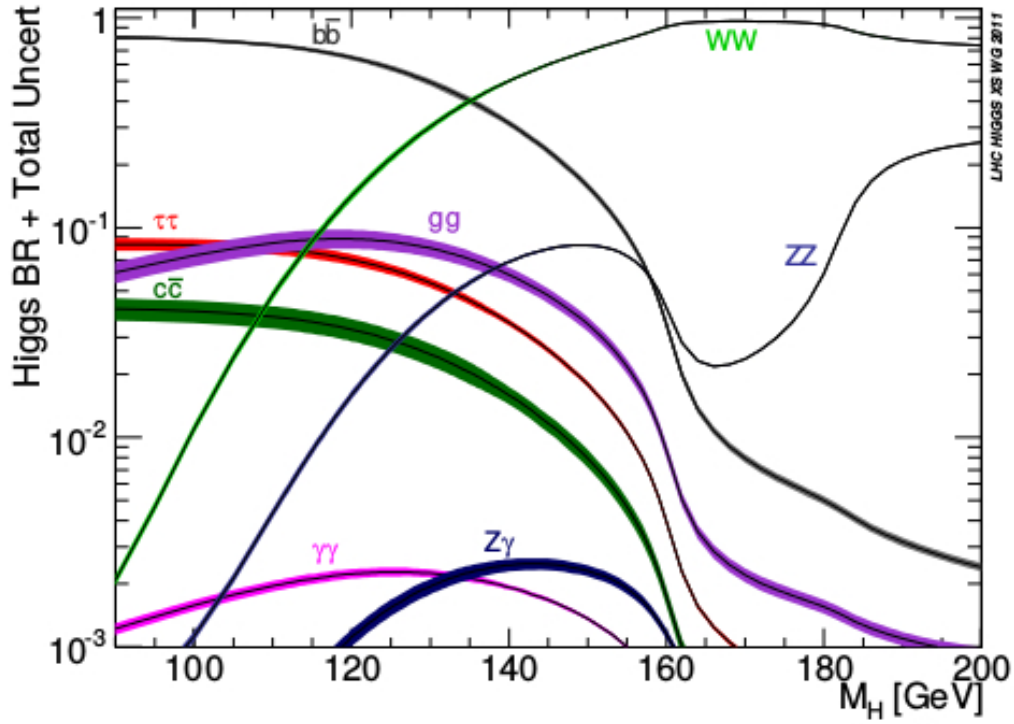
A scan over the mass of the gluino to demonstrate the expected suppression of 3 body decays relative to 2 body decays was also performed, see Figure 4. The result of this is that phenomenologically three body modes are only important when 2 body modes are unavailable. For this reason, SOFTSUSY only calculates 3 body modes when there are no two-body modes kinematically available and does not output the 3 body modes if the total 3 body decay width (sum of all the 3 body decay widths available) is less than the BR tolerance. More details on the other 3 body modes, the contributions included, approximations made and the level of agreement seen between SOFTSUSY and other decay calculators for each mode are given in Appendix G. There, the relevant expressions used by our decay calculator to determine their partial widths are also provided.

### 5.4. NMSSM Decays - $H_3$

Similar detailed checks were performed in the NMSSM sector and we provide some details here. For example, in Table 7 we present a quantitative comparison of the decays of the heaviest neutral CP even Higgs of the NMSSM,



(a) SOFTSUSY



(b) LHC Higgs Cross-Section Working Group

Figure 2: Branching ratios for an SM(-like) Higgs as calculated in (a) by SOFTSUSY and in (b) by the LHC Higgs Cross-Section Working Group in [44]. This demonstrates a verification of the partial widths output by SOFTSUSY.

SOFTSUSY		sPHENO-3.38		SUSYHIT-1.4		mode
PW/GeV	BR	PW/GeV	BR	PW/GeV	BR	
2.90e-04	2.26e-02	2.89e-04	2.32e-02	2.89e-04	2.32e-02	$\tilde{g} \rightarrow \tilde{Z}_1 u \bar{u}$
3.21e-04	2.51e-02	3.19e-04	2.56e-02	3.19e-04	2.56e-02	$\tilde{g} \rightarrow \tilde{Z}_2 u \bar{u}$
1.35e-07	1.06e-05	1.35e-07	1.08e-05	1.35e-07	1.08e-05	$\tilde{g} \rightarrow \tilde{Z}_3 u \bar{u}$
5.52e-06	4.31e-04	5.49e-06	4.40e-04	5.49e-06	4.40e-04	$\tilde{g} \rightarrow \tilde{Z}_4 u \bar{u}$
9.06e-05	7.07e-03	9.02e-05	7.22e-03	9.02e-05	7.24e-03	$\tilde{g} \rightarrow \tilde{Z}_1 d \bar{d}$
3.07e-04	2.40e-02	3.06e-04	2.45e-02	3.06e-04	2.45e-02	$\tilde{g} \rightarrow \tilde{Z}_2 d \bar{d}$
1.75e-07	1.36e-05	1.74e-07	1.39e-05	1.74e-07	1.40e-05	$\tilde{g} \rightarrow \tilde{Z}_3 d \bar{d}$
6.67e-06	5.21e-04	6.64e-06	5.31e-04	6.64e-06	5.33e-04	$\tilde{g} \rightarrow \tilde{Z}_4 d \bar{d}$
2.90e-04	2.26e-02	2.89e-04	2.32e-02	2.89e-04	2.32e-02	$\tilde{g} \rightarrow \tilde{Z}_1 c \bar{c}$
3.21e-04	2.51e-02	3.19e-04	2.56e-02	3.19e-04	2.56e-02	$\tilde{g} \rightarrow \tilde{Z}_2 c \bar{c}$
1.35e-07	1.05e-05	1.41e-07	1.13e-05	1.35e-07	1.08e-05	$\tilde{g} \rightarrow \tilde{Z}_3 c \bar{c}$
5.52e-06	4.31e-04	5.50e-06	4.40e-04	5.49e-06	4.40e-04	$\tilde{g} \rightarrow \tilde{Z}_4 c \bar{c}$
9.06e-05	7.07e-03	9.02e-05	7.22e-03	9.02e-05	7.24e-03	$\tilde{g} \rightarrow \tilde{Z}_1 s \bar{s}$
3.07e-04	2.40e-02	3.06e-04	2.45e-02	3.06e-04	2.45e-02	$\tilde{g} \rightarrow \tilde{Z}_2 s \bar{s}$
1.75e-07	1.36e-05	1.77e-07	1.42e-05	1.74e-07	1.40e-05	$\tilde{g} \rightarrow \tilde{Z}_3 s \bar{s}$
6.67e-06	5.21e-04	6.64e-06	5.32e-04	6.64e-06	5.33e-04	$\tilde{g} \rightarrow \tilde{Z}_4 s \bar{s}$
1.47e-03	1.15e-01	1.47e-03	1.17e-01	1.44e-03	1.15e-01	$\tilde{g} \rightarrow \tilde{Z}_1 t \bar{t}$
2.56e-04	1.99e-02	2.46e-04	1.97e-02	2.67e-04	2.15e-02	$\tilde{g} \rightarrow \tilde{Z}_2 t \bar{t}$
3.48e-04	2.71e-02	3.10e-04	2.48e-02	3.34e-04	2.68e-02	$\tilde{g} \rightarrow \tilde{Z}_3 t \bar{t}$
6.13e-04	4.79e-02	5.66e-04	4.53e-02	5.21e-04	4.18e-02	$\tilde{g} \rightarrow \tilde{Z}_4 t \bar{t}$
1.27e-04	9.93e-03	1.25e-04	1.00e-02	1.25e-04	1.00e-02	$\tilde{g} \rightarrow \tilde{Z}_1 b \bar{b}$
7.80e-04	6.09e-02	7.74e-04	6.20e-02	7.74e-04	6.21e-02	$\tilde{g} \rightarrow \tilde{Z}_2 b \bar{b}$
2.20e-05	1.72e-03	1.77e-05	1.42e-03	1.78e-05	1.43e-03	$\tilde{g} \rightarrow \tilde{Z}_3 b \bar{b}$
3.48e-05	2.72e-03	3.24e-05	2.60e-03	3.23e-05	2.60e-03	$\tilde{g} \rightarrow \tilde{Z}_4 b \bar{b}$
6.28e-04	4.90e-02	6.24e-04	5.00e-02	6.24e-04	5.01e-02	$\tilde{g} \rightarrow \tilde{W}_1^- u \bar{d}$
6.28e-04	4.90e-02	6.24e-04	5.00e-02	6.24e-04	5.01e-02	$\tilde{g} \rightarrow \tilde{W}_1^+ d \bar{u}$
6.28e-04	4.90e-02	6.24e-04	5.00e-02	6.24e-04	5.01e-02	$\tilde{g} \rightarrow \tilde{W}_1^- c \bar{s}$
6.28e-04	4.90e-02	6.24e-04	5.00e-02	6.24e-04	5.01e-02	$\tilde{g} \rightarrow \tilde{W}_1^+ s \bar{c}$
1.20e-05	9.36e-04	1.19e-05	9.56e-04	1.19e-05	9.58e-04	$\tilde{g} \rightarrow \tilde{W}_2^- u \bar{d}$
1.20e-05	9.36e-04	1.19e-05	9.56e-04	1.19e-05	9.58e-04	$\tilde{g} \rightarrow \tilde{W}_2^+ d \bar{u}$
1.20e-05	9.36e-04	1.19e-05	9.56e-04	1.19e-05	9.58e-04	$\tilde{g} \rightarrow \tilde{W}_2^- c \bar{s}$
1.20e-05	9.36e-04	1.19e-05	9.56e-04	1.19e-05	9.58e-04	$\tilde{g} \rightarrow \tilde{W}_2^+ s \bar{c}$
9.29e-04	7.25e-02	9.21e-04	7.38e-02	9.21e-04	7.39e-02	$\tilde{g} \rightarrow \tilde{W}_1^- t \bar{b}$
9.29e-04	7.25e-02	9.21e-04	7.38e-02	9.21e-04	7.39e-02	$\tilde{g} \rightarrow \tilde{W}_1^+ b \bar{t}$
1.35e-03	1.05e-01	1.27e-03	1.01e-01	1.27e-03	1.02e-01	$\tilde{g} \rightarrow \tilde{W}_2^- t \bar{b}$
1.35e-03	1.05e-01	1.27e-03	1.01e-01	1.27e-03	1.02e-01	$\tilde{g} \rightarrow \tilde{W}_2^+ b \bar{t}$

Table 5: The  $\tilde{g}$  decay partial widths and branching ratios as output by SOFTSUSY, sPHENO-3.3.8 [11] and SUSYHIT-1.4 [21] for the spectrum given in Figure 3, for which the gluino only has 3 body decay modes kinematically available. This CMSSM spectrum has  $m_0 = 1500$  GeV,  $m_{1/2} = 400$  GeV,  $\tan\beta = 10.37$ ,  $\text{sign}(\mu) = +1$ ,  $A_0 = -80$  GeV and was generated in sPHENO.

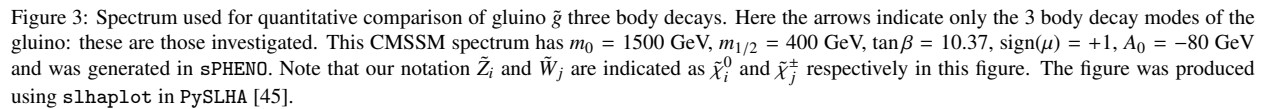


Table 6: The  $\tilde{g}$  decay partial widths and branching ratios to  $\tilde{Z}_i b \bar{b}$  as output by SOFTSUSY with `runmb` taken so that the  $b$  Yukawa coupling in SOFTSUSY matches that in sPHENO. These decays showed significant differences between the two programs, particularly for  $\tilde{Z}_3$  and  $\tilde{Z}_4$ , see Table 5. The agreement is now much improved, demonstrating that the differences result from a choice of the running  $b$  mass `runmb` and hence in the Yukawa coupling. This again used the parameters from sPHENO’s spectrum file directly in the SOFTSUSY decay calculator, with the addition relative to Table 5 of sPHENO’s running  $b$  mass.

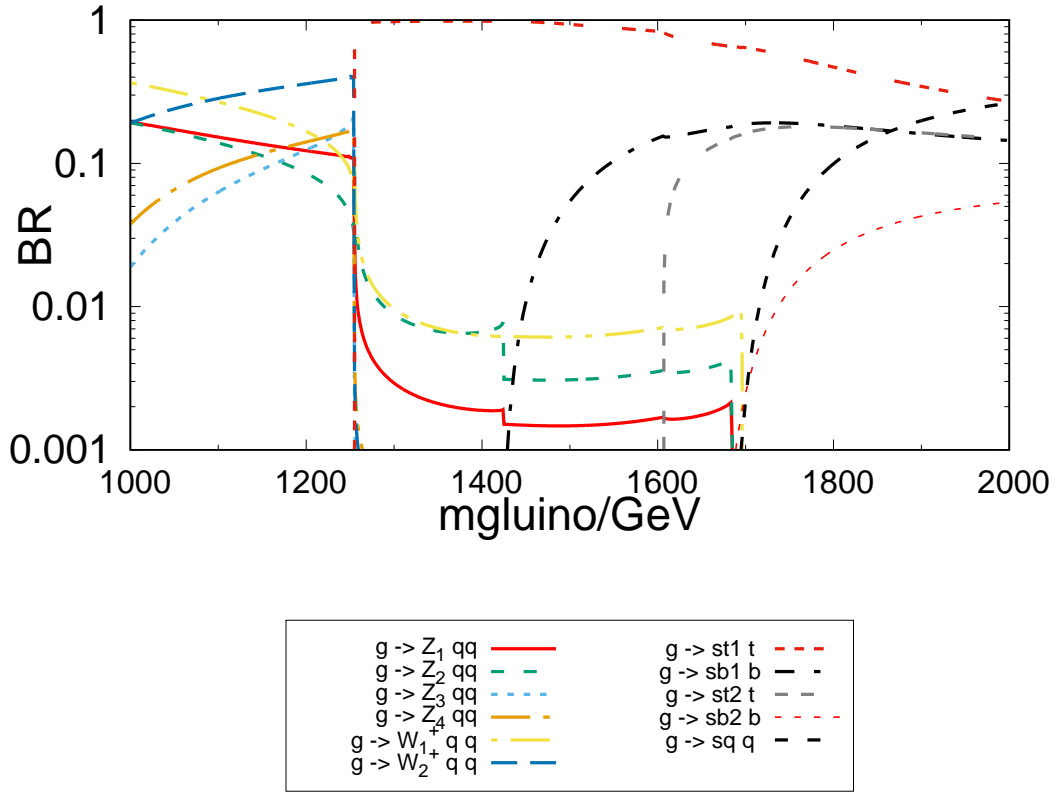


Figure 4: Branching ratios for the gluino  $\tilde{g}$  as it's mass is increased from 1 to 2 TeV. The suppression of 2 body modes relative to 3 body modes is clearly evident in the sudden drop in the 3 body branching ratios once the first 2 body mode  $\tilde{g} \rightarrow \tilde{t}_1 t$  is kinematically available. Note the “g” indicated in the plot are  $\tilde{g}$  (i.e. gluinos), whilst “Z<sub>i</sub>” are  $\tilde{Z}_i$  (i.e. neutralinos) and “W<sub>j</sub>” are  $\tilde{W}_j$  (i.e. charginos). “st” indicates stops  $\tilde{t}_i$ , “sb” indicate sbottoms  $\tilde{b}_i$ , “sq” are  $\tilde{q}$  squarks of the first two generations and “q” here are quarks of the first two generations.





SOFTSUSY with its quark masses and coupling constants run to $m_{H_3}$		SOFTSUSY with NMSSMTools-4.2.1 quark masses and running coupling constants		NMSSMTools-4.2.1		mode
PW/GeV	BR	PW/GeV	BR	PW/GeV	BR	
8.46e-06	2.55e-06	8.88e-06	2.84e-06	9.01e-06	2.82e-06	$H_3 \rightarrow c\bar{c}$
4.08e-04	1.23e-04	4.94e-04	1.58e-04	5.16e-04	1.62e-04	$H_3 \rightarrow s\bar{s}$
1.24e+00	3.72e-01	1.04e+00	3.32e-01	1.09e+00	3.40e-01	$H_3 \rightarrow b\bar{b}$
1.62e-01	4.88e-02	1.62e-01	5.18e-02	1.64e-01	5.14e-02	$H_3 \rightarrow t\bar{t}$
3.39e-04	1.02e-04	3.58e-04	1.15e-04	3.75e-04	1.18e-04	$H_3 \rightarrow \mu^-\mu^+$
1.03e-01	3.09e-02	1.01e-01	3.25e-02	1.06e-01	3.33e-02	$H_3 \rightarrow \tau^-\tau^+$
3.74e-02	1.13e-02	3.74e-02	1.20e-02	3.55e-02	1.11e-02	$H_3 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$
1.42e-01	4.27e-02	1.42e-01	4.54e-02	1.40e-01	4.38e-02	$H_3 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$
2.34e-03	7.05e-04	2.34e-03	7.50e-04	2.35e-03	7.35e-04	$H_3 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_3^0$
1.83e-01	5.51e-02	1.83e-01	5.86e-02	1.74e-01	5.45e-02	$H_3 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_4^0$
1.22e-01	3.68e-02	1.22e-01	3.92e-02	1.26e-01	3.94e-02	$H_3 \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0$
6.15e-03	1.85e-03	6.15e-03	1.97e-03	6.21e-03	1.95e-03	$H_3 \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_3^0$
2.81e-01	8.46e-02	2.81e-01	9.0e-02	2.89e-01	9.07e-02	$H_3 \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_4^0$
1.11e-04	3.35e-05	1.11e-04	3.56e-05	1.11e-04	3.48e-05	$H_3 \rightarrow \tilde{\chi}_3^0 \tilde{\chi}_3^0$
3.63e-01	1.09e-01	3.63e-01	1.16e-01	3.62e-01	1.13e-01	$H_3 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$
3.18e-01	9.58e-02	3.18e-01	1.02e-01	3.25e-01	1.02e-01	$H_3 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-$
3.18e-01	9.58e-02	3.18e-01	1.02e-01	3.25e-01	1.02e-01	$H_3 \rightarrow \tilde{\chi}_2^+ \tilde{\chi}_1^-$
6.97e-03	2.10e-03	6.97e-03	2.23e-03	7.11e-03	2.23e-03	$H_3 \rightarrow hh$
1.20e-05	3.63e-06	1.12e-05	3.58e-06	1.06e-05	3.31e-06	$H_3 \rightarrow \gamma\gamma$
8.81e-04	2.65e-04	7.61e-04	2.44e-04	8.00e-04	2.51e-04	$H_3 \rightarrow gg$
3.37e-05	1.02e-05	3.37e-05	1.08e-05	3.35e-05	1.05e-05	$H_3 \rightarrow Z\gamma$
1.42e-03	4.26e-04	<b>1.42e-03</b>	4.53e-04	<b>3.25e-03</b>	1.02e-03	$H_3 \rightarrow W^+W^-$
6.79e-04	2.04e-04	<b>6.79e-04</b>	2.17e-04	<b>1.56e-03</b>	4.89e-04	$H_3 \rightarrow ZZ$
2.79e-02	8.39e-03	2.79e-02	8.93e-03	2.61e-02	8.17e-03	$H_3 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_5^0$
8.61e-03	2.59e-03	8.61e-03	2.76e-03	8.89e-03	2.79e-03	$H_3 \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_5^0$
4.84e-05	1.46e-05	4.84e-05	1.55e-05	5.06e-05	1.59e-05	$H_3 \rightarrow hH$
3.32e+00	1.00e+00	3.12e+00	1.00e+00	3.19e+00	1.00e+00	Column totals

Table 7: The  $H_3$  decay partial widths and branching ratios as output by SOFTSUSY with the quark masses and coupling constants run to  $m_{H_3}$  as is the default, SOFTSUSY with the quark masses and coupling constants set temporarily to those of NMSSMTools for comparison of results, and NMSSMTools-4.2.1 [12, 24, 25]. For columns 3 and 4 this meant setting  $\alpha_{\text{phasAtMH3}} = 9.42 \times 10^{-2}$ ,  $m_b = 4.54$  GeV for  $H_3 \rightarrow b\bar{b}$ ,  $m_{\mu} = 0.106$  GeV for  $H_3 \rightarrow \mu\mu$ ,  $m_{\tau} = 1.78$  GeV for  $H_3 \rightarrow \tau\tau$ , using these same masses for  $H_3 \rightarrow \gamma\gamma$  as well as  $m_{\text{top}} = 170.9$  GeV and  $\alpha_{em} = 7.30 \times 10^{-3}$  (and the same values were used appropriately for  $H_3 \rightarrow gg$ ). The parameter point considered is the `nmssmSLHAnoZ3Input` file provided with the SOFTSUSY package. There is good agreement between the two programs with differences around 5 – 10%. Differences can be larger for the decays to quarks and 1-loop decays to  $\gamma\gamma$  and  $gg$  due to differences in the quark masses and coupling constants taken. Using the same values for these as in NMSSMTools significantly reduces these differences to around 5% or less, demonstrating this is the main source of the differences. This is illustrated in the improved agreement of the 3rd and 4th columns with the 5th and 6th columns. A more detailed description of the level of agreement and source of differences is given in the text.

the same package as the spectrum should make their calculation easier. The usual SLHA and SLHA2 conventions for input and output have been followed in order to facilitate ‘joining up’ SOFTSUSY with other observable calculating programs in a bug-free manner, for example with programs that perform Monte-Carlo event simulation.

## Acknowledgements

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## Appendix A. Running SOFTSUSY to Calculate Sparticle Decays

SOFTSUSY produces an executable called `softpoint.x`. It can be run by the command

```
./softpoint.x lesHouches < inOutFiles/lesHouchesInput
```

where the file `inOutFiles/lesHouchesInput` contains an ASCII file for input prepared in SUSY Les Houches Accord (SLHA) [33] or SLHA2 [34] format. A SOFTSUSY-specific Block of the SLHA input file is provided in the case that decays are required:

```
Block SOFTSUSY                # Optional SOFTSUSY-specific parameters
    0    1.000000000e+00        # Calculate decays in output (only for RPC (N)MSSM)
```

If this block is absent, or if the numerical value is instead `0.000000000e+00`, then decays will not be calculated. Other input options are available for changing the behaviour of the program as regards sparticle decays:

```
Block SOFTSUSY                # Optional SOFTSUSY-specific parameters
    24    1.000000000e-06        # If decay BR is below this number, don't output
    25    1.000000000e+00        # If set to 0, don't calculate 3-body decays (1=default)
    26    0.000000000e+00        # If set to 1, output partial widths (0=default)
```

Thus, parameter 24 under this block sets the smallest decay branching ratio that will be output (the default is, as listed,  $10^{-6}$ ) whereas parameter 25 can be used to instruct SOFTSUSY not to calculate the 3-body modes, which are more computationally intensive, requiring numerical integration. It should be noted that 3-body modes are not currently included in decays of the NMSSM so for such calculations, regardless of the value of 25, there will be no 3-body modes calculated. Parameter 26 is used to instruct SOFTSUSY to output the partial widths in addition to the branching ratios, this may be useful in making theoretical calculations or performing comparisons with other codes. These partial widths are output as a column beyond the comments # column so as to not interfere with the SLHA conventions.

If command-line input is required (see Refs. [2, 3] for a full description), the user can use the argument `--decays` to tell SOFTSUSY to perform the decay calculation. The command line argument `--minBR=<value>` tells SOFTSUSY which minimum branching ratio to print out in each decay table (where `<value>` is replaced by a numerical value between 0 and 1), whereas the command line argument `--dontCalculateThreeBody` tells SOFTSUSY *not* to calculate the 3-body decays of sparticles in order to save time. In addition the command line argument `--outputPartialWidths` is used to instruct SOFTSUSY to output the partial widths in the final column of the decay tables, beyond the comments so as to allow the output to be read directly into SLHA programs.

In the decay program SOFTSUSY itself there are also additional flags and switches which may be helpful to the user. There are flags at the start of the code named `flag<name>` where one must replace `<name>` with the particle name, when these flags have value 1 the particle decays are calculated, therefore by default all such flags are set to 1. These flags allow the user to turn off irrelevant decays for their analyses; for example in producing the scanning plots, such as Figure 4 in Section 5.3, all decays apart from those of the gluino  $\tilde{g}$  were turned off by setting all such flags to 0 apart from `flaggluino`, this ensured all decays output were gluino decays, allowing the plots to be produced more straightforwardly. Similarly there is a Boolean variable `QCDcorr`, which by default is `true`, which may be used to turn off QCD corrections. In case the user should want to run the parameters used to different scales, for example

#	PDG	Width	# Gluino decays			
DECAY	1000021	1.41325020e+01	# Gluino decays			
#	BR	NDA	PDG1	PDG2	Comments	PW
	2.13728751e-02	2	1	-1000001	# $\tilde{g} \rightarrow d \tilde{d}_L^*$	3.02052200e-01
	2.13728751e-02	2	-1	1000001	# $\tilde{g} \rightarrow db \tilde{d}_L$	3.02052200e-01
	4.53465485e-02	2	1	-2000001	# $\tilde{g} \rightarrow d \tilde{d}_R^*$	6.40860189e-01
	4.53465485e-02	2	-1	2000001	# $\tilde{g} \rightarrow db \tilde{d}_R$	6.40860189e-01
	2.26614800e-02	2	2	-1000002	# $\tilde{g} \rightarrow u \tilde{u}_L^*$	3.20263412e-01
	2.26614800e-02	2	-2	1000002	# $\tilde{g} \rightarrow ub \tilde{u}_L$	3.20263412e-01
	4.32957172e-02	2	2	-2000002	# $\tilde{g} \rightarrow u \tilde{u}_R^*$	6.11876811e-01
	4.32957172e-02	2	-2	2000002	# $\tilde{g} \rightarrow ub \tilde{u}_R$	6.11876811e-01
	2.13739410e-02	2	3	-1000003	# $\tilde{g} \rightarrow s \tilde{s}_L^*$	3.02067264e-01
	2.13739410e-02	2	-3	1000003	# $\tilde{g} \rightarrow sb \tilde{s}_L$	3.02067264e-01
	4.53479787e-02	2	3	-2000003	# $\tilde{g} \rightarrow s \tilde{s}_R^*$	6.40880401e-01
	4.53479787e-02	2	-3	2000003	# $\tilde{g} \rightarrow sb \tilde{s}_R$	6.40880401e-01
	2.26621068e-02	2	4	-1000004	# $\tilde{g} \rightarrow c \tilde{c}_L^*$	3.20272270e-01
	2.26621068e-02	2	-4	1000004	# $\tilde{g} \rightarrow cb \tilde{c}_L$	3.20272270e-01
	4.32969654e-02	2	4	-2000004	# $\tilde{g} \rightarrow c \tilde{c}_R^*$	6.11894452e-01
	4.32969654e-02	2	-4	2000004	# $\tilde{g} \rightarrow cb \tilde{c}_R$	6.11894452e-01
	7.37354606e-02	2	5	-1000005	# $\tilde{g} \rightarrow b \tilde{b}_1^*$	1.04206655e+00
	7.37354606e-02	2	-5	1000005	# $\tilde{g} \rightarrow bb \tilde{b}_1$	1.04206655e+00
	4.83477101e-02	2	5	-2000005	# $\tilde{g} \rightarrow b \tilde{b}_2^*$	6.83274111e-01
	4.83477101e-02	2	-5	2000005	# $\tilde{g} \rightarrow bb \tilde{b}_2$	6.83274111e-01
	1.12559217e-01	2	6	-1000006	# $\tilde{g} \rightarrow t \tilde{t}_1^*$	1.59074336e+00
	1.12559217e-01	2	-6	1000006	# $\tilde{g} \rightarrow tb \tilde{t}_1$	1.59074336e+00

[ ... ]

Table B.8: Sample Output produced by the SOFTSUSY decay calculator, here only the gluino,  $\tilde{g}$ , decay table is shown. The input file used was `lesHouchesInput` which is provided with the SOFTSUSY package and has  $m_0 = 125$  GeV,  $m_{1/2} = 500$  GeV,  $\tan\beta = 10$ ,  $\text{sign}(\mu) = +1$  and  $A_0 = 0$  in the CMSSM.

in performing comparisons with other decay calculators, it should be noted that running in SOFTSUSY is implemented using `MssmSoftsusy` and `NmssmSoftsusy` objects (detailed in references [2] and [3] respectively) and the `runto` command. If one alters the running scales within SOFTSUSY one must remember to instruct SOFTSUSY to recalculate the  $\overline{DR}$  parameters at this scale using `calcDRbarPars()`. Nonetheless, any changes made to the code are at the user's risk. Finally, given the dependence of many of the partial widths on the input parameters, and in particular on the quark masses used in the cases highlighted previously, users may wish to alter the quark masses  $m_q$ , this may be done in the `BLOCK SMINPUTS` of the SLHA/SLHA2 input file, the masses used within the decay calculator ("kinematic" and "running") will change accordingly.

## Appendix B. Sample Output

The output comes in the standard SLHA/SLHA2 format [33, 34]. A sample of the part relevant for decays is shown in Table B.8. In concordance with SLHA conventions, the widths/partial widths (PW) are output in units of GeV. The partial widths are output after the comments column so as not to interfere with reading in the decay tables into other programs; this outputting of partial widths can be switched off, as described in Appendix A, so the final column of partial widths in Table B.8 would then not appear. NDA lists the number of daughter particles and PDGi lists the Particle Data Group (PDG) code of daughter *i* (Section 43 of Ref. [46]). The comment at the end of each line after # lists the decay mode for easy perusal by the eye.

data variable	description
string name	particle name
double mass	particle mass
double PDG	particle PDG code
double No_of_Decays	Total Number of possible decays of particle
double No_1to2_Decays	Total Number of possible 2 body decays of particle
double No_1to3_Decays	Total Number of possible 3 body decays of particle
double No_grav_Decays	Total Number of possible decays of particle to LSP gravitinos
double No_NMSSM_Decays	Total Number of possible decays of particle in the NMSSM
double total_width	Total Decay Width of the particle
double two_width	Two-body decay partial width of the particle
double three_width	Three-body decay partial width of the particle
vector <vector<double>> Array_Decays	A Nx6 array, where N = No_of_Decays, containing the PDGs of the daughter particles in columns 0 and 1 (and 4 for 3-body decays), the partial widths of the decay modes in column 2, the number of daughters (NDA) in column 3 and the branching ratio for each decay mode in column 5.
vector <string> Array_Comments	A Nx1 array (vector), where N = No_of_Decays, containing a comment for each decay mode which is output with the decay table, detailing the decay mode involved, e.g. $\tilde{g} \rightarrow \bar{u}u_L$

Table C.9: The information contained in the `Particle` object for each of the decaying particles. PDG codes are given in the reference [34]. Note the numbers of decays contained in `double No_..._Decays` are the total number of such decays assuming none are kinematically forbidden. All these decays are checked by the program to see if they are allowed kinematically and calculated if so. All the numbers of decays are in the MSSM unless stated otherwise.

## Appendix C. Particle Class

As part of the extension of `SOFTSUSY` to include decay calculation a new class has been written, `Particle`. This is a container for all the relevant decay information of a particle and is used to output the decay tables. We display the class in Table C.9.

## Appendix D. Glossary: Reference Tables for Decays

We begin by listing the various decay modes included in `SOFTSUSY` in some tables, along with equation numbers for quick reference. We group the listings into several tables, each comprising a set of decays: in Table D.10, we list 2 body MSSM tree-level decays, in Table D.11, we list the MSSM Higgs boson decays, in Table D.12, the MSSM 3 body decays included, in Table D.13 we list the decays into gravitinos, in Table D.14 we list the NMSSM decays of neutralinos not already listed above and in Table D.15 we list NMSSM Higgs decay modes.

Gluino Decays		Slepton of first 2 generations decays	
$\tilde{g} \rightarrow q\tilde{q}_{L/R}$	F.1	$\tilde{l}_L \rightarrow l\tilde{Z}_i$	F.27
$\tilde{g} \rightarrow q\tilde{q}_{1/2}$	F.2	$\tilde{l}_R \rightarrow l\tilde{Z}_i$	F.28
<b>Squark of first 2 generations decays</b>		$\tilde{\nu}_l \rightarrow \nu_l\tilde{Z}_i$	F.29
$\tilde{q}_L \rightarrow \tilde{Z}_i q$	F.9	$\tilde{l}_L \rightarrow \nu_l\tilde{W}_j^-$	F.30
$\tilde{q}_{L/R} \rightarrow q\tilde{g}$	F.3	$\tilde{\nu}_l \rightarrow l\tilde{W}_j^+$	F.31
$\tilde{q}_L \rightarrow q'\tilde{W}_{1/2}^-$	F.5		
$\tilde{q}_R \rightarrow \tilde{Z}_i q$	F.10		
Squark of 3rd generation decays		Slepton of 3rd generation decays	
$\tilde{q}_{1/2} \rightarrow q\tilde{g}$	F.4	$\tilde{\tau}_{1/2} \rightarrow \tau\tilde{Z}_i$	F.32
$\tilde{b}_{1/2} \rightarrow \tilde{W}_j^- t$	F.6	$\tilde{\tau}_{1/2} \rightarrow \nu_\tau\tilde{W}_j^-$	F.36
$\tilde{t}_{1/2} \rightarrow \tilde{W}_j^+ b$	F.7	$\tilde{\nu}_\tau \rightarrow \tau\tilde{W}_j^+$	F.37
$\tilde{t}_{1/2} \rightarrow \tilde{Z}_i t$	F.11	$\tilde{\tau}_{1/2} \rightarrow \tilde{\nu}_\tau H^-$	F.38
$\tilde{b}_{1/2} \rightarrow \tilde{Z}_i b$	F.14	$\tilde{\tau}_{1/2} \rightarrow \tilde{\nu}_\tau W^-$	F.39
$\tilde{t}_{1/2} \rightarrow \tilde{b}_{1/2} W^+$	F.17	$\tilde{\nu}_\tau \rightarrow \tilde{Z}_i \nu_\tau$	F.29
$\tilde{t}_{1/2} \rightarrow \tilde{b}_{1/2} H^+$	F.18	$\tilde{\nu}_\tau \rightarrow \tilde{\tau}_{1/2} W^+$	F.39
$\tilde{t}_2 \rightarrow \phi\tilde{t}_1$	F.20	$\tilde{\nu}_\tau \rightarrow \tilde{\tau}_{1/2} H^+$	F.38
$\tilde{b}_2 \rightarrow \phi\tilde{b}_1$	F.23	$\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 Z$	F.40
$\tilde{q}_2 \rightarrow Z\tilde{q}_1$	F.26	$\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 \phi$	F.41
Chargino decays		Neutralino decays	
$\tilde{W}_i^+ \rightarrow \tilde{q}\tilde{q}'_L$	F.44	$\tilde{Z}_i \rightarrow \tilde{q}\tilde{q}'_{L/R}$	F.69
$\tilde{W}_i^+ \rightarrow \tilde{b}\tilde{t}_{1/2}$	F.45	$\tilde{Z}_i \rightarrow \tilde{l}\tilde{l}_{L/R}$	F.71
$\tilde{W}_i^+ \rightarrow \tilde{t}\tilde{b}_{1/2}$	F.48	$\tilde{Z}_i \rightarrow \tilde{t}\tilde{t}_{1/2}$	F.73
$\tilde{W}_i^+ \rightarrow \tilde{l}\tilde{l}_L$	F.52	$\tilde{Z}_i \rightarrow \tilde{b}\tilde{b}_{1/2}$	F.77
$\tilde{W}_i^+ \rightarrow \tilde{\tau}\tilde{\nu}_\tau$	F.54	$\tilde{Z}_i \rightarrow \tilde{\tau}\tilde{\tau}_{1/2}$	F.80
$\tilde{W}_i^+ \rightarrow \tilde{\tau}_{1/2}\nu_\tau$	F.57	$\tilde{Z}_i \rightarrow W\tilde{W}_{1/2}$	F.85
$\tilde{W}_1^+ \rightarrow W\tilde{Z}_j$	F.59	$\tilde{Z}_j \rightarrow H^+\tilde{W}_{1/2}$	F.87
$\tilde{W}_1^+ \rightarrow H^-\tilde{Z}_j$	F.61	$\tilde{Z}_i \rightarrow Z\tilde{Z}_j$	F.88
$\tilde{W}_2 \rightarrow Z\tilde{W}_1$	F.64	$\tilde{Z}_i \rightarrow h\tilde{Z}_j$	F.89
$\tilde{W}_2 \rightarrow \phi\tilde{W}_1$	F.66	$\tilde{Z}_i \rightarrow A\tilde{Z}_j$	F.91

Table D.10: MSSM 2 body decays included in the SOFTSUSY decay program, the references for the formulae in the appendices are given.  $\phi$  here is  $h/H/A$  i.e. any of the neutral Higgs bosons. The same references may be given for different decays in cases where the underlying formulae are the same and the necessary replacements for different outgoing particles are given with the formulae.

CP Even Higgs decays		CP Odd Higgs decays	
$h/H \rightarrow f\bar{f}$	F.93, J.1	$A \rightarrow f\bar{f}$	F.95, J.4
$h/H \rightarrow \tilde{Z}_i \tilde{Z}_j$	F.97	$A \rightarrow \tilde{Z}_i \tilde{Z}_j$	F.98
$h/H \rightarrow AZ$	F.107	$A \rightarrow hZ$	F.108
$h \rightarrow AA$	F.103	$A \rightarrow \tilde{f}_i \tilde{f}_j^*$	F.124
$H \rightarrow hh$	F.104	CP Even/Odd Higgs decays	
$H \rightarrow AA$	F.105	$\phi \rightarrow \tilde{W}_i^+ \tilde{W}_i^-$	F.99
$H \rightarrow H^+ H^-$	F.106	$\phi \rightarrow \tilde{W}_i^+ \tilde{W}_j^-$	F.101
$h \rightarrow \tilde{q}_{L/R} \tilde{q}_{L/R}^*$	F.109	$\phi \rightarrow \gamma\gamma$	F.145
$H \rightarrow \tilde{q}_{L/R} \tilde{q}_{L/R}^*$	F.111	$\phi \rightarrow gg$	F.190, J.10, J.11
$h \rightarrow \tilde{l}_{L/R} \tilde{l}_{L/R}^*$	F.113	$\phi \rightarrow Z\gamma$	F.191
$h \rightarrow \tilde{t}_i \tilde{t}_j^*$	F.115	Charged Higgs decays	
$h \rightarrow \tilde{b}_i \tilde{b}_j^*$	F.118	$H^+ \rightarrow f\bar{f}'$	F.126
$h \rightarrow \tilde{\tau}_i \tilde{\tau}_j^*$	F.121	$H^+ \rightarrow \tilde{Z}_i \tilde{W}_j$	F.127
$h/H \rightarrow ZZ^*$	F.139	$H^+ \rightarrow W^+ h$	F.134
$h/H \rightarrow WW^*$	F.140	$H^+ \rightarrow \tilde{f}_{L/R} \tilde{f}_{L/R}'$	F.135
$h/H \rightarrow WW$	F.143	$H^+ \rightarrow \tilde{f}_i \tilde{f}_j'$	F.137
$h/H \rightarrow ZZ$	F.144		

Table D.11: Higgs decays included in the SOFTSUSY decay program, the references for the formulae in the appendices are given. The same references may be given for different decays in cases where the underlying formulae are the same and the necessary replacements for different outgoing particles are described with the formulae. Multiple references are given for decays where QCD corrections are included, the first reference is the non-QCD corrected decay and the remainder are once QCD corrections are included.

Gluino $1 \rightarrow 3$ decays		Neutralino $1 \rightarrow 3$ decays	
$\tilde{g} \rightarrow q\bar{q}\tilde{Z}_i$	G.1	$\tilde{Z}_i \rightarrow \tilde{Z}_j f\bar{f}$	G.45
$\tilde{g} \rightarrow t\bar{t}\tilde{Z}_i$	G.6	$\tilde{Z}_i \rightarrow \tilde{W}_j f'\bar{f}$	G.176
$\tilde{g} \rightarrow b\bar{b}\tilde{Z}_i$	G.6	Chargino $1 \rightarrow 3$ decays	
$\tilde{g} \rightarrow q\bar{q}'\tilde{W}_i^-$	G.24	$\tilde{W}_j \rightarrow \tilde{Z}_i f'\bar{f}$	G.176
$\tilde{g} \rightarrow t\bar{b}\tilde{W}_i^-$	G.24		

Table D.12: Three body decays included in the SOFTSUSY decay program, the references for the formulae in the appendices are given. Not all 3 body decays are included as they are naturally suppressed with respect to the 2 body decays, for this reason we have aimed only to incorporate the most phenomenologically relevant 3 body decays, however more may be added in future versions. The same reference is given for neutralino decays to a chargino, fermion and anti-fermion as for the “reverse” decays of a chargino to a neutralino, fermion and anti-fermion as this just results in minus signs in several places in the partial width formulae, which are given in the appendix.

$\tilde{g} \rightarrow g\tilde{G}$	H.1	$\tilde{q} \rightarrow q\tilde{G}$	H.2
$\tilde{Z}_i \rightarrow \gamma\tilde{G}$	H.3	$\tilde{l} \rightarrow l\tilde{G}$	H.2
$\tilde{Z}_i \rightarrow \phi\tilde{G}$	H.5	$\tilde{Z}_i \rightarrow Z\tilde{G}$	H.4

Table D.13: The Next-to-Lightest Susy Particle (NLSP) decays to gravitinos included in the program along with the appendix references for their formulae.

Neutralino Decays		Decays into Neutralinos	
$\tilde{Z}_i \rightarrow \tilde{q}_{L/R} \bar{q}$	I.151	$\tilde{q}_{L/R} \rightarrow q \tilde{Z}_i$	I.176
$\tilde{Z}_i \rightarrow \tilde{t}_{1/2} \bar{t}$	I.155	$\tilde{t}_{1/2} \rightarrow t \tilde{Z}_i$	I.179
$\tilde{Z}_i \rightarrow \tilde{b}_{1/2} \bar{b}$	I.158	$\tilde{b}_{1/2} \rightarrow b \tilde{Z}_i$	I.182
$\tilde{Z}_i \rightarrow \tilde{\tau}_{1/2} \bar{\tau}$	I.161	$\tilde{\tau}_{1/2} \rightarrow \tau \tilde{Z}_i$	I.185
$\tilde{Z}_i \rightarrow W \tilde{W}_{1/2}$	I.164	$\tilde{\nu}_{\tau_{1/2}} \rightarrow \nu_{\tau} \tilde{Z}_i$	I.188
$\tilde{Z}_i \rightarrow H^{\pm} \tilde{W}_{1/2}$	I.167	$\tilde{W}_1 \rightarrow H^{\pm} \tilde{Z}_j$	I.189
$\tilde{Z}_i \rightarrow Z \tilde{Z}_j$	I.170	$\tilde{W}_1 \rightarrow W \tilde{Z}_j$	I.192
$\tilde{Z}_i \rightarrow h_k \tilde{Z}_j$	I.172		
$\tilde{Z}_i \rightarrow A_k \tilde{Z}_j$	I.174		

Table D.14: The NMSSM decays involving neutralinos that are included in the SOFTSUSY decay program. Note any decays not involving neutralinos or neutral Higgs Bosons are the same as in the MSSM.

CP Even Higgs Decays		CP Odd Higgs Decays	
$h_i \rightarrow f \bar{f}$	I.1, J.1	$A_i \rightarrow f \bar{f}$	I.95, J.4
$h_i \rightarrow \tilde{f}_{L/R} \tilde{f}_{L/R}$	I.3	$A_i \rightarrow \tilde{f}_L \tilde{f}_R$	I.97
$h_i \rightarrow \tilde{f}_{L/R} \tilde{f}_{R/L}$	I.6	$A_i \rightarrow \tilde{Z}_j \tilde{Z}_k$	I.99
$h_i \rightarrow \tilde{t}_j \tilde{t}_j$	I.8	$A_i \rightarrow \tilde{W}_j \tilde{W}_j$	I.101
$h_i \rightarrow \tilde{t}_1 \tilde{t}_2$	I.10	$A_i \rightarrow \tilde{W}_1 \tilde{W}_2$	I.103
$h_i \rightarrow \tilde{b}_j \tilde{b}_j$	I.8	$A_i \rightarrow h_j Z$	I.106
$h_i \rightarrow \tilde{b}_1 \tilde{b}_2$	I.10	$A_i \rightarrow H^{\pm} W$	I.108
$h_i \rightarrow \tilde{\tau}_j \tilde{\tau}_k$	I.14	$A_i \rightarrow \gamma \gamma$	I.112
$h_i \rightarrow \tilde{W}_j \tilde{W}_j$	I.18	$A_i \rightarrow Z \gamma$	I.118
$h_i \rightarrow \tilde{W}_1 \tilde{W}_2$	I.20	$A_i \rightarrow g g$	I.125, J.18
$h_i \rightarrow \tilde{Z}_j \tilde{Z}_k$	I.23	<b>Decays into Higgs Bosons</b>	
$h_i \rightarrow A_j A_k$	I.25	$\tilde{b}_2 \rightarrow \tilde{b}_1 h_i$	I.129
$h_i \rightarrow A_j Z$	I.27	$\tilde{t}_2 \rightarrow \tilde{t}_1 h_i$	I.133
$h_i \rightarrow H^+ H^-$	I.29	$\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 h_i$	I.137
$h_i \rightarrow W^+ H^-$	I.31	$\tilde{b}_2 \rightarrow \tilde{b}_1 A_i$	I.141
$h_i \rightarrow Z Z^*$	I.35	$\tilde{t}_2 \rightarrow \tilde{t}_1 A_i$	I.141
$h_i \rightarrow W W^*$	I.36	$\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 A_i$	I.141
$h_i \rightarrow Z Z$	I.35	$\tilde{W}_2 \rightarrow \tilde{W}_1 h_i$	I.145
$h_i \rightarrow W W$	I.36	$\tilde{W}_2 \rightarrow \tilde{W}_1 A_i$	I.148
$h_i \rightarrow \gamma \gamma$	I.44	$h_i \rightarrow h_j h_k$	I.32
$h_i \rightarrow g g$	I.83 J.15	$h_i \rightarrow h_j h_k$	I.32
$h_i \rightarrow Z \gamma$	I.71	$A_2 \rightarrow A h_i$	I.109

Table D.15: The NMSSM decays involving neutral Higgs Bosons that are included in the SOFTSUSY decay program, the references for the formulae in the appendices are given, where two references are given the first is for the leading order case and the second for the QCD-corrected case. Note any decays not involving neutralinos or neutral Higgs Bosons are the same as in the MSSM.



## Appendix E. Kinematic Functions

Here we begin the list of partial width expressions used in calculating the decay branching ratios in SOFTSUSY, we hope this provides a useful reference. With the exception of the 3 body decays, the majority of these widths were rederived as a form of validation.

The following are a list of commonly occurring functions that arise from the kinematics of the decays:  $\tilde{\lambda}^{1/2}$  appears as a result of the phase space integration:

$$\tilde{\lambda}^{1/2}(m_1, m_2, m_3) = \sqrt{(1 - (\frac{m_2 + m_3}{m_1})^2)(1 - (\frac{m_2 - m_3}{m_1})^2)} \quad (\text{E.1})$$

For loop integrals the real and imaginary parts of the loop give the following kinetic factor, where  $\tau_a = 4(\frac{m_a}{m_{h_i}})^2$ :

$$f(\tau) = \begin{cases} [\sin^{-1}(\frac{1}{\sqrt{\tau}})]^2, & \text{for } \tau \geq 1, \\ -\frac{1}{4}[\ln(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}) - i\pi]^2, & \text{for } \tau < 1, \end{cases} \quad (\text{E.2})$$

this is where the real and imaginary parts come from.

For the  $Z\gamma$  decay loops, the kinetic factor  $g(\tau)$  also occurs:

$$g(\tau) = \begin{cases} \sqrt{\tau - 1} \sin^{-1}(\frac{1}{\sqrt{\tau}}), & \text{for } \tau \geq 1, \\ \frac{1}{2} \sqrt{1 - \tau} [\ln(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}) - i\pi], & \text{for } \tau < 1 \end{cases} \quad (\text{E.3})$$

## Appendix F. MSSM Two Body Decay Formulae

Here we list for ease of reference the formulae for the partial widths of each of the  $1 \rightarrow 2$  decay modes incorporated into the decay calculator SOFTSUSY. The  $1 \rightarrow 2$  decay widths were all rederived, the book by Baer and Tata [39] was used as a guide, however differences exist relative to their formulae. The formulae provided in SUSYHIT [21, 22, 47] also provided a useful check.

### Appendix F.1. Gluinos

The partial widths for the decays of the gluinos to squarks and quarks are:

$$\Gamma(\tilde{g} \rightarrow q\tilde{q}_{L/R}) = \frac{\alpha_S}{4} \frac{1}{2m_{\tilde{g}}} (1 + \frac{m_q^2}{m_{\tilde{g}}^2} - \frac{m_{\tilde{q}_{L/R}}^2}{m_{\tilde{g}}^2}) \tilde{\lambda}^{1/2}(m_{\tilde{g}}, m_q, m_{\tilde{q}_{L/R}}), \quad (\text{F.1})$$

$$\Gamma(\tilde{g} \rightarrow q\tilde{q}_{1/2}) = \frac{\alpha_S}{4} \frac{1}{2m_{\tilde{g}}} [1 + \frac{m_q^2}{m_{\tilde{g}}^2} - \frac{m_{\tilde{q}_{L/R}}^2}{m_{\tilde{g}}^2} \mp 2 \sin 2\theta_q \frac{m_q}{m_{\tilde{g}}} ] \tilde{\lambda}^{1/2}(m_{\tilde{g}}, m_q, m_{\tilde{q}_{L/R}}), \quad (\text{F.2})$$

where the minus/plus sign applies for  $\tilde{q}_1/\tilde{q}_2$  respectively.

### Appendix F.2. Squarks

The partial widths for the decays of the squarks to quarks are:

$$\Gamma(\tilde{q}_{L/R} \rightarrow q\tilde{g}) = \frac{4\alpha_S}{3} \frac{1}{2m_{\tilde{q}_{L/R}}} (1 - \frac{m_q^2}{m_{\tilde{q}_{L/R}}^2} - \frac{m_{\tilde{g}}^2}{m_{\tilde{q}_{L/R}}^2}) \tilde{\lambda}^{1/2}(m_{\tilde{q}_{L/R}}, m_q, m_{\tilde{g}}), \quad (\text{F.3})$$

$$\Gamma(\tilde{q}_{1/2} \rightarrow q\tilde{g}) = \frac{4\alpha_S}{3} \frac{1}{2m_{\tilde{q}_{1/2}}} (1 - \frac{m_q^2}{m_{\tilde{q}_{1/2}}^2} - \frac{m_{\tilde{g}}^2}{m_{\tilde{q}_{1/2}}^2} \pm 2 \sin 2\theta_q \frac{m_q m_{\tilde{g}}}{m_{\tilde{q}_{1/2}}^2}) \tilde{\lambda}^{1/2}(m_{\tilde{q}_{1/2}}, m_q, m_{\tilde{g}}), \quad (\text{F.4})$$

where the minus/plus sign applies for  $\tilde{q}_1/\tilde{q}_2$  respectively.

$$\Gamma(\tilde{q}_L \rightarrow q' \tilde{W}_1^-) = \frac{g^2}{\sin^2 \theta_{L/R}} \frac{m_{\tilde{q}_L}}{16\pi} \left(1 - \frac{m_{\tilde{W}_1^-}^2}{m_{\tilde{q}_L}^2} - \frac{m_q^2}{m_{\tilde{q}_L}^2}\right) \tilde{\lambda}^{1/2}(m_{\tilde{q}_L}, m_q, m_{\tilde{W}_1^-}). \quad (\text{F.5})$$

Note here the ' indicates that the quark produced is the opposite type to the squark (so  $\tilde{d}_L$  produces an up quark for example) and  $\theta_L$  is for when up-type quarks (i.e. up or charm) are produced and  $\theta_R$  is for when down-type quarks are produced (i.e. down or strange). The expression (F.5) applies for the first two generations of quarks as no mixing has been accounted for. The formula for decay to  $\tilde{W}_2^-$  is similar but  $\sin \theta_{L/R} \rightarrow \cos \theta_{L/R}$ . The expressions with sfermion mixing, for the third generation of squarks, are given below.

$$\begin{aligned} \Gamma(\tilde{b}_1 \rightarrow \tilde{W}_1^- t) &= \frac{m_{\tilde{b}_1}}{16\pi} \left[ \{(-g \sin \theta_L \cos \theta_b + f_b \cos \theta_L \sin \theta_b)^2 + (-f_t \cos \theta_R \cos \theta_b)^2\} \right. \\ &\quad \times \left(1 - \frac{m_{\tilde{W}_1^-}^2}{m_{\tilde{b}_1}^2} - \frac{m_t^2}{m_{\tilde{b}_1}^2}\right) + 4 \frac{m_t m_{\tilde{W}_1^-}}{m_{\tilde{b}_1}^2} (-g \sin \theta_L \cos \theta_b + f_b \cos \theta_L \sin \theta_b) \\ &\quad \left. \times (-f_t \cos \theta_R \cos \theta_b) \right] \tilde{\lambda}^{1/2}(m_{\tilde{b}_1}, m_t, m_{\tilde{W}_1^-}), \end{aligned} \quad (\text{F.6})$$

$$\begin{aligned} \Gamma(\tilde{t}_1 \rightarrow \tilde{W}_1^+ b) &= \frac{m_{\tilde{t}_1}}{16\pi} \left[ \{(-g \sin \theta_R \cos \theta_t + f_t \cos \theta_R \sin \theta_t)^2 + (-f_b \cos \theta_L \cos \theta_t)^2\} \right. \\ &\quad \times \left(1 - \frac{m_{\tilde{W}_1^+}^2}{m_{\tilde{t}_1}^2} - \frac{m_b^2}{m_{\tilde{t}_1}^2}\right) + 4 \frac{m_b m_{\tilde{W}_1^+}}{m_{\tilde{t}_1}^2} (-g \sin \theta_R \cos \theta_t + f_t \cos \theta_R \sin \theta_t) \\ &\quad \left. \times (-f_b \cos \theta_L \cos \theta_t) \right] \tilde{\lambda}^{1/2}(m_{\tilde{t}_1}, m_b, m_{\tilde{W}_1^+}), \end{aligned} \quad (\text{F.7})$$

where

$$f_t = \frac{g m_t^{run}}{\sqrt{2} m_W \sin \beta}, \quad f_b = \frac{g m_b^{run}}{\sqrt{2} m_W \cos \beta}. \quad (\text{F.8})$$

(For decays of stops and sbottoms to  $\tilde{W}_2^+$ ,  $\sin \theta_{L/R} \rightarrow \cos \theta_{L/R}$  and  $\cos \theta_{L/R} \rightarrow -\sin \theta_{L/R}$ , and for decays of  $\tilde{b}_2$  and  $\tilde{t}_2$ ,  $\sin \theta_{t/b} \rightarrow \cos \theta_{t/b}$  and  $\cos \theta_{t/b} \rightarrow -\sin \theta_{t/b}$ .) The squark decays to neutralinos are given by:

$$\Gamma(\tilde{q}_L \rightarrow \tilde{Z}_i q) = \frac{1}{2} (\pm g N_{2i} + \frac{g'}{3} N_{1i})^2 \frac{m_{\tilde{q}_L}}{16\pi} \left(1 - \frac{m_{\tilde{Z}_i}^2}{m_{\tilde{q}_L}^2} - \frac{m_q^2}{m_{\tilde{q}_L}^2}\right) \tilde{\lambda}^{1/2}(m_{\tilde{q}_L}, m_q, m_{\tilde{Z}_i}), \quad (\text{F.9})$$

$$\Gamma(\tilde{q}_R \rightarrow \tilde{Z}_i q) = \frac{1}{2} (\frac{a}{3} g' N_{1i})^2 \frac{m_{\tilde{q}_R}}{16\pi} \left(1 - \frac{m_{\tilde{Z}_i}^2}{m_{\tilde{q}_R}^2} - \frac{m_q^2}{m_{\tilde{q}_R}^2}\right) \tilde{\lambda}^{1/2}(m_{\tilde{q}_R}, m_q, m_{\tilde{Z}_i}), \quad (\text{F.10})$$

where  $a = 4$  for up type squarks and  $a = -2$  for down type squarks).  $N_{ji}$  are neutralino mixing matrix elements. Decays of  $\tilde{t}_1$ ,  $\tilde{b}_1$ ,  $\tilde{t}_2$  and  $\tilde{b}_2$  are similar except for the mixing of the  $L$  and  $R$  parts so we get a linear combination of the two prefactors involving the neutralino mixing matrix elements  $N_{ji}$  with weights which are sines and cosines of the mixing angle  $\theta_{t/b}$ . In addition the Higgsino components of the neutralinos become important:

$$\Gamma(\tilde{t}_1 \rightarrow \tilde{Z}_i t) = \frac{m_{\tilde{t}_1}}{8\pi} \tilde{\lambda}^{1/2}(m_{\tilde{t}_1}, m_t, m_{\tilde{Z}_i}) \left[ \frac{1}{2} \left\{ a^2 \left(1 - \left(\frac{m_t + m_{\tilde{Z}_i}}{m_{\tilde{t}_1}}\right)^2\right) + b^2 \left(1 - \left(\frac{m_t - m_{\tilde{Z}_i}}{m_{\tilde{t}_1}}\right)^2\right) \right\} \right], \quad (\text{F.11})$$

where

$$a = \frac{1}{2} \left[ \frac{1}{\sqrt{2}} \cos \theta_t [-g N_{2i} - \frac{g'}{3} N_{1i}] - f_u \sin \theta_t N_{4i} + \frac{4}{3\sqrt{2}} g' N_{1i} \sin \theta_t - f_u N_{4i} \cos \theta_t \right], \quad (\text{F.12})$$

$$b = \frac{1}{2} \left[ \frac{1}{\sqrt{2}} \cos \theta_t [-g N_{2i} - \frac{g'}{3} N_{1i}] - f_u \sin \theta_t N_{4i} - \frac{4}{3\sqrt{2}} g' N_{1i} \sin \theta_t + f_u N_{4i} \cos \theta_t \right]. \quad (\text{F.13})$$

Whilst

$$\Gamma(\tilde{b}_1 \rightarrow \tilde{Z}_i b) = \frac{m_{\tilde{b}_1}}{8\pi} \tilde{\lambda}^{1/2}(m_{\tilde{b}_1}, m_b, m_{\tilde{Z}_i}) \left[ a^2 \left(1 - \left(\frac{m_{\tilde{Z}_i} + m_b}{m_{\tilde{b}_1}}\right)^2\right) + b^2 \left(1 - \left(\frac{m_{\tilde{Z}_i} - m_b}{m_{\tilde{b}_1}}\right)^2\right) \right], \quad (\text{F.14})$$

where

$$a = \frac{1}{2} \left[ \frac{1}{\sqrt{2}} (\cos \theta_b [-N_{1i} \frac{g'}{3} + N_{3i} g]) - \sin \theta_b N_{3i} f_d - \frac{2}{3\sqrt{2}} \sin \theta_b N_{1i} g' - \cos \theta_b f_d N_{3i} \right], \quad (\text{F.15})$$

$$b = \frac{1}{2} \left[ \frac{1}{\sqrt{2}} (\cos \theta_b [-N_{1i} \frac{g'}{3} + N_{3i} g]) - \sin \theta_b N_{3i} f_d + \frac{2}{3\sqrt{2}} \sin \theta_b N_{1i} g' + \cos \theta_b f_d N_{3i} \right]. \quad (\text{F.16})$$

As usual if we instead consider  $\tilde{q}_2$ , make the changes  $m_{q_1} \rightarrow m_{q_2}$ ,  $\cos \theta_q \rightarrow \sin \theta_q$  and  $\sin \theta_q \rightarrow -\cos \theta_q$ .

$$\Gamma(\tilde{t}_1 \rightarrow \tilde{b}_1 W^+) = \frac{g^2}{32\pi} \frac{m_{\tilde{t}_1}^3}{m_W^2} \tilde{\lambda}^{3/2}(m_{\tilde{t}_1}, m_W, m_{\tilde{b}_1}) \cos^2 \theta_t \cos^2 \theta_b. \quad (\text{F.17})$$

For  $\tilde{t}_2$ ,  $\cos \theta_t \rightarrow \sin \theta_t$ , whereas for  $\tilde{b}_2$  then  $\cos \theta_b \rightarrow \sin \theta_b$ . If the sbottoms are the initial states and stops are in the final state then exchange  $m_{\tilde{t}_i}$  and  $m_{\tilde{b}_i}$ . For the decays to charged Higgs bosons:

$$\Gamma(\tilde{t}_1 \rightarrow \tilde{b}_1 H^+) = \frac{g^2}{32\pi m_{\tilde{t}_1} m_W^2} A^2 \tilde{\lambda}^{1/2}(m_{\tilde{t}_1}, m_{H^+}, m_{\tilde{b}_1}), \quad (\text{F.18})$$

where here

$$A = m_t m_b (\tan \beta + \cot \beta) \sin \theta_t \sin \theta_b + m_t (\mu + A_t \cot \beta) \sin \theta_t \cos \theta_b + m_b (\mu + A_b \tan \beta) \sin \theta_b \cos \theta_t + (m_b^2 \tan \beta + m_t^2 \cot \beta - m_W^2 \sin 2\beta) \cos \theta_t \cos \theta_b. \quad (\text{F.19})$$

If instead we have  $\tilde{t}_2$  then  $\cos \theta_t \rightarrow \sin \theta_t$  and if we have  $\tilde{b}_2$  then  $\cos \theta_b \rightarrow \sin \theta_b$  and again if the sbottoms are the initial states and the stops are the final state then exchange  $m_{\tilde{t}_i}$  and  $m_{\tilde{b}_i}$ .

$$\Gamma(\tilde{t}_2 \rightarrow \phi \tilde{t}_1) = \frac{A_\phi^2}{16\pi m_{\tilde{t}_2}} \tilde{\lambda}^{1/2}(m_{\tilde{t}_2}, m_\phi, m_{\tilde{t}_1}), \quad (\text{F.20})$$

where

$$A_h = \frac{gm_W}{4} \sin(\beta + \alpha) \left[ 1 - \frac{5}{3} \frac{g'^2}{g^2} \right] \sin 2\theta_t + \frac{gm_t}{2m_W \sin \beta} \cos 2\theta_t (A_t \cos \alpha + \mu \sin \alpha), \quad (\text{F.21})$$

$A_H$  is similar but we must transform  $\cos \alpha \rightarrow -\sin \alpha$  and  $\sin \alpha \rightarrow \cos \alpha$ , whilst

$$A_A = \frac{gm_t}{2m_W} (A_t \cot \beta + \mu). \quad (\text{F.22})$$

For  $\tilde{b}_2$  decaying to a Higgs and  $\tilde{b}_1$ :

$$\Gamma(\tilde{b}_2 \rightarrow \phi \tilde{b}_1) = \frac{B_\phi^2}{16\pi m_{\tilde{b}_2}} \tilde{\lambda}^{1/2}(m_{\tilde{b}_2}, m_\phi, m_{\tilde{b}_1}), \quad (\text{F.23})$$

where

$$B_h = gm_W \sin(\alpha + \beta) \frac{1}{4} \left[ -1 + \frac{1}{3} \frac{g'^2}{g^2} \sin 2\theta_b \right] + gm_b \cos 2\theta_b \frac{1}{2m_W \cos \beta} [-A_b \sin \alpha - \mu \cos \alpha], \quad (\text{F.24})$$

$B_H$  is similar but again we must transform  $\cos \alpha \rightarrow -\sin \alpha$  and  $\sin \alpha \rightarrow \cos \alpha$ , whilst

$$B_A = \frac{gm_b}{2m_W} (A_b \tan \beta + \mu). \quad (\text{F.25})$$

For third generation squark decays to Z bosons we have the following (note that the amplitude is proportional to the sine squared of the mixing so this does not occur for the first two generations):

$$\Gamma(\tilde{q}_2 \rightarrow Z \tilde{q}_1) = \frac{g^2 m_{\tilde{q}_2}^3}{64\pi m_Z^2 \cos^2 \theta_W} \tilde{\lambda}^{3/2}(m_{\tilde{q}_2}, m_{\tilde{q}_1}, m_Z) \cos^2 \theta_q \sin^2 \theta_q. \quad (\text{F.26})$$

### Appendix F.3. Sleptons

$$\Gamma(\tilde{L}_L \rightarrow l\tilde{Z}_i) = \frac{1}{2}[gN_{2i} + g'N_{1i}]^2 \frac{m_{\tilde{L}_L}^2}{16\pi} \left(1 - \frac{m_{\tilde{Z}_i}^2}{m_{\tilde{L}_L}^2} - \frac{m_l^2}{m_{\tilde{L}_L}^2}\right) \tilde{\lambda}^{1/2}(m_{\tilde{L}_L}, m_l, m_{\tilde{Z}_i}). \quad (\text{F.27})$$

$$\Gamma(\tilde{L}_R \rightarrow l\tilde{Z}_i) = \frac{1}{2}[g'N_{1i}]^2 \frac{m_{\tilde{L}_R}^2}{16\pi} \left(1 - \frac{m_{\tilde{Z}_i}^2}{m_{\tilde{L}_R}^2} - \frac{m_l^2}{m_{\tilde{L}_R}^2}\right) \tilde{\lambda}^{1/2}(m_{\tilde{L}_R}, m_l, m_{\tilde{Z}_i}). \quad (\text{F.28})$$

$$\Gamma(\tilde{\nu}_l \rightarrow \nu_l \tilde{Z}_i) = \frac{1}{2}[gN_{2i} - g'N_{1i}]^2 \frac{m_{\tilde{\nu}_l}^2}{16\pi} \left(1 - \frac{m_{\tilde{Z}_i}^2}{m_{\tilde{\nu}_l}^2}\right)^2. \quad (\text{F.29})$$

$$\Gamma(\tilde{L}_L \rightarrow \nu_l \tilde{W}_1^-) = \frac{g^2 \sin^2 \theta_L}{16\pi} m_{\tilde{L}_L}^2 \left(1 - \frac{m_{\tilde{W}_1}^2}{m_{\tilde{L}_L}^2}\right)^2. \quad (\text{F.30})$$

For decays to  $\tilde{W}_2$  make the replacement  $\sin \theta_L \rightarrow \cos \theta_L$ .

$$\Gamma(\tilde{\nu}_l \rightarrow l\tilde{W}_1^+) = \frac{g^2 \sin^2 \theta_R}{16\pi} m_{\tilde{\nu}_l}^2 \left(1 - \frac{m_{\tilde{W}_1}^2}{m_{\tilde{\nu}_l}^2} - \frac{m_l^2}{m_{\tilde{\nu}_l}^2}\right)^2 \tilde{\lambda}^{1/2}(m_{\tilde{\nu}_l}, m_l, m_{\tilde{W}_1}). \quad (\text{F.31})$$

For decays to  $\tilde{W}_2$  make the replacement  $\sin \theta_R \rightarrow \cos \theta_R$ .

$$\Gamma(\tilde{\tau}_1 \rightarrow \tau \tilde{Z}_i) = \frac{m_{\tilde{\tau}_1}}{8\pi} \left[ a^2 \left(1 - \left(\frac{m_\tau + m_{\tilde{Z}_i}}{m_{\tilde{\tau}_1}}\right)^2\right) + b^2 \left(1 - \left(\frac{m_\tau - m_{\tilde{Z}_i}}{m_{\tilde{\tau}_1}}\right)^2\right) \right] \tilde{\lambda}^{1/2}(m_{\tilde{\tau}_1}, m_\tau, m_{\tilde{Z}_i}), \quad (\text{F.32})$$

where

$$a = \frac{1}{2} \left[ \frac{1}{\sqrt{2}} \sin \theta_\tau (gN_{2i} + g'N_{1i}) + f_\tau N_{3i} \cos \theta_\tau - \sqrt{2} g' N_{1i} \cos \theta_\tau + f_\tau N_{3i} \sin \theta_\tau \right], \quad (\text{F.33})$$

$$b = \frac{1}{2} \left[ \frac{1}{\sqrt{2}} \sin \theta_\tau (gN_{2i} + g'N_{1i}) + f_\tau N_{3i} \cos \theta_\tau + \sqrt{2} g' N_{1i} \cos \theta_\tau + f_\tau N_{3i} \sin \theta_\tau \right], \quad (\text{F.34})$$

and

$$f_\tau = \frac{gm_\tau}{\sqrt{2}m_W \cos \beta}. \quad (\text{F.35})$$

For  $\tilde{\tau}_2$  decaying replace  $m_{\tilde{\tau}_1} \rightarrow m_{\tilde{\tau}_2}$ ,  $\cos \theta_\tau \rightarrow \sin \theta_\tau$  and  $\sin \theta_\tau \rightarrow -\cos \theta_\tau$ .

$$\Gamma(\tilde{\tau}_1 \rightarrow \nu_\tau \tilde{W}_1^-) = [-g \sin \theta_L \sin \theta_\tau - f_\tau \cos \theta_L \cos \theta_\tau]^2 m_{\tilde{\tau}_1}^2 \left(1 - \frac{m_{\tilde{W}_1}^2}{m_{\tilde{\tau}_1}^2}\right)^2. \quad (\text{F.36})$$

For decays to  $\tilde{W}_2$  make the replacements  $m_{\tilde{W}_1} \rightarrow m_{\tilde{W}_2}$ ,  $\sin \theta_L \rightarrow \cos \theta_L$  and  $\cos \theta_L \rightarrow -\sin \theta_L$ , meanwhile for  $\tilde{\tau}_2$  decays change  $m_{\tilde{\tau}_1} \rightarrow m_{\tilde{\tau}_2}$ ,  $\cos \theta_\tau \rightarrow \sin \theta_\tau$  and  $\sin \theta_\tau \rightarrow -\cos \theta_\tau$ .

$$\begin{aligned} \Gamma(\tilde{\nu}_\tau \rightarrow \tau \tilde{W}_1^+) &= \frac{m_{\tilde{\nu}_\tau}}{16\pi} \left[ (g^2 \sin^2 \theta_R + f_\tau^2 \cos^2 \theta_L) \left(1 - \frac{m_{\tilde{W}_1}^2}{m_{\tilde{\nu}_\tau}^2} - \frac{m_\tau^2}{m_{\tilde{\nu}_\tau}^2}\right) \right. \\ &\quad \left. - 4 \frac{m_\tau m_{\tilde{W}_1}}{m_{\tilde{\nu}_\tau}^2} g \sin \theta_R f_\tau \cos \theta_L \right] \tilde{\lambda}^{1/2}(m_{\tilde{\nu}_\tau}, m_\tau, m_{\tilde{W}_1}). \end{aligned} \quad (\text{F.37})$$

For decays to  $\tilde{W}_2$  then make the replacements  $m_{\tilde{W}_1} \rightarrow m_{\tilde{W}_2}$ ,  $\sin \theta_R \rightarrow \cos \theta_R$  and  $\cos \theta_L \rightarrow -\sin \theta_L$ .

$$\Gamma(\tilde{\tau}_1 \rightarrow \tilde{\nu}_\tau H^-) = \frac{g^2}{32\pi m_W^2 m_{\tilde{\tau}_1}} \left[ m_\tau^2 \tan \beta \sin \theta_\tau - m_\tau (\mu + A_\tau \tan \beta) \cos \theta_\tau \right]^2 \tilde{\lambda}^{1/2}(m_{\tilde{\tau}_1}, m_{\tilde{\nu}_\tau}, m_H^-). \quad (\text{F.38})$$

For  $\tilde{\tau}_2$  decays then one must make the changes  $m_{\tilde{\tau}_1} \rightarrow m_{\tilde{\tau}_2}$ ,  $\cos \theta_\tau \rightarrow \sin \theta_\tau$  and  $\sin \theta_\tau \rightarrow -\cos \theta_\tau$  as usual.

$$\Gamma(\tilde{\tau}_1 \rightarrow \tilde{\nu}_\tau W^-) = \frac{g^2 \sin^2 \theta_\tau m_{\tilde{\tau}_1}^3}{32\pi m_W^2} \tilde{\lambda}^{3/2}(m_{\tilde{\tau}_1}, m_{\tilde{\nu}_\tau}, m_W). \quad (\text{F.39})$$

The equations for  $\tilde{\tau}_{1/2} \rightarrow \tilde{\nu}_\tau W^-$  and  $\tilde{\tau}_{1/2} \rightarrow \tilde{\nu}_\tau H^-$  can be used for  $\tilde{\nu}_\tau \rightarrow \tilde{\tau}_{1/2} W^-$  and  $\tilde{\nu}_\tau \rightarrow \tilde{\tau}_{1/2} H^-$  by interchanging the  $m_{\tilde{\tau}_{1/2}} \leftrightarrow m_{\tilde{\nu}_\tau}$ .

$$\Gamma(\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 Z) = \frac{g^2 \sin^2 \theta_\tau \cos^2 \theta_\tau m_{\tilde{\tau}_2}^3}{64\pi m_Z^2 \cos^2 \theta_W} \tilde{\lambda}^{3/2}(m_{\tilde{\tau}_2}, m_{\tilde{\tau}_1}, m_Z), \quad (\text{F.40})$$

$$\Gamma(\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 \phi) = \frac{\tilde{A}_\phi^2}{16\pi m_{\tilde{\tau}_2}} \tilde{\lambda}^{1/2}(m_{\tilde{\tau}_2}, m_{\tilde{\tau}_1}, m_\phi), \quad (\text{F.41})$$

where  $\tilde{A}_h$  is

$$\tilde{A}_h = \frac{-gm_w}{4} \sin(\alpha + \beta) \sin 2\theta_\tau \left[ -1 + 3 \frac{g'^2}{g^2} \right] + \frac{gm_\tau}{2m_W \cos \beta} \cos 2\theta_\tau (\mu \cos \alpha + A_\tau \sin \alpha), \quad (\text{F.42})$$

$\tilde{A}_H$  is the same as  $\tilde{A}_h$  but with the changes  $\cos \alpha \rightarrow -\sin \alpha$  and  $\sin \alpha \rightarrow \cos \alpha$ , meanwhile  $\tilde{A}_A$  is:

$$\tilde{A}_A = \frac{gm_\tau}{2m_W} (\mu + A_\tau \tan \beta). \quad (\text{F.43})$$

#### Appendix F.4. Charginos

$$\Gamma(\tilde{W}_1 \rightarrow \tilde{q} \tilde{q}'_L) = \frac{3m_{\tilde{W}_1}}{32\pi} (g^2 \sin^2 \theta_{L/R}) \left( 1 - \frac{m_{\tilde{q}_L}^2}{m_{\tilde{W}_1}^2} + \frac{m_q^2}{m_{\tilde{W}_1}^2} \right) \tilde{\lambda}^{1/2}(m_{\tilde{W}_1}, m_q, m_{\tilde{q}_L}). \quad (\text{F.44})$$

Note here the ' on the squark indicates it's of the opposite SU(2) type to the quark, e.g. if the quark is an up then the squark is a  $\tilde{d}_L$ . Also note that  $\theta_L$  occurs when up-type quarks (i.e. up or charm) are produced and  $\theta_R$  is when down-type quarks are produced (i.e. down or strange). (The formula for decay of  $\tilde{W}_2^-$  is similar but we must change  $\sin \theta_{L/R} \rightarrow \cos \theta_{L/R}$ ).

$$\Gamma(\tilde{W}_1^+ \rightarrow \tilde{b} \tilde{t}_1) = \frac{3m_{\tilde{W}_1}}{32\pi} \left[ (\mathcal{A}^2 + C^2 \sin^2 \theta_t) \left( 1 - \frac{m_{\tilde{t}_1}^2}{m_{\tilde{W}_1}^2} + \frac{m_b^2}{m_{\tilde{W}_1}^2} \right) + 4\mathcal{A}C \sin \theta_t \frac{m_b}{m_{\tilde{W}_1}} \right] \tilde{\lambda}^{1/2}(m_{\tilde{W}_1}, m_b, m_{\tilde{t}_1}), \quad (\text{F.45})$$

where

$$\mathcal{A} = g \sin \theta_R \cos \theta_t - f_u \cos \theta_R \sin \theta_t, \quad (\text{F.46})$$

$$C = -f_d \cos \theta_L. \quad (\text{F.47})$$

For  $\tilde{t}_2$  take  $\cos \theta_t \rightarrow \sin \theta_t$ ,  $\sin \theta_t \rightarrow -\cos \theta_t$  and  $m_{\tilde{t}_1} \rightarrow m_{\tilde{t}_2}$ . For  $\tilde{W}_2$  take  $\cos \theta_R \rightarrow -\sin \theta_R$ ,  $\cos \theta_L \rightarrow -\sin \theta_L$  and  $\sin \theta_R \rightarrow \cos \theta_R$ , and also  $m_{\tilde{W}_1} \rightarrow m_{\tilde{W}_2}$ .

$$\Gamma(\tilde{W}_1^+ \rightarrow \tilde{t} \tilde{b}_1) = \frac{3m_{\tilde{W}_1}}{32\pi} \left[ (\mathcal{A}^2 + C^2 \cos^2 \theta_b) \left( 1 - \frac{m_{\tilde{b}_1}^2}{m_{\tilde{W}_1}^2} + \frac{m_t^2}{m_{\tilde{W}_1}^2} \right) + 4\mathcal{A}C \cos \theta_b \frac{m_t}{m_{\tilde{W}_1}} \right] \tilde{\lambda}^{1/2}(m_{\tilde{W}_1}, m_t, m_{\tilde{b}_1}), \quad (\text{F.48})$$

where now

$$\mathcal{A} = -g \sin \theta_L \cos \theta_b + f_d \cos \theta_L \sin \theta_b, \quad (\text{F.49})$$

$$C = f_u \cos \theta_R, \quad (\text{F.50})$$

$$f_t = \frac{gm_t^{\text{run}}}{\sqrt{2}m_W \sin \beta}, \quad f_b = \frac{gm_b^{\text{run}}}{\sqrt{2}m_W \cos \beta}. \quad (\text{F.51})$$

For  $\tilde{b}_2$  take  $\cos \theta_b \rightarrow \sin \theta_b$ ,  $\sin \theta_b \rightarrow -\cos \theta_b$  and  $m_{\tilde{b}_1} \rightarrow m_{\tilde{b}_2}$ . For  $\tilde{W}_2$  take  $\cos \theta_R \rightarrow -\sin \theta_R$ ,  $\cos \theta_L \rightarrow -\sin \theta_L$  and  $\sin \theta_L \rightarrow \cos \theta_L$ , and also  $m_{\tilde{W}_1} \rightarrow m_{\tilde{W}_2}$ .

$$\Gamma(\tilde{W}_i^+ \rightarrow \tilde{l}l) = \frac{m_{\tilde{W}_i}}{32\pi} A^2 \left(1 - \frac{m_{\tilde{l}_L}^2}{m_{\tilde{W}_i}^2} + \frac{m_l^2}{m_{\tilde{W}_i}^2}\right) \tilde{\lambda}^{1/2}(m_{\tilde{W}_i}, m_l, m_{\tilde{l}_L}), \quad (\text{F.52})$$

where

$$A = \begin{cases} -g \sin \theta_{L/R}, & \text{for } \tilde{W}_1. \\ -g \cos \theta_{L/R}, & \text{for } \tilde{W}_2. \end{cases} \quad (\text{F.53})$$

$\theta_L$  is used for decays to  $\nu_l$  and  $\theta_R$  for decays to  $\tilde{\nu}_{l_L}$ .

$$\Gamma(\tilde{W}_i^+ \rightarrow \tilde{\tau}\nu_\tau) = \frac{m_{\tilde{W}_i}}{32\pi} \left[ (A^2 + B^2) \left(1 - \frac{m_{\tilde{\nu}_\tau}^2}{m_{\tilde{W}_i}^2} + \frac{m_\tau^2}{m_{\tilde{W}_i}^2}\right) + 4AB \frac{m_\tau}{m_{\tilde{W}_i}} \right] \tilde{\lambda}^{1/2}(m_{\tilde{W}_i}, m_\tau, m_{\tilde{\nu}_\tau}), \quad (\text{F.54})$$

where

$$A = \begin{cases} g \sin \theta_R, & \text{for } \tilde{W}_1, \\ g \cos \theta_R, & \text{for } \tilde{W}_2, \end{cases} \quad (\text{F.55})$$

$$B = \begin{cases} -f_\tau \cos \theta_L, & \text{for } \tilde{W}_1, \\ f_\tau \sin \theta_L, & \text{for } \tilde{W}_2, \end{cases} \quad (\text{F.56})$$

and  $f_\tau$  has been given before in (F.35).

$$\Gamma(\tilde{W}_i^+ \rightarrow \tilde{\tau}_1 \nu_\tau) = \frac{m_{\tilde{W}_i}}{32\pi} \mathcal{A}^2 \left(1 - \frac{m_{\tilde{\tau}_1}^2}{m_{\tilde{W}_i}^2}\right)^2, \quad (\text{F.57})$$

where

$$\mathcal{A} = -g \sin \theta_L \sin \theta_\tau - f_\tau \cos \theta_L \cos \theta_\tau. \quad (\text{F.58})$$

For  $\tilde{W}_2$  make the replacements  $\cos \theta_L \rightarrow -\sin \theta_L$ ,  $\sin \theta_L \rightarrow \cos \theta_L$  and  $m_{\tilde{W}_1} \rightarrow m_{\tilde{W}_2}$ . For  $\tilde{\tau}_2$  make the replacements  $\cos \theta_\tau \rightarrow \sin \theta_\tau$ ,  $\sin \theta_\tau \rightarrow -\cos \theta_\tau$  and  $m_{\tilde{\tau}_1} \rightarrow m_{\tilde{\tau}_2}$ .

$$\Gamma(\tilde{W}_1^+ \rightarrow W\tilde{Z}_j) = \frac{g^2}{16\pi|m_{\tilde{W}_1}|} \tilde{\lambda}^{1/2}(m_{\tilde{W}_1}, m_W, m_{\tilde{Z}_j}) \left[ (X^2 + Y^2) (m_{\tilde{W}_1}^2 + m_{\tilde{Z}_j}^2 - m_W^2) + \frac{1}{m_W^2} \{ (m_{\tilde{W}_1}^2 - m_{\tilde{Z}_j}^2)^2 - m_W^4 \} - 6(X^2 - Y^2) m_{\tilde{W}_1} m_{\tilde{Z}_j} \right], \quad (\text{F.59})$$

where

$$X = \frac{1}{2} [\cos \theta_R N_{4j} \frac{1}{\sqrt{2}} - \sin \theta_R N_{2j} - \cos \theta_L N_{3j} \frac{1}{\sqrt{2}} - \sin \theta_L N_{2j}]. \quad (\text{F.60})$$

$Y$  is the same as  $X$  except the first two terms change sign. For  $\tilde{W}_2$  transform  $\cos \theta_L \rightarrow -\sin \theta_L$ ,  $\sin \theta_L \rightarrow \cos \theta_L$ ,  $\cos \theta_R \rightarrow -\sin \theta_R$ ,  $\sin \theta_R \rightarrow \cos \theta_R$  and change  $m_{\tilde{W}_1} \rightarrow m_{\tilde{W}_2}$ .

$$\Gamma(\tilde{W}_1^+ \rightarrow H^+ \tilde{Z}_j) = \frac{1}{16\pi|m_{\tilde{W}_1}|} \tilde{\lambda}^{1/2}(m_{\tilde{W}_1}, m_{H^+}, m_{\tilde{Z}_j}) [(a^2 + b^2)(m_{\tilde{W}_1}^2 + m_{\tilde{Z}_j}^2 - m_{H^+}^2) + 2(a^2 - b^2)m_{\tilde{W}_1} m_{\tilde{Z}_j}], \quad (\text{F.61})$$

where

$$\begin{aligned} a &= \frac{1}{2} (-\cos \beta A_2 + \sin \beta A_4), \\ b &= \frac{1}{2} (-\cos \beta A_2 - \sin \beta A_4), \end{aligned} \quad (\text{F.62})$$

and

$$\begin{aligned} A_2 &= -\frac{1}{\sqrt{2}}[gN_{2j} + g'N_{1j}] \cos \theta_R - gN_{4j} \sin \theta_R, \\ A_4 &= -\frac{1}{\sqrt{2}}[gN_{2j} + g'N_{1j}] \cos \theta_L + gN_{3j} \sin \theta_L. \end{aligned} \quad (\text{F.63})$$

For  $\tilde{W}_2$  change  $\cos \theta_L \rightarrow -\sin \theta_L$ ,  $\sin \theta_L \rightarrow \cos \theta_L$ ,  $\cos \theta_R \rightarrow -\sin \theta_R$ ,  $\sin \theta_R \rightarrow \cos \theta_R$  and  $m_{\tilde{W}_1} \rightarrow m_{\tilde{W}_2}$ .

$$\begin{aligned} \Gamma(\tilde{W}_2 \rightarrow Z\tilde{W}_1) &= \frac{1}{64\pi m_{\tilde{W}_2}} \tilde{\lambda}^{1/2}(m_{\tilde{W}_2}, m_Z, m_{\tilde{W}_1}) \frac{(gg')^2}{g^2 + g'^2} \left[ \left( \frac{g}{g'} + \frac{g'}{g} \right)^2 [(x^2 + y^2)(m_{\tilde{W}_2}^2 + m_{\tilde{W}_1}^2 - m_Z^2 + \frac{1}{m_W^2}((m_{\tilde{W}_1}^2 - m_{\tilde{W}_2}^2)^2 - m_Z^4)) \right. \\ &\quad \left. + 6(x^2 - y^2)m_{\tilde{W}_1}m_{\tilde{W}_2}] \right], \end{aligned} \quad (\text{F.64})$$

where

$$x = \frac{1}{2}(\sin \theta_L \cos \theta_L - \sin \theta_R \cos \theta_R), \quad (\text{F.65})$$

and  $y$  is the same as  $x$  except the 2nd term changes sign.

$$\Gamma(\tilde{W}_2 \rightarrow \phi\tilde{W}_1) = \frac{g^2}{32\pi m_{\tilde{W}_2}} \tilde{\lambda}^{1/2}(m_{\tilde{W}_2}, m_\phi, m_{\tilde{W}_1}) \left[ (S_\phi^2 + P_\phi^2)(m_{\tilde{W}_2}^2 + m_{\tilde{W}_1}^2 - m_\phi^2) + 2(S_\phi^2 - P_\phi^2)m_{\tilde{W}_1}m_{\tilde{W}_2} \right], \quad (\text{F.66})$$

where

$$S_h = \frac{1}{2}(-\sin \theta_R \sin \theta_L \sin \alpha - \cos \theta_L \cos \theta_R \cos \alpha + \sin \theta_L \sin \theta_R \cos \alpha + \cos \theta_L \cos \theta_R \sin \alpha), \quad (\text{F.67})$$

$P_h$  is the same as  $S_h$  except the first and second terms gain an extra minus sign (become +).  $S_H$ ,  $P_H$  are the same as  $S_h$  and  $P_h$  if you take  $\sin \alpha \rightarrow -\cos \alpha$  and  $\cos \alpha \rightarrow \sin \alpha$ .

$$S_A = \frac{1}{2}(\sin \theta_R \sin \theta_L \sin \beta - \cos \theta_L \cos \theta_R \cos \beta - \sin \theta_L \sin \theta_R \cos \beta + \cos \theta_L \cos \theta_R \sin \beta). \quad (\text{F.68})$$

Again  $P_A$  is the same as  $S_A$  except the first two terms gain an additional minus sign.

#### Appendix F.5. Neutralinos

$$\Gamma(\tilde{Z}_i \rightarrow \tilde{u}\tilde{u}_{L/R}) = \frac{3C^2|m_{Z_i}|}{32\pi} \tilde{\lambda}^{1/2}(m_{Z_i}, m_u, m_{\tilde{u}_{L/R}}) \left( 1 + \frac{m_u^2}{m_{Z_i}^2} - \frac{m_{\tilde{u}_{L/R}}^2}{m_{Z_i}^2} \right), \quad (\text{F.69})$$

where

$$C = \begin{cases} \frac{1}{\sqrt{2}}(-gN_{2i} - \frac{g'}{3}N_{1i}), & \text{for } \tilde{u}_L, \\ \frac{-4}{3\sqrt{2}}g'N_{1i}, & \text{for } \tilde{u}_R. \end{cases} \quad (\text{F.70})$$

Neutralino decays to charm and  $\tilde{c}_{L/R}$  have a similar expression. For decays to down and  $\tilde{d}_{L/R}$  or to strange and  $\tilde{s}_{L/R}$  then  $C$  for the  $L$  component is the same as above except  $g \rightarrow -g$  and  $C$  for the  $R$  component has a factor of 2 rather than -4 in the numerator of the prefactor. The masses must also be changed appropriately. Note the difference between the LH and RH squark comes from the LH squark coupling to both the zino and wino components of the neutralinos whereas the RH squark couples only to the zino components.

$$\Gamma(\tilde{Z}_i \rightarrow \tilde{l}\tilde{l}_{L/R}) = \frac{C^2|m_{Z_i}|}{32\pi} \tilde{\lambda}^{1/2}(m_{Z_i}, m_l, m_{\tilde{l}_{L/R}}) \left( 1 + \frac{m_l^2}{m_{Z_i}^2} - \frac{m_{\tilde{l}_{L/R}}^2}{m_{Z_i}^2} \right), \quad (\text{F.71})$$

where

$$C = \begin{cases} \frac{1}{\sqrt{2}}(gN_{2i} + g'N_{1i}), & \text{for } \tilde{l}_L, \\ \sqrt{2}g'N_{1i}, & \text{for } \tilde{l}_R. \end{cases} \quad (\text{F.72})$$

Again the difference here between the  $L$  and  $R$  sleptons is due to the  $L$  sleptons coupling to the wino and zino components of the neutralinos whilst the  $R$  sleptons couple only to the zino components.

$$\Gamma(\tilde{Z}_i \rightarrow \tilde{t}\tilde{t}_1) = \frac{3|m_{\tilde{Z}_i}|}{16\pi} \tilde{\lambda}^{1/2}(m_{\tilde{Z}_i}, m_t, m_{\tilde{t}_1}) \left[ a^2 \left\{ \left(1 + \frac{m_t}{m_{\tilde{Z}_i}}\right)^2 - \left(\frac{m_{\tilde{t}_1}}{m_{\tilde{Z}_i}}\right)^2 \right\} + b^2 \left\{ \left(1 - \frac{m_t}{m_{\tilde{Z}_i}}\right)^2 - \left(\frac{m_{\tilde{t}_1}}{m_{\tilde{Z}_i}}\right)^2 \right\} \right], \quad (\text{F.73})$$

where

$$a = \frac{1}{2}(\alpha + \beta), \quad b = \frac{1}{2}(\alpha - \beta), \quad (\text{F.74})$$

$$\alpha = \cos \theta_t \frac{1}{\sqrt{2}} [-gN_{2i} - \frac{g'}{3}N_{1i}] - f_t \sin \theta_t N_{4i}, \quad (\text{F.75})$$

$$\beta = \frac{4}{3\sqrt{2}} g' N_{1i} \sin \theta_t - f_t \cos \theta_t N_{4i}. \quad (\text{F.76})$$

For  $\tilde{t}_2$  take  $\cos \theta_t \rightarrow \sin \theta_t$ ,  $\sin \theta_t \rightarrow -\cos \theta_t$  and  $m_{\tilde{t}_1} \rightarrow m_{\tilde{t}_2}$ .

$$\Gamma(\tilde{Z}_i \rightarrow \tilde{b}\tilde{b}_1) = \frac{3|m_{\tilde{Z}_i}|}{16\pi} \tilde{\lambda}^{1/2}(m_{\tilde{Z}_i}, m_b, m_{\tilde{b}_1}) \left[ a^2 \left\{ \left(1 + \frac{m_b}{m_{\tilde{Z}_i}}\right)^2 - \left(\frac{m_{\tilde{b}_1}}{m_{\tilde{Z}_i}}\right)^2 \right\} + b^2 \left\{ \left(1 - \frac{m_b}{m_{\tilde{Z}_i}}\right)^2 - \left(\frac{m_{\tilde{b}_1}}{m_{\tilde{Z}_i}}\right)^2 \right\} \right], \quad (\text{F.77})$$

where  $a$  and  $b$  are as before but the  $\alpha$  and  $\beta$  are different:

$$\alpha = \cos \theta_b \frac{1}{\sqrt{2}} [-\frac{g'}{3}N_{1i} + gN_{2i}] - f_b \sin \theta_b N_{3i}, \quad (\text{F.78})$$

$$\beta = -\sin \theta_b \frac{2}{3\sqrt{2}} g' N_{1i} - \cos \theta_b f_b N_{3i}. \quad (\text{F.79})$$

For  $\tilde{b}_2$  take  $\cos \theta_b \rightarrow \sin \theta_b$ ,  $\sin \theta_b \rightarrow -\cos \theta_b$  and  $m_{\tilde{b}_1} \rightarrow m_{\tilde{b}_2}$ .

$$\Gamma(\tilde{Z}_i \rightarrow \tilde{\tau}\tilde{\tau}_1) = \frac{|m_{\tilde{Z}_i}|}{16\pi} \tilde{\lambda}^{1/2}(m_{\tilde{Z}_i}, m_\tau, m_{\tilde{\tau}_1}) \left[ a^2 \left\{ \left(1 - \left(\frac{m_\tau + m_{\tilde{\tau}_1}}{m_{\tilde{Z}_i}}\right)^2 \right\} + b^2 \left\{ \left(1 - \left(\frac{m_\tau - m_{\tilde{\tau}_1}}{m_{\tilde{Z}_i}}\right)^2 \right\} \right\} \right], \quad (\text{F.80})$$

where

$$a = \frac{1}{2}(\alpha + \beta), \quad (\text{F.81})$$

$$b = \frac{1}{2}(\beta - \alpha), \quad (\text{F.82})$$

$$\alpha = \frac{1}{\sqrt{2}} \sin \theta_\tau [gN_{2i} + g'N_{1i}] + f_\tau N_{3i} \cos \theta_\tau, \quad (\text{F.83})$$

$$\beta = -\sqrt{2} g' N_{1i} \cos \theta_\tau + f_\tau N_{3i} \sin \theta_\tau. \quad (\text{F.84})$$

For  $\tilde{\tau}_2$  make the replacements  $\cos \theta_\tau \rightarrow \sin \theta_\tau$ ,  $\sin \theta_\tau \rightarrow -\cos \theta_\tau$  and  $m_{\tilde{\tau}_1} \rightarrow m_{\tilde{\tau}_2}$ .

$$\Gamma(\tilde{Z}_i \rightarrow W\tilde{W}_1) = \frac{g^2}{16\pi|m_{\tilde{Z}_i}|} \tilde{\lambda}^{1/2}(m_{\tilde{Z}_i}, m_W, m_{\tilde{W}_1}) \left[ (X^2 + Y^2) (m_{\tilde{Z}_i}^2 + m_{\tilde{W}_1}^2 - m_W^2 + \frac{1}{m_W^2} \{ (m_{\tilde{Z}_i}^2 - m_{\tilde{W}_1}^2)^2 - m_W^4 \}) - 6(X^2 - Y^2)m_{\tilde{Z}_i}m_{\tilde{W}_1} \right], \quad (\text{F.85})$$

where

$$X = \frac{1}{2} [\cos \theta_R N_{4i} \frac{1}{\sqrt{2}} - \sin \theta_R N_{2i} - \cos \theta_L N_{3i} \frac{1}{\sqrt{2}} - \sin \theta_L N_{2i}], \quad (\text{F.86})$$

and  $Y$  is the same as  $X$  except the first two terms get an extra minus sign. For  $\tilde{W}_2$  change  $\cos \theta_L \rightarrow -\sin \theta_L$ ,  $\sin \theta_L \rightarrow \cos \theta_L$ ,  $\cos \theta_R \rightarrow -\sin \theta_R$ ,  $\sin \theta_R \rightarrow \cos \theta_R$  and  $m_{\tilde{W}_1} \rightarrow m_{\tilde{W}_2}$ .

$$\Gamma(\tilde{Z}_j \rightarrow H^+\tilde{W}_1) = \frac{1}{16\pi|m_{\tilde{Z}_j}|} \tilde{\lambda}^{1/2}(m_{\tilde{Z}_j}, m_{H^+}, m_{\tilde{W}_1}) \left[ (a^2 + b^2)(m_{\tilde{Z}_j}^2 + m_{\tilde{W}_1}^2 - m_{H^+}^2) + 2(a^2 - b^2)m_{\tilde{Z}_j}m_{\tilde{W}_1} \right], \quad (\text{F.87})$$



where  $a$  and  $b$  and then  $A_2$  and  $A_4$  are exactly as given for the decay  $\tilde{W}_1 \rightarrow H^+ Z_j$  in (F.61).

$$\Gamma(\tilde{Z}_i \rightarrow Z\tilde{Z}_j) = \frac{g^2 + g'^2}{64\pi|m_{\tilde{Z}_i}|} \tilde{\lambda}^{1/2}(m_{\tilde{Z}_i}, m_Z, m_{\tilde{Z}_j}) \{N_{4i}N_{4j} - N_{3i}N_{3j}\}^2 \times \left[ m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2 - m_Z^2 + \frac{1}{m_Z^2} [(m_{\tilde{Z}_i}^2 - m_{\tilde{Z}_j}^2)^2 - m_Z^4] + 6m_{\tilde{Z}_i}m_{\tilde{Z}_j} \right]. \quad (\text{F.88})$$

$$\Gamma(\tilde{Z}_i \rightarrow h\tilde{Z}_j) = \frac{(X_{ij}^h + X_{ji}^h)^2}{16\pi|m_{\tilde{Z}_i}|} \tilde{\lambda}^{1/2}(m_{\tilde{Z}_i}, m_h, m_{\tilde{Z}_j}) [m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2 - m_h^2 + 2m_{\tilde{Z}_i}m_{\tilde{Z}_j}], \quad (\text{F.89})$$

where

$$X_{ij}^h = \frac{1}{2} [N_{3i} \sin \alpha + N_{4i} \cos \alpha] (-gN_{2j} + g'N_{1j}), \quad (\text{F.90})$$

$X_{ji}^h$  is the same but with  $i \leftrightarrow j$ . For  $\tilde{Z}_i \rightarrow H\tilde{Z}_j$  the formula is the same except one must change  $\sin \alpha \rightarrow -\cos \alpha$ ,  $\cos \alpha \rightarrow \sin \alpha$  and  $m_h \rightarrow m_H$ .

$$\Gamma(\tilde{Z}_i \rightarrow A\tilde{Z}_j) = \frac{(X_{ij}^A + X_{ji}^A)^2}{16\pi|m_{\tilde{Z}_i}|} \tilde{\lambda}^{1/2}(m_{\tilde{Z}_i}, m_A, m_{\tilde{Z}_j}) [m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2 - m_A^2 - 2m_{\tilde{Z}_i}m_{\tilde{Z}_j}], \quad (\text{F.91})$$

where

$$X_{ij}^A = \frac{1}{2} [N_{3i} \sin \beta - N_{4i} \cos \beta] (-gN_{2j} + g'N_{1j}), \quad (\text{F.92})$$

and  $X_{ji}^A$  is the same but with  $i \leftrightarrow j$ .

#### Appendix F.6. Higgs Sector

Once more, the partial widths for all of the Higgs decays incorporated into SOFTSUSY were rederived, including for the three-body and 1-loop decays, however the majority of them can also be found in “The Higgs Hunter’s Guide” [28].

$$\Gamma(h \rightarrow q\bar{q}) = \frac{3g^2 m_h}{32\pi} \left( \frac{m_q}{m_W} \right)^2 \left( 1 - 4 \frac{m_q^2}{m_h^2} \right)^{\frac{3}{2}} \mathcal{J}^2, \quad (\text{F.93})$$

where

$$\mathcal{J} = \begin{cases} \frac{\cos \alpha}{\sin \beta}, & \text{for up type quarks (u,c,t),} \\ \frac{\sin \alpha}{\cos \beta}, & \text{for down type quarks (d,s,b).} \end{cases} \quad (\text{F.94})$$

This is similar for  $H \rightarrow qq$  except we must make the replacements  $\sin \alpha \rightarrow -\cos \alpha$ ,  $\cos \alpha \rightarrow \sin \alpha$  and  $m_h \rightarrow m_H$ .

$$\Gamma(A \rightarrow q\bar{q}) = \frac{3g^2 \mathcal{J}_A^2}{32\pi} \left( \frac{m_q}{m_W} \right)^2 m_A \sqrt{1 - 4 \left( \frac{m_q}{m_A} \right)^2}, \quad (\text{F.95})$$

where

$$\mathcal{J}_A = \begin{cases} 1/(\tan \beta), & \text{for up type quarks (u,c,t),} \\ \tan \beta, & \text{for down type quarks (d,s,b).} \end{cases} \quad (\text{F.96})$$

$$\Gamma(h \rightarrow \tilde{Z}_i \tilde{Z}_j) = \frac{|m_h|}{8\pi} (X_{ij}^h + X_{ji}^h)^2 \tilde{\lambda}^{\frac{1}{2}}(m_h, m_{\tilde{Z}_i}, m_{\tilde{Z}_j}) \left( 1 - \left( \frac{m_{\tilde{Z}_i} + m_{\tilde{Z}_j}}{m_h} \right)^2 \right), \quad (\text{F.97})$$

with an extra prefactor of  $\frac{1}{2}$  if  $i = j$  (as the above formula includes a prefactor of 2 from  $\tilde{Z}_i \tilde{Z}_j$  being indistinguishable from  $\tilde{Z}_j \tilde{Z}_i$ ). Here  $X_{ij}^h$  is as in eq. (F.90) and  $X_{ji}^h$  is the same but with  $i \leftrightarrow j$ . Again similar formulae exist for  $H \rightarrow \tilde{Z}_i \tilde{Z}_j$  except we transform  $\sin \alpha \rightarrow -\cos \alpha$ ,  $\cos \alpha \rightarrow \sin \alpha$  and  $m_h \rightarrow m_H$ .

$$\Gamma(A \rightarrow \tilde{Z}_i \tilde{Z}_j) = \frac{|m_A|}{8\pi} (X_{ij}^A + X_{ji}^A)^2 \tilde{\lambda}^{\frac{1}{2}}(m_A, m_{\tilde{Z}_i}, m_{\tilde{Z}_j}) \left( 1 - \left( \frac{m_{\tilde{Z}_i} + m_{\tilde{Z}_j}}{m_A} \right)^2 \right), \quad (\text{F.98})$$

here  $X_{ij}^A$  is as given in eq. (F.92) and  $X_{ji}^A$  is the same but with  $i \leftrightarrow j$ .

$$\Gamma(\phi \rightarrow \tilde{W}_i^+ \tilde{W}_i^-) = \frac{g^2 |m_\phi|}{4\pi} S^2 \tilde{\lambda}^a(m_\phi, m_{\tilde{W}_i}, m_{\tilde{W}_i}), \quad (\text{F.99})$$

where  $a = 3/2$  for  $\phi = h, H$  or  $a = 1/2$  for  $A$  and  $S$  is given by:

$$S = \begin{cases} \frac{1}{2}(-\sin \alpha \sin \theta_R \cos \theta_L + \cos \alpha \sin \theta_L \cos \theta_R), & \text{for: } h \rightarrow \tilde{W}_1 \tilde{W}_1, \\ \frac{1}{2}(\sin \alpha \cos \theta_R \sin \theta_L - \cos \alpha \cos \theta_L \sin \theta_R), & \text{for: } h \rightarrow \tilde{W}_2 \tilde{W}_2, \\ \frac{1}{2}(\cos \alpha \sin \theta_R \cos \theta_L + \sin \alpha \sin \theta_L \cos \theta_R), & \text{for: } H \rightarrow \tilde{W}_1 \tilde{W}_1, \\ -\frac{1}{2}(\cos \alpha \cos \theta_R \sin \theta_L + \sin \alpha \cos \theta_L \sin \theta_R), & \text{for: } H \rightarrow \tilde{W}_2 \tilde{W}_2, \\ \frac{1}{2}(\sin \beta \sin \theta_R \cos \theta_L + \cos \beta \sin \theta_L \cos \theta_R), & \text{for: } A \rightarrow \tilde{W}_1 \tilde{W}_1, \\ -\frac{1}{2}(\sin \beta \cos \theta_R \sin \theta_L + \cos \beta \cos \theta_L \sin \theta_R), & \text{for: } A \rightarrow \tilde{W}_2 \tilde{W}_2. \end{cases} \quad (\text{F.100})$$

$$\Gamma(\phi \rightarrow \tilde{W}_i^+ \tilde{W}_j^-) = \frac{g^2}{16\pi |m_\phi|} \tilde{\lambda}^{\frac{1}{2}}(m_\phi, m_{\tilde{W}_i}, m_{\tilde{W}_j}) \left[ S^2 \left( 1 - \left( \frac{m_{\tilde{W}_i} + m_{\tilde{W}_j}}{m_\phi} \right)^2 \right) + P^2 \left( 1 - \left( \frac{m_{\tilde{W}_i} - m_{\tilde{W}_j}}{m_\phi} \right)^2 \right) \right], \quad (\text{F.101})$$

where

$$S = \begin{cases} \frac{1}{2}(\sin \alpha \sin \theta_R \sin \theta_L + \cos \alpha \cos \theta_L \cos \theta_R - \sin \theta_L \sin \theta_R \cos \alpha - \cos \theta_L \cos \theta_R \sin \alpha), & \text{for: } \phi = h, \\ \frac{1}{2}(-\cos \alpha \sin \theta_R \sin \theta_L + \sin \alpha \cos \theta_L \cos \theta_R - \sin \theta_L \sin \theta_R \sin \alpha + \cos \theta_L \cos \theta_R \cos \alpha), & \text{for: } \phi = H, \\ \frac{1}{2}(-\sin \beta \sin \theta_R \sin \theta_L + \cos \beta \cos \theta_L \cos \theta_R + \sin \theta_L \sin \theta_R \cos \beta - \cos \theta_L \cos \theta_R \sin \beta), & \text{for: } \phi = A. \end{cases} \quad (\text{F.102})$$

$P$  is the same as  $S$  except the signs of the first two terms are reversed.

$$\Gamma(h \rightarrow AA) = \frac{g^2 m_W^2}{128\pi |m_h| \cos^4(\theta_W)} \tilde{\lambda}^{\frac{1}{2}}(m_h, m_A, m_A) \sin^2(\alpha + \beta) \cos^2 2\beta. \quad (\text{F.103})$$

$$\Gamma(H \rightarrow hh) = \frac{g^2 m_W^2}{128\pi |m_H| \cos^4(\theta_W)} \tilde{\lambda}^{\frac{1}{2}}(m_H, m_h, m_h) \left[ \cos 2\alpha \cos(\alpha + \beta) - 2 \sin 2\alpha \sin(\alpha + \beta) \right]^2. \quad (\text{F.104})$$

$$\Gamma(H \rightarrow AA) = \frac{g^2 m_W^2}{128\pi |m_H| \cos^4(\theta_W)} \tilde{\lambda}^{\frac{1}{2}}(m_H, m_A, m_A) \cos^2 2\beta \cos^2(\alpha + \beta). \quad (\text{F.105})$$

$$\Gamma(H \rightarrow H^+ H^-) = \frac{g^2 m_W^2}{16\pi |m_H|} \tilde{\lambda}^{\frac{1}{2}}(m_H, m_{H^+}, m_{H^-}) \left[ \cos(\beta - \alpha) - \frac{\cos(\alpha + \beta) \cos 2\beta}{2 \cos^2 \theta_W} \right]^2. \quad (\text{F.106})$$

$$\Gamma(h \rightarrow AZ) = \frac{g^2 |m_h^3| \cos^2(\beta - \alpha)}{64\pi \cos^2 \theta_W m_Z^2} \tilde{\lambda}^{\frac{3}{2}}(m_h, m_Z, m_A). \quad (\text{F.107})$$

The decay  $H \rightarrow AZ$  follows the same formula but with the changes  $\cos(\beta - \alpha) \rightarrow \sin(\beta - \alpha)$  and  $m_h \rightarrow m_H$ .

$$\Gamma(A \rightarrow hZ) = \frac{g^2 |m_A^3| \cos^2(\beta - \alpha)}{64\pi \cos^2 \theta_W m_Z^2} \tilde{\lambda}^{\frac{3}{2}}(m_A, m_Z, m_h). \quad (\text{F.108})$$

The decay  $A \rightarrow HZ$  is not included as it's largely ruled out by SUSY mass constraints.

$$\Gamma(h \rightarrow \tilde{q}_{L/R} \tilde{q}_{L/R}^*) = \frac{3}{16\pi m_h} \tilde{\lambda}^{1/2}(m_h, m_{\tilde{q}_{L/R}}, m_{\tilde{q}_{L/R}}) C_{h\tilde{q}_{L/R}\tilde{q}_{L/R}}^2, \quad (\text{F.109})$$

where

$$C_{h\tilde{q}_{L/R}\tilde{q}_{L/R}} = \begin{cases} g \left[ m_W \left( \frac{1}{2} - \frac{1}{6} \frac{g'^2}{g^2} \right) \sin(\beta + \alpha) - \frac{m_u^2 \cos \alpha}{m_W \sin \beta} \right], & \text{for: } \tilde{u}_L \tilde{\bar{u}}_L, \\ g \left[ m_W \left( -\frac{1}{2} - \frac{1}{6} \frac{g'^2}{g^2} \right) \sin(\beta + \alpha) + \frac{m_d^2 \sin \alpha}{m_W \cos \beta} \right], & \text{for: } \tilde{d}_L \tilde{\bar{d}}_L, \\ g \left[ \frac{2}{3} m_W \frac{g'^2}{g^2} \sin(\beta + \alpha) - \frac{m_u^2 \cos \alpha}{m_W \sin \beta} \right], & \text{for: } \tilde{u}_R \tilde{\bar{u}}_R, \\ g \left[ \frac{-m_W}{3} \frac{g'^2}{g^2} \sin(\beta + \alpha) + \frac{m_d^2 \sin \alpha}{m_W \cos \beta} \right], & \text{for: } \tilde{d}_R \tilde{\bar{d}}_R, \\ \frac{g m_u}{2 m_W \sin \beta} (\mu \sin \alpha + A_u \cos \alpha), & \text{for: } \tilde{u}_L \tilde{\bar{u}}_R \text{ or } \tilde{u}_R \tilde{\bar{u}}_L, \\ \frac{g m_d}{2 m_W \cos \beta} (-\mu \cos \alpha - A_d \sin \alpha), & \text{for: } \tilde{d}_L \tilde{\bar{d}}_R \text{ or } \tilde{d}_R \tilde{\bar{d}}_L. \end{cases} \quad (\text{F.110})$$

$$\Gamma(H \rightarrow \tilde{q}_{L/R} \tilde{q}_{L/R}^*) = \frac{3}{16\pi m_H} \tilde{\lambda}^{1/2}(m_H, m_{\tilde{q}_{L/R}}, m_{\tilde{q}_{L/R}}) C_{H\tilde{q}_{L/R}\tilde{q}_{L/R}}^2, \quad (\text{F.111})$$

where

$$C_{H\tilde{q}_{L/R}\tilde{q}_{L/R}} = \begin{cases} g \left[ -m_W \left( \frac{1}{2} - \frac{1}{6} \frac{g'^2}{g^2} \right) \cos(\beta + \alpha) - \frac{m_u^2 \sin \alpha}{m_W \sin \beta} \right], & \text{for: } \tilde{u}_L \tilde{u}_L, \\ g \left[ m_W \left( \frac{1}{2} + \frac{1}{6} \frac{g'^2}{g^2} \right) \cos(\beta + \alpha) - \frac{m_d^2 \cos \alpha}{m_W \cos \beta} \right], & \text{for: } \tilde{d}_L \tilde{d}_L, \\ g \left[ \frac{-2m_W}{3} \frac{g'^2}{g^2} \cos(\beta + \alpha) - \frac{m_u^2 \sin \alpha}{m_W \cos \beta} \right], & \text{for: } \tilde{u}_R \tilde{u}_R, \\ g \left[ \frac{m_W}{3} \frac{g'^2}{g^2} \cos(\beta + \alpha) - \frac{m_d^2 \cos \alpha}{m_W \cos \beta} \right], & \text{for: } \tilde{d}_R \tilde{d}_R, \\ \frac{gm_u}{2m_W \sin \beta} (-\mu \cos \alpha + A_u \sin \alpha), & \text{for: } \tilde{u}_L \tilde{u}_R \text{ or } \tilde{u}_R \tilde{u}_L, \\ \frac{gm_d}{2m_W \cos \beta} (-\mu \sin \alpha + A_d \cos \alpha), & \text{for: } \tilde{d}_L \tilde{d}_R \text{ or } \tilde{d}_R \tilde{d}_L. \end{cases} \quad (\text{F.112})$$

$$\Gamma(h \rightarrow \tilde{l}_{L/R} \tilde{l}_{L/R}) = \frac{1}{16\pi m_h} \tilde{\lambda}^{1/2}(m_h, m_{\tilde{l}_{L/R}}, m_{\tilde{l}_{L/R}}) C_{h\tilde{l}_{L/R}\tilde{l}_{L/R}}^2, \quad (\text{F.113})$$

where

$$C_{h\tilde{l}_{L/R}\tilde{l}_{L/R}} = \begin{cases} g \left[ m_W \left( \frac{1}{2} + \frac{1}{2} \frac{g'^2}{g^2} \right) \right] \sin(\beta + \alpha), & \text{for } \tilde{\nu}_L \tilde{\nu}_L^*, \\ g \left[ m_W \left( -\frac{1}{2} + \frac{1}{2} \frac{g'^2}{g^2} \right) \sin(\alpha + \beta) + \frac{m_e^2 \sin \alpha}{m_W \cos \beta} \right], & \text{for } \tilde{e}_L \tilde{e}_L^*, \\ g \left[ -m_W \frac{g'^2}{g^2} \sin(\alpha + \beta) + \frac{m_e^2 \sin \alpha}{m_W \cos \beta} \right], & \text{for } \tilde{e}_R \tilde{e}_R^*, \\ \frac{gm_e}{2m_W \cos \beta} (-\mu \cos \alpha - A_e \sin \alpha), & \text{for } \tilde{e}_L \tilde{e}_R^* \text{ or } \tilde{e}_R \tilde{e}_L^*. \end{cases} \quad (\text{F.114})$$

For third generation sfermions, the formulae are more complicated as a result of sfermion mixing and Yukawa coupling effects:

$$\Gamma(h \rightarrow \tilde{t}_i \tilde{t}_j^*) = \frac{3}{16\pi m_h} \tilde{\lambda}^{1/2}(m_h, m_{\tilde{t}_i}, m_{\tilde{t}_j}) C_{h\tilde{t}_i \tilde{t}_j}^2, \quad (\text{F.115})$$

where here  $i$  and  $j$  can each be 1 or 2 independently of each other. The coupling depends on  $i$  and  $j$ , for  $\tilde{t}_1 \tilde{t}_1^*$  (i.e.  $i = j = 1$ ):

$$C_{h\tilde{t}_1 \tilde{t}_1} = \cos^2 \theta_t C_{h\tilde{t}_L \tilde{t}_L} + \sin^2 \theta_t C_{h\tilde{t}_R \tilde{t}_R} - 2 \sin \theta_t \cos \theta_t C_{h\tilde{t}_L \tilde{t}_R}, \quad (\text{F.116})$$

where  $C_{h\tilde{t}_L \tilde{t}_L}$ ,  $C_{h\tilde{t}_R \tilde{t}_R}$  and  $C_{h\tilde{t}_L \tilde{t}_R}$  are the corresponding couplings of  $\tilde{u}_L \tilde{u}_L$ ,  $\tilde{u}_R \tilde{u}_R$  and  $\tilde{u}_L \tilde{u}_R$ , respectively with the changes  $m_u \rightarrow m_t$  and  $A_u \rightarrow A_t$ . For  $\tilde{t}_2 \tilde{t}_2^*$  make the replacements  $\cos \theta_t \rightarrow \sin \theta_t$ ,  $\sin \theta_t \rightarrow -\cos \theta_t$ ,  $m_{\tilde{t}_1} \rightarrow m_{\tilde{t}_2}$ . For  $\tilde{t}_1 \tilde{t}_2^*$  or  $\tilde{t}_2 \tilde{t}_1^*$ :

$$C_{h\tilde{t}_1 \tilde{t}_2} = (C_{h\tilde{t}_L \tilde{t}_L} - C_{h\tilde{t}_R \tilde{t}_R}) \cos \theta_t \sin \theta_t + C_{h\tilde{t}_L \tilde{t}_R} \cos 2\theta_t. \quad (\text{F.117})$$

For  $H \rightarrow \tilde{t}_i \tilde{t}_j^*$  everything is as above but one must transform  $\sin \alpha \rightarrow -\cos \alpha$  and  $\cos \alpha \rightarrow \sin \alpha$  and  $m_h \rightarrow m_H$ .

$$\Gamma(h \rightarrow \tilde{b}_i \tilde{b}_j^*) = \frac{3}{16\pi m_h} \tilde{\lambda}^{1/2}(m_h, m_{\tilde{b}_i}, m_{\tilde{b}_j}) C_{h\tilde{b}_i \tilde{b}_j}^2. \quad (\text{F.118})$$

For  $\tilde{b}_1 \tilde{b}_1^*$ , i.e.  $i = j = 1$ :

$$C_{h\tilde{b}_1 \tilde{b}_1} = C_{h\tilde{b}_L \tilde{b}_L} \cos^2 \theta_b + C_{h\tilde{b}_R \tilde{b}_R} \sin^2 \theta_b - 2 \cos \theta_b \sin \theta_b C_{h\tilde{b}_L \tilde{b}_R}, \quad (\text{F.119})$$

where  $C_{h\tilde{b}_L \tilde{b}_L}$ ,  $C_{h\tilde{b}_R \tilde{b}_R}$  and  $C_{h\tilde{b}_L \tilde{b}_R}$  correspond to the couplings given for  $\tilde{d}_L \tilde{d}_L$ ,  $\tilde{d}_R \tilde{d}_R$  and  $\tilde{d}_L \tilde{d}_R$  with the changes  $m_d \rightarrow m_b$  and  $A_d \rightarrow A_b$ . For  $\tilde{b}_2 \tilde{b}_2^*$  it's the same except one must change  $\cos \theta_b \rightarrow \sin \theta_b$ ,  $\sin \theta_b \rightarrow -\cos \theta_b$ ,  $m_{\tilde{b}_1} \rightarrow m_{\tilde{b}_2}$ . For  $\tilde{b}_1 \tilde{b}_2^*$  or  $\tilde{b}_2 \tilde{b}_1^*$ :

$$C_{h\tilde{b}_1 \tilde{b}_2} = (C_{h\tilde{b}_L \tilde{b}_L} - C_{h\tilde{b}_R \tilde{b}_R}) \sin \theta_b \cos \theta_b + C_{h\tilde{b}_L \tilde{b}_R} \cos 2\theta_b. \quad (\text{F.120})$$

For  $H \rightarrow \tilde{b}_i \tilde{b}_j^*$  everything is as above with the replacements  $\sin \alpha \rightarrow -\cos \alpha$  and  $\cos \alpha \rightarrow \sin \alpha$  and  $m_h \rightarrow m_H$ .

$$\Gamma(h \rightarrow \tilde{\tau}_1 \tilde{\tau}_1^*) = \frac{1}{16\pi m_h} \tilde{\lambda}^{1/2} C_{h\tilde{\tau}_1 \tilde{\tau}_1}^2, \quad (\text{F.121})$$

where

$$C_{h\tilde{\tau}_1 \tilde{\tau}_1} = C_{h\tilde{\tau}_L \tilde{\tau}_L} \sin^2 \theta_\tau + C_{h\tilde{\tau}_R \tilde{\tau}_R} \cos^2 \theta_\tau + 2 \cos \theta_\tau \sin \theta_\tau C_{h\tilde{\tau}_L \tilde{\tau}_R}. \quad (\text{F.122})$$

$h \rightarrow \tilde{\tau}_2 \tilde{\tau}_2^*$  is the same with the replacements  $\cos \theta_\tau \rightarrow \sin \theta_\tau$ ,  $\sin \theta_\tau \rightarrow -\cos \theta_\tau$  and  $m_{\tilde{\tau}_1} \rightarrow m_{\tilde{\tau}_2}$ . For  $h \rightarrow \tilde{\tau}_1 \tilde{\tau}_2^*$  or  $\tilde{\tau}_2 \tilde{\tau}_1^*$  the coupling is instead given by:

$$C_{h\tilde{\tau}_1\tilde{\tau}_2} = (C_{h\tilde{\tau}_R\tilde{\tau}_R} - C_{h\tilde{\tau}_L\tilde{\tau}_L}) \cos \theta_\tau \sin \theta_\tau + C_{h\tilde{\tau}_L\tilde{\tau}_R} \cos 2\theta_\tau. \quad (\text{F.123})$$

$C_{h\tilde{\tau}_L\tilde{\tau}_L}$ ,  $C_{h\tilde{\tau}_R\tilde{\tau}_R}$  and  $C_{h\tilde{\tau}_L\tilde{\tau}_R}$  are identical to the corresponding couplings of  $\tilde{e}_L\tilde{e}_L^*$ ,  $\tilde{e}_R\tilde{e}_R^*$  and  $\tilde{e}_L\tilde{e}_R^*$  respectively, with the expected replacements. For  $H \rightarrow \tilde{\tau}_i \tilde{\tau}_j^*$  everything is as above with the changes  $\sin \alpha \rightarrow -\cos \alpha$ ,  $\cos \alpha \rightarrow \sin \alpha$  and  $m_h \rightarrow m_H$ .

$$\Gamma(A \rightarrow \tilde{f}_i \tilde{f}_j^*) = \frac{N_c}{16\pi m_A} \tilde{\lambda}^{1/2} C_{A\tilde{f}_i\tilde{f}_j}^2, \quad (\text{F.124})$$

note  $i \neq j$  by CP conservation, and  $N_c$  is 3 for squarks and 1 for sleptons. The coupling is given by:

$$C_{A\tilde{f}_i\tilde{f}_j} = \begin{cases} \frac{gm_f}{2m_W}(\mu + A_f \cot \beta), & \text{for } u\text{-type sfermions } \tilde{u}, \tilde{c}, \tilde{t}, \tilde{\nu}, \\ \frac{gm_f}{2m_W}(\mu + A_f \tan \beta), & \text{for } d\text{-type sfermions } \tilde{d}, \tilde{s}, \tilde{b}, \tilde{l}. \end{cases} \quad (\text{F.125})$$

$$\Gamma(H^+ \rightarrow q\bar{q}') = \frac{3g^2 CKM^2}{32\pi m_W^2 m_{H^+}} \tilde{\lambda}^{1/2}(m_{H^+}, m_{q_1}, m_{q_2}) \left\{ [m_{q_1}^2 \tan^2 \beta + \frac{m_{q_2}^2}{\tan^2 \beta}] (m_{H^+}^2 - m_{q_1}^2 - m_{q_2}^2) - 4m_{q_1}^2 m_{q_2}^2 \right\}, \quad (\text{F.126})$$

here  $m_{q_1}$  is the mass of the  $u$ -type quark and  $m_{q_2}$  is the mass of the  $d$ -type quark.

$$\Gamma(H^+ \rightarrow \tilde{Z}_i \tilde{W}_j) = \frac{1}{8\pi m_{H^+}} \tilde{\lambda}^{1/2}(m_{H^+}, m_{\tilde{Z}_i}, m_{\tilde{W}_j}) \left[ (a^2 + b^2)(m_{H^+}^2 - m_{\tilde{Z}_i}^2 - m_{\tilde{W}_j}^2) - 2(a^2 - b^2)m_{\tilde{Z}_i} m_{\tilde{W}_j} \right], \quad (\text{F.127})$$

where for  $j = 1$  i.e.  $\tilde{W}_1$ :

$$a = \frac{1}{2}(-\cos \beta A_2 + \sin \beta A_4), \quad b = \frac{1}{2}(-\cos \beta A_2 - \sin \beta A_4), \quad (\text{F.128})$$

for  $j = 2$  i.e.  $\tilde{W}_2$ :

$$a = \frac{1}{2}(-\cos \beta A_1 + \sin \beta A_3), \quad b = \frac{1}{2}(-\cos \beta A_1 - \sin \beta A_3). \quad (\text{F.129})$$

The  $A_i$  are:

$$A_1 = \frac{1}{\sqrt{2}}[gN_{2i} + g'N_{1i}] \sin \theta_R - gN_{4i} \cos \theta_R, \quad (\text{F.130})$$

$$A_2 = \frac{1}{\sqrt{2}}[-gN_{2i} - g'N_{1i}] \cos \theta_R - gN_{4i} \sin \theta_R, \quad (\text{F.131})$$

$$A_3 = \frac{1}{\sqrt{2}}[gN_{2i} + g'N_{1i}] \sin \theta_L + gN_{3i} \cos \theta_L, \quad (\text{F.132})$$

$$A_4 = \frac{1}{\sqrt{2}}[-gN_{2i} - g'N_{1i}] \cos \theta_L + gN_{3i} \sin \theta_L. \quad (\text{F.133})$$

$$\Gamma(H^+ \rightarrow W^+ h) = \frac{g^2 \cos^2(\beta - \alpha) m_{H^+}^3}{64\pi m_W^2} \tilde{\lambda}^{3/2}(m_{H^+}, m_W, m_h). \quad (\text{F.134})$$

$$\Gamma(H^+ \rightarrow \tilde{q}_{L/R} \tilde{q}'_{L/R}) = \frac{3B}{16\pi m_{H^+}} \tilde{\lambda}^{1/2}(m_{H^+}, m_{\tilde{q}_{L/R}}, m_{\tilde{q}'_{L/R}}), \quad (\text{F.135})$$

where  $B$  is the coupling and is given by:

$$B = \begin{cases} \frac{g}{\sqrt{2}} \left[ -m_W \sin 2\beta + \frac{1}{m_W} (m_d^2 \tan \beta + m_u \cot \beta) \right], & \text{for } \tilde{u}_L \tilde{d}_L, \\ g m_u m_d (\tan \beta + \cot \beta) \frac{1}{\sqrt{2} m_W}, & \text{for } \tilde{u}_R \tilde{d}_R, \\ \frac{-g m_d}{\sqrt{2} m_W} (A_d \tan \beta + \mu), & \text{for } \tilde{u}_L \tilde{d}_R, \\ \frac{-g m_u}{\sqrt{2} m_W} (A_u \cot \beta + \mu) & \text{for } \tilde{u}_R \tilde{d}_L. \end{cases} \quad (\text{F.136})$$

$$\Gamma(H^+ \rightarrow \tilde{q}_i \tilde{q}'_j) = \frac{3}{16\pi m_{H^+}} \tilde{\lambda}^{1/2}(m_{H^+}, m_{\tilde{q}_i}, m_{\tilde{q}'_j}) C^2, \quad (\text{F.137})$$

note  $q$  is the top squark and  $q'$  the bottom squark; for  $i = j = 1$  we have  $\tilde{t}_1 \tilde{b}_1$  and:

$$C = \cos \theta_t \cos \theta_b B_{\tilde{u}_L \tilde{d}_L} + \sin \theta_t \sin \theta_b B_{\tilde{u}_R \tilde{d}_R} - \cos \theta_t \sin \theta_b B_{\tilde{u}_L \tilde{d}_R} - \sin \theta_t \cos \theta_b B_{\tilde{u}_R \tilde{d}_L}, \quad (\text{F.138})$$

for a  $\tilde{b}_2$  we take  $\cos \theta_b \rightarrow \sin \theta_b$ ,  $\sin \theta_b \rightarrow -\cos \theta_b$ ; for a  $\tilde{t}_2$  we take  $\cos \theta_t \rightarrow \sin \theta_t$ ,  $\sin \theta_t \rightarrow -\cos \theta_t$ . Note, the same formulae as in eq.s (F.135) and (F.137) can be used for decays of  $H^\pm$  to sleptons, however for staus, because of the conventions used, one must use  $\theta_\tau - \pi/2$  which means the replacements  $\cos \theta_\tau \rightarrow \sin \theta_\tau$  and  $\sin \theta_\tau \rightarrow -\cos \theta_\tau$  are necessary in  $C$  in eq. (F.138).

Decays to two vector bosons are somewhat more complicated. Included in SOFTSUSY are the cases both where the Higgs has mass  $m_{h/H} > 2m_V$ , and so decays to two on-shell vector bosons, and also the case where the Higgs has mass  $m_V < m_{h/H} < 2m_V$ , so that it may only undergo a decay to one on-shell vector boson and one off-shell vector boson, which then decays into a fermion anti-fermion pair, i.e.  $h/H \rightarrow VV^* \rightarrow V f \bar{f}$ . This is technically a  $1 \rightarrow 3$  decay but is included here as it is computed exactly without the need for numerical integration, unlike the  $1 \rightarrow 3$  decays listed later. To obtain the formulae for  $h/H \rightarrow VV^*$ , one therefore sums over all possible outgoing  $f \bar{f}$  into which the  $V^*$  may decay. First consider the case where  $m_V < m_{h/H} < 2m_V$  so we have decays  $h/H \rightarrow VV^* \rightarrow V f \bar{f}$ , this is how the SM-like lightest Higgs,  $h$ , will decay:

$$\Gamma(h/H \rightarrow ZZ^*) = \frac{g^4 m_{h/H} c_{h/HVV}^2}{2048\pi^3} \cos^4 \theta_W F(\epsilon_Z) \left[ 7 - \frac{40}{3} \sin^2 \theta_W + \frac{160}{9} \sin^4 \theta_W \right], \quad (\text{F.139})$$

$$\Gamma(h/H \rightarrow WW^*) = \frac{3g^4 m_{h/H} c_{h/HVV}^2}{512\pi^3} F(\epsilon_W), \quad (\text{F.140})$$

here

$$\epsilon_V = \frac{m_V}{m_{h/H}}, \quad c_{hVV} = \sin(\beta - \alpha), \quad c_{HVV} = \cos(\beta - \alpha), \quad (\text{F.141})$$

and

$$F(\epsilon_V) = \frac{3(1 - 8\epsilon_V^2 + 20\epsilon_V^4)}{\sqrt{4\epsilon_V^2 - 1}} \cos^{-1} \left[ \frac{3\epsilon_V^2 - 1}{2\epsilon_V^3} \right] - (1 - \epsilon_V^2) \left( \frac{47}{2} \epsilon_V^2 - \frac{13}{2} + \frac{1}{\epsilon_V^2} \right) - 3(1 - 6\epsilon_V^2 + 4\epsilon_V^4) \log(\epsilon_V). \quad (\text{F.142})$$

If however  $m_{h/H} > 2m_V$  then the decay to two on-shell vector bosons occurs instead and the formulae are:

$$\Gamma(h/H \rightarrow WW) = \frac{g^2 m_{h/H}^3}{64\pi m_W^2} \tilde{\lambda}^{\frac{1}{2}}(m_{h/H}, m_W, m_W) (1 - r^2 + \frac{3}{4} r^4) c_{h/HWW}^2, \quad (\text{F.143})$$

$$\Gamma(h/H \rightarrow ZZ) = \frac{g^2 m_{h/H}^3}{128\pi m_W^2} \tilde{\lambda}^{\frac{1}{2}}(m_{h/H}, m_Z, m_Z) (1 - r^2 + \frac{3}{4} r^4) c_{h/HZZ}^2, \quad (\text{F.144})$$

where  $r = 2 \frac{m_V}{m_{h/H}}$ .

One loop decays to  $\gamma\gamma$ :

The function  $f(\tau)$  appears, it is given previously in eq. (E.2),

$$\Gamma(\phi \rightarrow \gamma\gamma) = \frac{g^2 \alpha_{em}^2 m_\phi^3}{1024\pi^3 m_W^2} \|\Sigma I_{loop}^\phi\|^2, \quad (\text{F.145})$$

where the  $I_{loop}^\phi$  are the amplitudes of the contributions of different particles in the loop to the decay  $\phi \rightarrow \gamma\gamma$ . The top contributions are:

$$I_t^h = \frac{4 \cos \alpha}{3 \sin \beta} \left[ -2\tau \{1 + (1 - \tau)f(\tau)\} \right], \quad (\text{F.146})$$

$$I_t^H = \frac{4}{3} \frac{\sin \alpha}{\sin \beta} \left[ -2\tau \{1 + (1 - \tau)f(\tau)\} \right], \quad (\text{F.147})$$

$$I_t^A = -\frac{8}{3} \tau f(\tau) \cot \beta. \quad (\text{F.148})$$

The stop contributions for  $h \rightarrow \gamma\gamma$  are:

$$I_{\tilde{t}_1}^h = \frac{4}{3} \tau (1 - \tau f(\tau)) \left[ R_{\tilde{t}_L \tilde{t}_L}^1 \cos^2 \theta_t + R_{\tilde{t}_R \tilde{t}_R}^1 \sin^2 \theta_t - 2 R_{\tilde{t}_L \tilde{t}_R}^1 \cos \theta_t \sin \theta_t \right], \quad (\text{F.149})$$

$$I_{\tilde{t}_2}^h = \frac{4}{3} \tau (1 - \tau f(\tau)) \left[ R_{\tilde{t}_L \tilde{t}_L}^2 \sin^2 \theta_t + R_{\tilde{t}_R \tilde{t}_R}^2 \cos^2 \theta_t + 2 R_{\tilde{t}_L \tilde{t}_R}^2 \cos \theta_t \sin \theta_t \right], \quad (\text{F.150})$$

where

$$R_{\tilde{t}_L \tilde{t}_L}^i = R_{\tilde{t}_L} \frac{m_W}{g m_{\tilde{t}_i}} = \left[ m_W \left( \frac{1}{2} - \frac{1}{6} \tan^2 \theta_W \right) \sin(\alpha + \beta) - \frac{m_t^2 \cos \alpha}{m_W \sin \beta} \right] \frac{m_W}{m_{\tilde{t}_i}}, \quad (\text{F.151})$$

$$R_{\tilde{t}_R \tilde{t}_R}^i = R_{\tilde{t}_R} \frac{m_W}{g m_{\tilde{t}_i}} = \left[ m_W \frac{2}{3} \tan^2 \theta_W \sin(\alpha + \beta) - \frac{m_t^2 \cos \alpha}{m_W \sin \beta} \right] \frac{m_W}{m_{\tilde{t}_i}}, \quad (\text{F.152})$$

$$R_{\tilde{t}_L \tilde{t}_R}^i = R_{\tilde{t}_L \tilde{t}_R} \frac{m_W}{g m_{\tilde{t}_i}} = \frac{m_t}{2 m_W \sin \beta} (\mu \sin \alpha + A_t \cos \alpha) \frac{m_W}{m_{\tilde{t}_i}}. \quad (\text{F.153})$$

The stop contributions for  $H$  are the same but the  $R_{\tilde{t}_L \tilde{t}_L}^i$ ,  $R_{\tilde{t}_R \tilde{t}_R}^i$ ,  $R_{\tilde{t}_L \tilde{t}_R}^i$  (note  $i = 1, 2$  it represents whether the loop is a  $\tilde{t}_1$  or a  $\tilde{t}_2$  loop) are different:

$$R_{\tilde{t}_L \tilde{t}_L}^i = \frac{m_W}{m_{\tilde{t}_i}} \left[ -m_W \left( \frac{1}{2} - \frac{1}{6} \tan^2 \theta_W \right) \cos(\alpha + \beta) - \frac{m_t^2 \sin \alpha}{m_W \sin \beta} \right], \quad (\text{F.154})$$

$$R_{\tilde{t}_R \tilde{t}_R}^i = \frac{m_W}{m_{\tilde{t}_i}} \left[ -m_W \frac{2}{3} \tan^2 \theta_W \cos(\alpha + \beta) - \frac{m_t^2 \sin \alpha}{m_W \sin \beta} \right], \quad (\text{F.155})$$

$$R_{\tilde{t}_L \tilde{t}_R}^i = \frac{m_W}{m_{\tilde{t}_i}} \frac{m_t}{2 m_W \sin \beta} (-\mu \cos \alpha + A_t \sin \alpha). \quad (\text{F.156})$$

$A \rightarrow \gamma\gamma$  has no stop loop contribution because of CP conservation, i.e.  $I_{\tilde{t}_{1/2}}^A = 0$ . The bottom contributions are:

$$I_b^h = -\frac{1}{3} \left[ -2\tau \{1 + (1 - \tau)f(\tau)\} \right] \frac{\sin \alpha}{\cos \beta}, \quad (\text{F.157})$$

$$I_b^H = \frac{1}{3} \left[ -2\tau \{1 + (1 - \tau)f(\tau)\} \right] \frac{\cos \alpha}{\cos \beta}, \quad (\text{F.158})$$

$$I_b^A = -\frac{1}{3} \{2\tau f(\tau)\} \tan \beta. \quad (\text{F.159})$$

The sbottom contributions to  $h \rightarrow \gamma\gamma$  are:

$$I_{\tilde{b}_1}^h = \frac{1}{3} \tau \{1 - \tau f(\tau)\} \left[ R_{\tilde{b}_L \tilde{b}_L}^1 \cos^2 \theta_b + R_{\tilde{b}_R \tilde{b}_R}^1 \sin^2 \theta_b - 2 \sin \theta_b \cos \theta_b R_{\tilde{b}_L \tilde{b}_R}^1 \right], \quad (\text{F.160})$$

$$I_{\tilde{b}_2}^h = \frac{1}{3} \tau \{1 - \tau f(\tau)\} \left[ R_{\tilde{b}_L \tilde{b}_L}^2 \sin^2 \theta_b + R_{\tilde{b}_R \tilde{b}_R}^2 \cos^2 \theta_b + 2 \sin \theta_b \cos \theta_b R_{\tilde{b}_L \tilde{b}_R}^2 \right], \quad (\text{F.161})$$

where here

$$R_{\tilde{b}_L \tilde{b}_L}^i = \frac{m_W}{m_{\tilde{b}_i}} \left[ m_W \left( -\frac{1}{2} - \frac{1}{6} \tan^2 \theta_W \right) \sin(\alpha + \beta) + \frac{m_b^2 \sin \alpha}{m_W \cos \beta} \right], \quad (\text{F.162})$$

$$R_{\tilde{b}_R \tilde{b}_R}^i = \frac{m_W}{m_{\tilde{b}_i}} \left[ -\frac{1}{3} m_W \tan^2 \theta_W \sin(\alpha + \beta) + \frac{m_b^2 \sin \alpha}{m_W \cos \beta} \right], \quad (\text{F.163})$$

$$R_{\tilde{b}_L \tilde{b}_R}^i = \frac{m_W}{m_{\tilde{b}_i}} \frac{m_b}{2 m_W \cos \beta} (-\mu \cos \alpha - A_b \sin \alpha). \quad (\text{F.164})$$

For  $H \rightarrow \gamma\gamma$  the sbottom contributions are the same except the  $R_{\tilde{b}_L\tilde{b}_L}^i, R_{\tilde{b}_R\tilde{b}_R}^i, R_{\tilde{b}_L\tilde{b}_R}^i$  change:

$$R_{\tilde{b}_L\tilde{b}_L}^i = \frac{m_W}{gm_{\tilde{b}_i}} g \left[ m_W \left( \frac{1}{2} + \frac{1}{6} \tan^2 \theta_W \right) \cos(\alpha + \beta) - \frac{m_b^2 \cos \alpha}{m_W \cos \beta} \right], \quad (\text{F.165})$$

$$R_{\tilde{b}_R\tilde{b}_R}^i = \frac{m_W}{gm_{\tilde{b}_i}} g \left[ m_W \left( \frac{1}{3} \tan^2 \theta_W \right) \cos(\alpha + \beta) - \frac{m_b^2 \cos \alpha}{m_W \cos \beta} \right], \quad (\text{F.166})$$

$$R_{\tilde{b}_L\tilde{b}_R}^i = \frac{m_W}{gm_{\tilde{b}_i}} g \frac{m_b}{2m_W \cos \beta} [-\mu \sin \alpha + A_b \cos \alpha]. \quad (\text{F.167})$$

$A \rightarrow \gamma\gamma$  has no sbottom loop contribution because of CP conservation, i.e.  $I_{\tilde{b}_{1/2}}^A = 0$ . The charm loop contributions are given by:

$$I_c^h = \frac{4}{3} \left[ -2\tau \{1 + (1 - \tau)f(\tau)\} \right] \frac{\cos \alpha}{\sin \beta}, \quad (\text{F.168})$$

$$I_c^H = \frac{4}{3} \left[ -2\tau \{1 + (1 - \tau)f(\tau)\} \right] \frac{\sin \alpha}{\sin \beta}, \quad (\text{F.169})$$

$$I_c^A = -\frac{4}{3} (2\tau f(\tau)) \cot \beta. \quad (\text{F.170})$$

$\tau$  loop contributions are given by:

$$I_\tau^h = 2\tau [1 + (1 - \tau)f(\tau)] \frac{\sin \alpha}{\cos \beta}, \quad (\text{F.171})$$

$$I_\tau^H = -2\tau [1 + (1 - \tau)f(\tau)] \frac{\cos \alpha}{\cos \beta}, \quad (\text{F.172})$$

$$I_\tau^A = -2\tau f(\tau) \tan \beta. \quad (\text{F.173})$$

$\tilde{\tau}_i$  contributions to  $h \rightarrow \gamma\gamma$  are:

$$I_{\tilde{\tau}_1} = \tau \{1 - \tau f(\tau)\} \left[ R_{\tilde{\tau}_L\tilde{\tau}_L}^1 \sin^2 \theta_\tau + R_{\tilde{\tau}_R\tilde{\tau}_R}^1 \cos^2 \theta_\tau + 2 \sin \theta_\tau \cos \theta_\tau R_{\tilde{\tau}_L\tilde{\tau}_R}^1 \right], \quad (\text{F.174})$$

$$I_{\tilde{\tau}_2} = \tau \{1 - \tau f(\tau)\} \left[ R_{\tilde{\tau}_L\tilde{\tau}_L}^2 \cos^2 \theta_\tau + R_{\tilde{\tau}_R\tilde{\tau}_R}^2 \sin^2 \theta_\tau - 2 \sin \theta_\tau \cos \theta_\tau R_{\tilde{\tau}_L\tilde{\tau}_R}^2 \right], \quad (\text{F.175})$$

here

$$R_{\tilde{\tau}_L\tilde{\tau}_L}^i = \frac{m_W}{m_{\tilde{\tau}_i}} \left[ m_W \left( -\frac{1}{2} + \frac{1}{2} \tan^2 \theta_W \right) \sin(\alpha + \beta) + \frac{m_\tau^2 \sin \alpha}{m_W \cos \beta} \right], \quad (\text{F.176})$$

$$R_{\tilde{\tau}_R\tilde{\tau}_R}^i = \frac{m_W}{m_{\tilde{\tau}_i}} \left[ -m_W \tan^2 \theta_W \sin(\alpha + \beta) + \frac{m_\tau^2 \sin \alpha}{m_W \cos \beta} \right], \quad (\text{F.177})$$

$$R_{\tilde{\tau}_L\tilde{\tau}_R}^i = \frac{m_W}{m_{\tilde{\tau}_i}} \frac{m_\tau}{2m_W \cos \beta} (-\mu \cos \alpha - A_\tau \sin \alpha). \quad (\text{F.178})$$

For  $H \rightarrow \gamma\gamma$  via  $\tilde{\tau}_i$  it's the same except the  $R_{\tilde{\tau}_L\tilde{\tau}_L}^i, R_{\tilde{\tau}_R\tilde{\tau}_R}^i, R_{\tilde{\tau}_L\tilde{\tau}_R}^i$  differ:

$$R_{\tilde{\tau}_L\tilde{\tau}_L}^i = \frac{m_W}{m_{\tilde{\tau}_i}} \left[ m_W \left( \frac{1}{2} - \frac{1}{2} \tan^2 \theta_W \right) \cos(\alpha + \beta) - \frac{m_\tau^2 \cos \alpha}{m_W \cos \beta} \right], \quad (\text{F.179})$$

$$R_{\tilde{\tau}_R\tilde{\tau}_R}^i = \frac{m_W}{m_{\tilde{\tau}_i}} \left[ m_W \tan^2 \theta_W \cos(\alpha + \beta) - \frac{m_\tau^2 \cos \alpha}{m_W \cos \beta} \right], \quad (\text{F.180})$$

$$R_{\tilde{\tau}_L\tilde{\tau}_R}^i = \frac{m_W}{m_{\tilde{\tau}_i}} \frac{m_\tau}{2m_W \cos \beta} (-\mu \sin \alpha + A_\tau \cos \alpha). \quad (\text{F.181})$$

$A \rightarrow \gamma\gamma$  has no stau loop contribution because of CP conservation, i.e.  $I_{\tilde{\tau}_{1/2}}^A = 0$ . The  $W$  loop contributions are:

$$I_W^h = \left[ 2 + 3\tau + 3\tau(2 - \tau)f(\tau) \right] \sin(\beta - \alpha), \quad (\text{F.182})$$

$$I_W^H = [2 + 3\tau + 3\tau(2 - \tau)f(\tau)] \cos(\beta - \alpha). \quad (\text{F.183})$$

$A \rightarrow \gamma\gamma$  has no  $W$  loop contribution because of CP conservation, i.e.  $I_W^A = 0$ .  $H^+$  loop contributions are:

$$I_{H^+}^h = \tau\{1 - \tau f(\tau)\} \frac{m_W^2}{m_{H^+}^2} \left[ \sin(\beta - \alpha) + \frac{\cos 2\beta \sin(\beta + \alpha)}{2 \cos^2 \theta_W} \right], \quad (\text{F.184})$$

$$I_{H^+}^H = \tau\{1 - \tau f(\tau)\} \frac{m_W^2}{m_{H^+}^2} \left[ \cos(\beta - \alpha) + \frac{\cos 2\beta \cos(\beta + \alpha)}{2 \cos^2 \theta_W} \right]. \quad (\text{F.185})$$

$A \rightarrow \gamma\gamma$  has no  $H^+$  loop contribution because of CP conservation, i.e.  $I_{H^+}^A = 0$ .  $\tilde{W}_i^+$  loop contributions are:

$$I_{\tilde{W}_1^+}^h = [-2\tau\{1 + (1 - \tau)f(\tau)\}] \frac{m_W}{m_{\tilde{W}_1^+}} \sqrt{2}(-\sin \alpha \sin \theta_R \cos \theta_L + \cos \alpha \sin \theta_L \cos \theta_R), \quad (\text{F.186})$$

$$I_{\tilde{W}_2^+}^h = [-2\tau\{1 + (1 - \tau)f(\tau)\}] \frac{m_W}{m_{\tilde{W}_2^+}} \sqrt{2}(\sin \alpha \cos \theta_R \sin \theta_L - \cos \alpha \cos \theta_L \sin \theta_R). \quad (\text{F.187})$$

For  $\tilde{W}_i^+$  contributions to  $H$  everything is the same except the replacements  $\cos \alpha \rightarrow \sin \alpha$  and  $\sin \alpha \rightarrow -\cos \alpha$  are required. For  $A$  the  $\tilde{W}_i^+$  contributions have:

$$I_{\tilde{W}_1^+}^A = -2\tau f(\tau) \frac{m_W}{m_{\tilde{W}_1^+}} \sqrt{2}(\sin \theta_R \cos \theta_L \sin \beta + \sin \theta_L \cos \theta_R \cos \beta), \quad (\text{F.188})$$

$$I_{\tilde{W}_2^+}^A = 2\tau f(\tau) \frac{m_W}{m_{\tilde{W}_2^+}} \sqrt{2}(\cos \theta_R \sin \theta_L \sin \beta + \cos \theta_L \sin \theta_R \cos \beta). \quad (\text{F.189})$$

$\phi \rightarrow gg$

The coloured particle loop contribution for  $\phi \rightarrow gg$  are exactly the same, except the prefactor changes and the bottom and sbottom contributions get multiplied by 4 in their amplitudes. There can be no uncoloured particles in the loop so there are no  $W$ , charged Higgs, chargino, lepton or slepton contributions; only quark and squark loop contributions:

$$\Gamma(\phi \rightarrow gg) = \frac{g_s^4 g^2}{1024\pi^3 16\pi^2} \frac{m_\phi^3}{m_W^2} \frac{9}{8} \Sigma |I_{loop}^\phi|^2, \quad (\text{F.190})$$

with the  $I_b^\phi \rightarrow 4I_b^\phi$ ,  $I_{\tilde{b}_i}^\phi \rightarrow 4I_{\tilde{b}_i}^\phi$  and the remaining  $I_{loop}^\phi$  as in the  $\phi \rightarrow \gamma\gamma$  decays.

Loop decays to  $Z\gamma$ : Throughout the function  $g(\tau)$  appears, it is given previously in eq. (E.3),

$$\Gamma(\phi \rightarrow Z\gamma) = \frac{m_\phi^3}{32\pi} \left(1 - \frac{m_Z^2}{m_\phi^2}\right)^3 \frac{g^6 \sin^4 \theta_W}{256\pi^4 m_W^2} \Sigma |I_{loop}^\phi|^2. \quad (\text{F.191})$$

$I_1(\tau_a, \tau_{aZ})$  and  $I_2(\tau_a, \tau_{aZ})$  also occur frequently, where  $\tau_{aZ} = 4(\frac{m_a}{m_Z})^2$  cf  $\tau_a = 4(\frac{m_a}{m_{h_i}})^2$ , they are as follows:

$$I_1(\tau_a, \tau_{aZ}) = \frac{\tau_a \tau_{aZ}}{2(\tau_a - \tau_{aZ})} + \frac{\tau_a^2 \tau_{aZ}^2}{2(\tau_a - \tau_{aZ})^2} [f(\tau_a) - f(\tau_{aZ})] + \frac{\tau_a^2 \tau_{aZ}}{(\tau_a - \tau_{aZ})^2} [g(\tau_a) - g(\tau_{aZ})], \quad (\text{F.192})$$

$$I_2(\tau_a, \tau_{aZ}) = -\frac{\tau_a \tau_{aZ}}{2(\tau_a - \tau_{aZ})} [f(\tau_a) - f(\tau_{aZ})]. \quad (\text{F.193})$$

The fermion loop contributions are:

$$I_t^h = 3 \frac{\cos \alpha}{\sin \beta} \frac{-\frac{4}{3}(\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W)}{\sin \theta_W \cos \theta_W} (I_1(\tau_t, \tau_{tZ}) - I_2(\tau_t, \tau_{tZ})), \quad (\text{F.194})$$



The expression for  $I_c^h$  is analogous.

$$I_b^h = -3 \frac{\sin \alpha}{\cos \beta} \frac{\frac{2}{3}(-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W)}{\sin \theta_W \cos \theta_W} (I_1(\tau_b, \tau_{bZ}) - I_2(\tau_b, \tau_{bZ})), \quad (\text{F.195})$$

A similar expression gives  $I_s^h$ . For the  $I_f^H$  transform  $\cos \alpha \rightarrow \sin \alpha$ ,  $\sin \alpha \rightarrow -\cos \alpha$ .

$$I_t^A = 3 \cot \beta \frac{\frac{4}{3}(\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W)}{\sin \theta_W \cos \theta_W} I_2(\tau_t, \tau_{tZ}), \quad (\text{F.196})$$

Again an analogous expression gives  $I_c^A$ .

$$I_b^A = -3 \tan \beta \frac{\frac{2}{3}(-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W)}{\sin \theta_W \cos \theta_W} I_2(\tau_b, \tau_{bZ}). \quad (\text{F.197})$$

$I_s^A$  is given similarly. The  $W$  loop contributions are given by:

$$I_W^h = -\frac{\sin(\beta - \alpha)}{\tan \theta_W} \left[ 4(3 - \tan^2 \theta_W) I_2(\tau_W, \tau_{WZ}) + \left\{ \left(1 + \frac{2}{\tau_W}\right) \tan^2 \theta_W - \left(5 + \frac{2}{\tau_W}\right) \right\} I_1(\tau_W, \tau_{WZ}) \right]. \quad (\text{F.198})$$

$I_W^H$  is the same but with the change  $\sin(\beta - \alpha) \rightarrow \cos(\beta - \alpha)$ .  $I_W^A = 0$  by CP conservation.  $H^+$  contributions are:

$$I_{H^+}^h = \left[ \sin(\beta - \alpha) + \frac{\cos 2\beta \sin(\beta + \alpha)}{2 \cos^2 \theta_W} \right] \frac{(1 - 2 \sin^2 \theta_W)}{\cos \theta_W \sin \theta_W} I_1(\tau_{H^+}, \tau_{H^+Z}) \frac{m_W^2}{m_{H^+}^2}. \quad (\text{F.199})$$

For  $H$ , the  $I_{H^+}^H$  are the same except the replacements  $\sin(\beta - \alpha) \rightarrow \cos(\beta - \alpha)$  and  $\sin(\beta + \alpha) \rightarrow -\cos(\beta + \alpha)$  which occur because of the transformations  $\cos \alpha \rightarrow \sin \alpha$  and  $\sin \alpha \rightarrow -\cos \alpha$ . Meanwhile  $I_{H^+}^A = 0$  by CP conservation.

## Appendix G. MSSM Three Body Decay Formulae

The following decay modes are included in SOFTSUSY:

1.  $h \rightarrow VV^* \rightarrow V f \bar{f}$
2.  $\tilde{g} \rightarrow \tilde{Z}_i q \bar{q}$
3.  $\tilde{g} \rightarrow \tilde{W}_i q \bar{q}'$
4.  $\tilde{Z}_i \rightarrow \tilde{Z}_j f \bar{f}$  where  $i > j$
5.  $\tilde{Z}_i \rightarrow \tilde{W}_j f \bar{f}'$
6.  $\tilde{W}_j \rightarrow \tilde{Z}_i f \bar{f}'$

The modes included are the most phenomenologically relevant modes, the formulae used were not rederived although are written in our notation and are restructured to match the calculations performed in SOFTSUSY. The formulae are as provided in sPHENO-3.3.8 [11, 31], which were based on the calculations in [30, 48].

$h \rightarrow VV^* \rightarrow V f \bar{f}$  Detailed previously, see eqs. (F.140) (F.139).

### Appendix G.1. Gluino $1 \rightarrow 3$ Decays

$\tilde{g} \rightarrow \tilde{Z}_i q \bar{q}$

First the formulae for the decays to a neutralino and a quark-antiquark pair of the first two generations; in this formula the Yukawa coupling contributions, squark mixing effects and final state quark masses have been neglected as they are negligible. The formulae after this for the third generation quarks include all such effects.

$$\Gamma(\tilde{g} \rightarrow q \bar{q} \tilde{Z}_i) = \frac{\alpha_s}{8\pi^2} \left[ |A_{\tilde{Z}_i}^q|^2 (\psi_L \pm \phi_L) + |B_{\tilde{Z}_i}^q|^2 (\psi_R \pm \phi_R) \right] \quad (\text{G.1})$$

Here which of the  $\pm$  signs to take depends on the signs of the neutralino and gluino masses; the '+' sign applies for the case when both masses have the same sign so  $m_{\tilde{Z}_i} > 0$  and  $m_{\tilde{g}} > 0$  (or when they are both less than 0) and the '-'

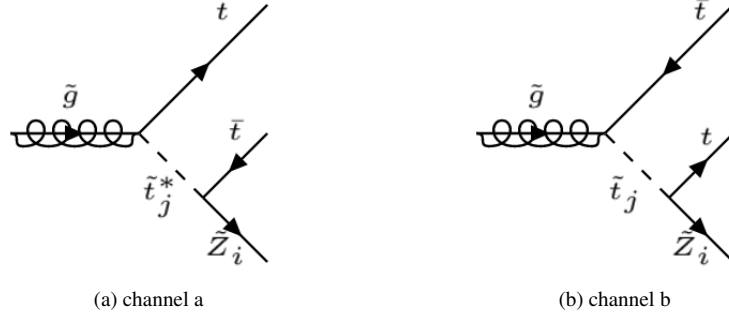


Figure G.6: Feynman diagrams for the 3 body decay of a gluino into a neutralino and a top anti-top pair, as mediated by stops  $\tilde{t}_{1/2}$ .  $i = 1, 2, 3, 4$  and  $j = 1, 2$ .

sign applies when one (but not both) of  $m_{\tilde{Z}_i}$  and  $m_{\tilde{g}}$  are negative. The signs essentially account for the fact that the couplings should become complex as the masses become negative. Here the  $\psi_{L/R}$  and  $\phi_{L/R}$  are integrals related to the  $\tilde{\psi}$  and  $\tilde{\phi}$  integrals given later in eq.s (G.74) and (G.78) by:

$$\psi_{L/R} = \frac{1}{\pi^2 m_{\tilde{g}}} \tilde{\psi}(m_{\tilde{g}}, m_{\tilde{q}_{L/R}}, m_{\tilde{q}_{L/R}}, m_{\tilde{Z}_i}). \quad (\text{G.2})$$

$$\phi_{L/R} = \frac{1}{\pi^2 m_{\tilde{g}}} \tilde{\psi}(m_{\tilde{g}}, m_{\tilde{q}_{L/R}}, m_{\tilde{q}_{L/R}}, m_{\tilde{Z}_i}). \quad (\text{G.3})$$

The  $A_{\tilde{Z}_i}^q$  and  $B_{\tilde{Z}_i}^q$  are couplings which depend upon if the quarks are “up-type” or “down-type” in  $SU(2)_L$  and are:

$$A_{\tilde{Z}_i}^q = \begin{cases} \frac{1}{\sqrt{2}}(-gN_{2i} - \frac{g'}{3}N_{1i}), & \text{for “up-type” quarks,} \\ \frac{1}{\sqrt{2}}(gN_{2i} - \frac{g'}{3}N_{1i}), & \text{for “down-type” quarks,} \end{cases} \quad (\text{G.4})$$

$$B_{\tilde{Z}_i}^q = \begin{cases} \frac{-4}{3\sqrt{2}}g'N_{1i}, & \text{for “up-type” quarks,} \\ \frac{2}{3\sqrt{2}}g'N_{1i}, & \text{for “down-type” quarks.} \end{cases} \quad (\text{G.5})$$

For the more complicated case of decays to third generation quarks; the Yukawa coupling contributions, squark-mixing effects and final state quark masses are all included as they can have significant effects. The decay is mediated by either  $\tilde{t}_1$  or  $\tilde{t}_2$  in the t or u channel, giving 4 Feynman diagrams (2 shown below as  $j = 1, 2$ ) and 6 (i.e.  ${}^4C_2$ ) interferences. The six interferences can be split into 2 “diagonal” contributions ( $\tilde{t}_1$  t -  $\tilde{t}_1$  u interference and  $\tilde{t}_2$  t -  $\tilde{t}_2$  u interference) and 4 “non-diagonal” contributions ( $\tilde{t}_1$  t -  $\tilde{t}_2$  t,  $\tilde{t}_1$  t -  $\tilde{t}_2$  u,  $\tilde{t}_1$  u -  $\tilde{t}_2$  t and  $\tilde{t}_1$  u -  $\tilde{t}_2$  u interferences). The possibility of negative neutralino masses (which can be absorbed into imaginary couplings) is also included. The formulae are adopted from sPHENIX, it should be noted that differences exist between these formulae and those present in Baer and Tata’s book [39].

The formulae for the case of  $\tilde{g} \rightarrow \tilde{Z}_i t \bar{t}$  are given, from this decays to other quarks can be obtained by making the appropriate replacements.

$$\Gamma(\tilde{g} \rightarrow t \bar{t} \tilde{Z}_i) = \frac{\alpha_s}{8\pi^4 m_{\tilde{g}}} [\Gamma_{\tilde{t}_1} + \Gamma_{\tilde{t}_2} + \Gamma_{\tilde{t}_1 \tilde{t}_2}]. \quad (\text{G.6})$$

The  $\Gamma_{\tilde{t}_1}$ ,  $\Gamma_{\tilde{t}_2}$ ,  $\Gamma_{\tilde{t}_1 \tilde{t}_2}$  can be split up into different contributions:

$$\Gamma_{\tilde{t}_1} = \Gamma_{LL}(\tilde{t}_1) \cos^2 \theta_t + \Gamma_{RR}(\tilde{t}_1) \sin^2 \theta_t - \sin \theta_t \cos \theta_t [\Gamma_{L_1 R_1}(\tilde{t}_1) + \Gamma_{L_1 R_2}(\tilde{t}_1) + \Gamma_{L_2 R_1}(\tilde{t}_1) + \Gamma_{L_2 R_2}(\tilde{t}_1)]. \quad (\text{G.7})$$

$$\Gamma_{\tilde{t}_2} = \Gamma_{LL}(\tilde{t}_2) \sin^2 \theta_t + \Gamma_{RR}(\tilde{t}_2) \cos^2 \theta_t + \sin \theta_t \cos \theta_t [\Gamma_{L_1 R_1}(\tilde{t}_2) + \Gamma_{L_1 R_2}(\tilde{t}_2) + \Gamma_{L_2 R_1}(\tilde{t}_2) + \Gamma_{L_2 R_2}(\tilde{t}_2)]. \quad (\text{G.8})$$

$$\Gamma_{\tilde{t}_1 \tilde{t}_2} = [\Gamma_{LL}(\tilde{t}_1, \tilde{t}_2) + \Gamma_{RR}(\tilde{t}_1, \tilde{t}_2)] \sin \theta_t \cos \theta_t + \Gamma_{LR}(\tilde{t}_1, \tilde{t}_2) \cos^2 \theta_t + \Gamma_{RL}(\tilde{t}_1, \tilde{t}_2) \sin^2 \theta_t. \quad (\text{G.9})$$

The moduli of the complex couplings are as follows:

$$|\alpha_1^{\tilde{t}}| = \tilde{A}_{\tilde{Z}_i}^t \cos \theta_t - f_t N_{4i} \sin \theta_t, \quad |\beta_1^{\tilde{t}}| = f_t N_{4i} \cos \theta_t + \tilde{B}_{\tilde{Z}_i}^t \sin \theta_t, \quad (\text{G.10})$$

$$|\alpha_1^{\tilde{b}}| = \tilde{A}_{\tilde{Z}_i}^t \sin \theta_t + f_t N_{4i} \cos \theta_t, \quad |\beta_1^{\tilde{b}}| = f_t N_{4i} \sin \theta_t - \tilde{B}_{\tilde{Z}_i}^t \cos \theta_t, \quad (\text{G.11})$$

where

$$\tilde{A}_{\tilde{Z}_i}^t = -\frac{g}{\sqrt{2}} N_{2i} - \frac{g'}{3\sqrt{2}} N_{1i}, \quad (\text{G.12})$$

$$\tilde{B}_{\tilde{Z}_i}^t = -\frac{4}{3} \frac{g'}{\sqrt{2}} N_{1i}, \quad (\text{G.13})$$

$$f_t = \frac{gm_t^{rum}}{\sqrt{2}M_W \sin \beta}. \quad (\text{G.14})$$

The couplings themselves are then complex and depend upon the sign of the corresponding neutralino's mass. They are of the form  $(a, b)$ , where this represents the complex number  $a + bi$ . For positive masses the couplings are just:

$$a^{\tilde{t}} = (|\alpha_1^{\tilde{t}}|, 0), \quad b^{\tilde{t}} = (|\beta_1^{\tilde{t}}|, 0), \quad a^{\tilde{b}} = (|\alpha_1^{\tilde{b}}|, 0), \quad b^{\tilde{b}} = (|\beta_1^{\tilde{b}}|, 0). \quad (\text{G.15})$$

Meanwhile, for negative neutralino masses the effect of our field redefinition is to multiply the corresponding row of the neutralino mixing matrix by  $-i$  therefore the couplings are then:

$$a^{\tilde{t}} = (0, -|\alpha_1^{\tilde{t}}|), \quad b^{\tilde{t}} = (0, -|\beta_1^{\tilde{t}}|), \quad a^{\tilde{b}} = (0, -|\alpha_1^{\tilde{b}}|), \quad b^{\tilde{b}} = (0, -|\beta_1^{\tilde{b}}|). \quad (\text{G.16})$$

In addition to account for differences in interactions for positive and negative neutralino masses as a result of this coupling difference and the extra associated  $\gamma_5$  matrices we must also include factors of:

$$(-1)^{\theta_i} = \begin{cases} +1, & \text{for positive neutralino masses,} \\ -1, & \text{for negative neutralino masses.} \end{cases} \quad (\text{G.17})$$

The formula we use for  $\Gamma_{\tilde{t}_1}$  is:

$$\begin{aligned} \Gamma_{\tilde{t}_1} = & (-1)^{\theta_i} \left[ (a^{\tilde{t}})^2 + b^{\tilde{t}}^2 \right] \tilde{\psi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{Z}_i}) + 4a^{\tilde{t}} b^{\tilde{t}} m_t m_{\tilde{Z}_i} \tilde{\chi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{Z}_i}) \\ & - 4 \sin \theta_t \cos \theta_t (a^{\tilde{t}})^2 + b^{\tilde{t}}^2 m_{\tilde{g}} m_t X(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{Z}_i}) \\ & - 8 \sin \theta_t \cos \theta_t a^{\tilde{t}} b^{\tilde{t}} m_{\tilde{g}} m_t^2 m_{\tilde{Z}_i} \tilde{\zeta}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{Z}_i}) \\ & - 2 \sin \theta_t \cos \theta_t a^{\tilde{t}} b^{\tilde{t}} Y(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{Z}_i}) + \{a^{\tilde{t}})^2 \cos^2 \theta_t + b^{\tilde{t}}^2 \sin^2 \theta_t\} \tilde{\phi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{Z}_i}) \\ & - 2m_t^2 \sin \theta_t \cos \theta_t a^{\tilde{t}} b^{\tilde{t}} \xi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{Z}_i}) + m_{\tilde{g}} m_t a^{\tilde{t}} b^{\tilde{t}} \xi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{Z}_i}) \\ & - m_{\tilde{g}} m_t a^{\tilde{t}} b^{\tilde{t}} m_{\tilde{Z}_i}^2 \tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{Z}_i}) + m_{\tilde{g}} m_t^2 m_{\tilde{Z}_i} \{a^{\tilde{t}})^2 \sin^2 \theta_t + b^{\tilde{t}}^2 \cos^2 \theta_t\} \tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{Z}_i}) \\ & - m_{\tilde{Z}_i} m_t \sin \theta_t \cos \theta_t (a^{\tilde{t}})^2 + b^{\tilde{t}}^2 m_{\tilde{g}}^2 \tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{Z}_i}) \\ & + m_{\tilde{Z}_i} m_t \sin \theta_t \cos \theta_t (a^{\tilde{t}})^2 + b^{\tilde{t}}^2 \xi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{Z}_i}) \Big]. \end{aligned} \quad (\text{G.18})$$

The  $\Gamma_{\tilde{t}_2}$  formula we use is:

$$\begin{aligned}
\Gamma_{\tilde{t}_2} = & (-1)^{\theta_t} \left[ (a^{\tilde{t}_2^2} + b^{\tilde{t}_2^2}) \tilde{\psi}(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) + 4a^{\tilde{t}_2} b^{\tilde{t}_2} m_t m_{\tilde{Z}_i} \tilde{\chi}(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \right. \\
& + 4m_{\tilde{g}} m_t \sin \theta_t \cos \theta_t (a^{\tilde{t}_2^2} + b^{\tilde{t}_2^2}) X(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& + 8 \sin \theta_t \cos \theta_t a^{\tilde{t}_2} b^{\tilde{t}_2} m_{\tilde{g}}^2 m_{\tilde{Z}_i} \zeta(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) + 2 \sin \theta_t \cos \theta_t a^{\tilde{t}_2} b^{\tilde{t}_2} Y(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& + \{a^{\tilde{t}_2^2} \sin^2 \theta_t + \cos^2 \theta_t b^{\tilde{t}_2^2}\} \tilde{\phi}(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) + 2m_t^2 \sin \theta_t \cos \theta_t a^{\tilde{t}_2} b^{\tilde{t}_2} \xi(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& + m_{\tilde{g}} m_t a^{\tilde{t}_2} b^{\tilde{t}_2} \xi(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) - m_{\tilde{g}} m_t m_{\tilde{Z}_i}^2 a^{\tilde{t}_2} b^{\tilde{t}_2} \tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& + m_{\tilde{g}} m_{\tilde{Z}_i} m_t^2 \{a^{\tilde{t}_2^2} \cos^2 \theta_t + b^{\tilde{t}_2^2} \sin^2 \theta_t\} \tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& - m_{\tilde{Z}_i} m_t \sin \theta_t \cos \theta_t (a^{\tilde{t}_2^2} + b^{\tilde{t}_2^2}) \xi(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& \left. + m_{\tilde{g}}^2 m_{\tilde{Z}_i} m_t \sin \theta_t \cos \theta_t (a^{\tilde{t}_2^2} + b^{\tilde{t}_2^2}) \tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \right]. \tag{G.19}
\end{aligned}$$

For  $\Gamma_{\tilde{t}_1 \tilde{t}_2}$ , our formula, again extracted from sPHENO-3.3.8, is:

$$\begin{aligned}
\Gamma_{\tilde{t}_1 \tilde{t}_2} = & (-1)^{\theta_t} \left[ 4m_{\tilde{g}} m_t (\cos^2 \theta_t - \sin^2 \theta_t) (a^{\tilde{t}_1} a^{\tilde{t}_2} + b^{\tilde{t}_1} b^{\tilde{t}_2}) X(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \right. \\
& + 4m_{\tilde{g}}^2 m_{\tilde{Z}_i} (a^{\tilde{t}_1} b^{\tilde{t}_2} + b^{\tilde{t}_1} a^{\tilde{t}_2}) (\cos^2 \theta_t - \sin^2 \theta_t) \zeta(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& + 2\{b^{\tilde{t}_1} a^{\tilde{t}_1} \cos^2 \theta_t - \sin^2 \theta_t b^{\tilde{t}_2} a^{\tilde{t}_1}\} Y(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& + 2 \sin \theta_t \cos \theta_t (a^{\tilde{t}_1} a^{\tilde{t}_2} - b^{\tilde{t}_1} b^{\tilde{t}_2}) \tilde{\phi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& + 2m_t m_{\tilde{Z}_i} (a^{\tilde{t}_1} a^{\tilde{t}_2} - b^{\tilde{t}_1} b^{\tilde{t}_2}) \chi'(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& + 2m_t^2 \{\cos^2 \theta_t a^{\tilde{t}_1} b^{\tilde{t}_2} - \sin^2 \theta_t b^{\tilde{t}_1} a^{\tilde{t}_2}\} \xi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& + 2m_{\tilde{g}}^2 m_{\tilde{Z}_i} (b^{\tilde{t}_1} b^{\tilde{t}_2} - a^{\tilde{t}_1} a^{\tilde{t}_2}) \sin \theta_t \cos \theta_t \tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& - 4 \sin \theta_t \cos \theta_t m_{\tilde{g}} m_t (a^{\tilde{t}_1} b^{\tilde{t}_2} - b^{\tilde{t}_1} a^{\tilde{t}_2}) \chi'(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& - 2 \sin \theta_t \cos \theta_t m_{\tilde{g}} m_t (a^{\tilde{t}_1} b^{\tilde{t}_2} - b^{\tilde{t}_1} a^{\tilde{t}_2}) \xi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& + 2m_t m_{\tilde{Z}_i} \{\sin^2 \theta_t a^{\tilde{t}_1} a^{\tilde{t}_2} - \cos^2 \theta_t b^{\tilde{t}_1} b^{\tilde{t}_2}\} \xi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& + 2m_{\tilde{g}}^3 m_t (a^{\tilde{t}_1} b^{\tilde{t}_2} - b^{\tilde{t}_1} a^{\tilde{t}_2}) \sin \theta_t \cos \theta_t \tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \\
& \left. - 2m_{\tilde{g}}^2 m_t m_{\tilde{Z}_i} \{\sin^2 \theta_t a^{\tilde{t}_1} a^{\tilde{t}_2} - \cos^2 \theta_t b^{\tilde{t}_1} b^{\tilde{t}_2}\} \tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}_i}) \right]. \tag{G.20}
\end{aligned}$$

For the 3 body decay of  $\tilde{g} \rightarrow \tilde{Z}_i b \bar{b}$  then the formulae are exactly as above but with the replacements:  $m_{\tilde{t}_i} \rightarrow m_{\tilde{b}_i}$ ,

$$\begin{aligned}
\tilde{A}_{\tilde{Z}_i}^t & \rightarrow \tilde{A}_{\tilde{Z}_i}^b, \\
\tilde{B}_{\tilde{Z}_i}^t & \rightarrow \tilde{B}_{\tilde{Z}_i}^b, \\
f_t & \rightarrow f_b, \\
N_{4i} & \rightarrow N_{3i}, \\
\theta_t & \rightarrow \theta_b, \\
m_t & \rightarrow m_b.
\end{aligned}$$

In our program both the  $\tilde{g} \rightarrow \tilde{Z}_i t \bar{t}$  and  $\tilde{g} \rightarrow \tilde{Z}_i b \bar{b}$  decays are implemented in the same function, just depending on whether its decaying to tops or bottoms different couplings are used as described above.

$$\tilde{A}_{\tilde{Z}_i}^b = \frac{g}{\sqrt{2}} N_{2i} - \frac{g'}{3\sqrt{2}} N_{1i}, \tag{G.21}$$

$$\tilde{B}_{\tilde{Z}_i}^b = \frac{2}{3} \frac{g'}{\sqrt{2}} N_{1i}, \tag{G.22}$$

$$f_b = \frac{gm_b^{run}}{\sqrt{2} M_W \cos \beta}. \tag{G.23}$$

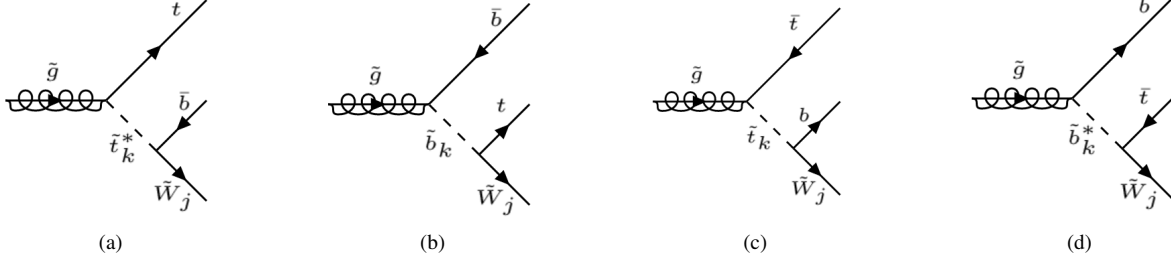


Figure G.7: Feynman diagrams for the 3 body decay of a gluino into a chargino and a top-bottom pair, as mediated by stops  $\tilde{t}_{1/2}$  and sbottoms  $\tilde{b}_{1/2}$ ,  $j, k = 1, 2$ .

Note that the integrals used in these equations,  $\tilde{\psi}, \tilde{\chi}, X, \zeta, \tilde{\phi}, \xi, \tilde{\rho}, Y, \chi'$  are given below in eq: (G.74), (G.75), (G.76), (G.77), (G.78), (G.79), (G.80) (G.81), (G.82) respectively, when the formulae for  $\tilde{Z}_i \rightarrow \tilde{Z}_j f \bar{f}$  are given.

$$\tilde{g} \rightarrow \tilde{W}_i q \bar{q}'$$

The decay of a gluino into a chargino, quark( $q$ ) and antiquark( $\bar{q}'$ ) can occur via intermediate squarks of either the  $q$  or  $q'$ , therefore there are four possible intermediates in the case where intra-generational squark mixing effects are included. For example,  $\tilde{g} \rightarrow \tilde{W}_j t \bar{b}$  may proceed via  $\tilde{t}_1, \tilde{t}_2, \tilde{b}_1$  or  $\tilde{b}_2$ . Again there are both  $t$  and  $u$  channel contributions which may contribute to  $\tilde{g} \rightarrow \tilde{W}_j t \bar{b}$  or  $\tilde{g} \rightarrow \tilde{W}_j b \bar{t}$ . The 8 diagrams are therefore shown as a set of 4 ( $k = 1, 2$  for each intermediate shown) in Fig G.7. There are therefore 4 squared contributions to each of  $t \bar{b}$  and  $b \bar{t}$  as well as  $\tilde{t}_1 \tilde{t}_1, \tilde{t}_1 \tilde{t}_2, \tilde{t}_2 \tilde{t}_2, \tilde{b}_1 \tilde{b}_1, \tilde{b}_1 \tilde{b}_2, \tilde{b}_2 \tilde{b}_2, \tilde{t}_1 \tilde{b}_1, \tilde{t}_1 \tilde{b}_2, \tilde{t}_2 \tilde{b}_1$  and  $\tilde{t}_2 \tilde{b}_2$  interferences. Note “diagonal” interferences such as  $\tilde{t}_1 \tilde{t}_1$  are included with non-interference squared terms into the  $\Gamma_{\tilde{t}_1}$  type contributions. In the formulae, the Yukawa couplings, intra-generational squark mixing and final state fermion masses are all accounted for; however whilst the bottom quark mass  $m_b$  is included in the phase space, it is neglected from the squared matrix element, this drops any  $\tilde{b}_k \tilde{b}_l$  interferences as these are proportional to  $m_b$ . The approach used follows that in Baer and Tata’s book ‘*Weak Scale Supersymmetry*’ [39] with the formulae we use taken from sPHENO [11, 31], based on the calculations in reference [30]. The formulae used, as in sPHENO, differ in a few places from those in Baer and Tata.

$$\Gamma(\tilde{g} \rightarrow t \bar{b} \tilde{W}_i^-) = \frac{\alpha_s}{16\pi^2 m_{\tilde{g}}} (\Gamma_{\tilde{t}_1} + \Gamma_{\tilde{t}_2} + \Gamma_{\tilde{t}_1 \tilde{t}_2} + \Gamma_{\tilde{b}_1} + \Gamma_{\tilde{b}_2} + \Gamma_{\tilde{t}_1 \tilde{b}_1} + \Gamma_{\tilde{t}_1 \tilde{b}_2} + \Gamma_{\tilde{t}_2 \tilde{b}_1} + \Gamma_{\tilde{t}_2 \tilde{b}_2}). \quad (\text{G.24})$$

The *chargino – quark – squark* couplings are given below, remember we have the chargino mixing angle  $\theta_L$  and  $\theta_R$  pre-transformed so that  $\theta_{L/R} \rightarrow -\theta_{L/R} + \pi/2$  in order to use the convention where the lightest mass chargino eigenstate  $\tilde{W}_1$  appears first in the multiplet:

$$\begin{aligned} \alpha_{\tilde{W}_1}^{\tilde{t}_1} &= -g \sin \theta_L \cos \theta_t + f_t \cos \theta_R \sin \theta_t, \\ \beta_{\tilde{W}_1}^{\tilde{t}_1} &= -f_b \cos \theta_L \cos \theta_t, \\ \alpha_{\tilde{W}_1}^{\tilde{b}_1} &= -g \sin \theta_L \cos \theta_b + f_b \cos \theta_L \sin \theta_b, \\ \beta_{\tilde{W}_1}^{\tilde{b}_1} &= -f_t \cos \theta_R \cos \theta_b, \\ \alpha_{\tilde{W}_2}^{\tilde{t}_1} &= -g \cos \theta_L \cos \theta_t - f_t \sin \theta_R \sin \theta_t, \\ \beta_{\tilde{W}_2}^{\tilde{t}_1} &= f_b \sin \theta_L \cos \theta_t, \\ \alpha_{\tilde{W}_2}^{\tilde{b}_1} &= -g \cos \theta_L \cos \theta_b - f_b \sin \theta_L \sin \theta_b, \\ \beta_{\tilde{W}_2}^{\tilde{b}_1} &= f_t \sin \theta_R \cos \theta_b. \end{aligned} \quad (\text{G.25})$$

We obtain the couplings for  $\tilde{t}_2$  and  $\tilde{b}_2$  by changing  $\cos \theta_q \rightarrow \sin \theta_q$  and  $\sin \theta_q \rightarrow -\cos \theta_q$ . The  $\tilde{W}_2$  couplings are obtained from those of  $\tilde{W}_1$  by making the replacements  $\cos \theta_{L/R} \rightarrow -\sin \theta_{L/R}$  and  $\sin \theta_{L/R} \rightarrow \cos \theta_{L/R}$ .

The contributions in (G.24) are as follows:

$$\Gamma_{\tilde{t}_1} = [(\alpha_{\tilde{W}_i}^{\tilde{t}_1})^2 + (\beta_{\tilde{W}_i}^{\tilde{t}_1})^2] [G_1(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{W}_i}) - \sin 2\theta_t G_8(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\tilde{W}_i})], \quad (\text{G.26})$$

$$\Gamma_{\tilde{t}_2} = [(\alpha_{\tilde{W}_i}^{\tilde{t}_2})^2 + (\beta_{\tilde{W}_i}^{\tilde{t}_2})^2] [G_1(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{W}_i}) + \sin 2\theta_t G_8(m_{\tilde{g}}, m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_{\tilde{W}_i})], \quad (\text{G.27})$$

$$\Gamma_{\tilde{b}_1} = [(\alpha_{\tilde{W}_i}^{\tilde{b}_1})^2 + (\beta_{\tilde{W}_i}^{\tilde{b}_1})^2] G_2(m_{\tilde{g}}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) + \alpha_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{b}_1} G_3(m_{\tilde{g}}, m_{\tilde{b}_1}, m_{\tilde{W}_i}), \quad (\text{G.28})$$

$$\Gamma_{\tilde{b}_2} = [(\alpha_{\tilde{W}_i}^{\tilde{b}_2})^2 + (\beta_{\tilde{W}_i}^{\tilde{b}_2})^2] G_2(m_{\tilde{g}}, m_{\tilde{b}_2}, m_{\tilde{W}_i}) + \alpha_{\tilde{W}_i}^{\tilde{b}_2} \beta_{\tilde{W}_i}^{\tilde{b}_2} G_3(m_{\tilde{g}}, m_{\tilde{b}_2}, m_{\tilde{W}_i}), \quad (\text{G.29})$$

$$\Gamma_{\tilde{t}_1 \tilde{t}_2} = 2(\alpha_{\tilde{W}_i}^{\tilde{t}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_2} + \beta_{\tilde{W}_i}^{\tilde{t}_1} \beta_{\tilde{W}_i}^{\tilde{t}_2}) \cos 2\theta_t G_8(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{W}_i}), \quad (\text{G.30})$$

$$\begin{aligned} \Gamma_{\tilde{t}_1 \tilde{b}_1} = & (\cos \theta_t \sin \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1} + \sin \theta_t \cos \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1}) G_6(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) \\ & - (\cos \theta_t \cos \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1} + \sin \theta_t \sin \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1}) G_4(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) \\ & - (\cos \theta_t \cos \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1} + \sin \theta_t \sin \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1}) G_5(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) \\ & - (\cos \theta_t \sin \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1} + \sin \theta_t \cos \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1}) G_7(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}), \end{aligned} \quad (\text{G.31})$$

$$\begin{aligned} \Gamma_{\tilde{t}_1 \tilde{b}_2} = & (-\cos \theta_t \cos \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1} + \sin \theta_t \sin \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1}) G_6(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) \\ & - (\cos \theta_t \sin \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1} - \sin \theta_t \cos \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1}) G_4(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) \\ & - (\cos \theta_t \sin \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1} - \sin \theta_t \cos \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1}) G_5(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) \\ & - (-\cos \theta_t \cos \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1} + \sin \theta_t \sin \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1}) G_7(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}), \end{aligned} \quad (\text{G.32})$$

$$\begin{aligned} \Gamma_{\tilde{t}_2 \tilde{b}_1} = & (\sin \theta_t \sin \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1} - \cos \theta_t \cos \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1}) G_6(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) \\ & - (\sin \theta_t \cos \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1} - \cos \theta_t \sin \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1}) G_4(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) \\ & - (\sin \theta_t \cos \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1} - \cos \theta_t \sin \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1}) G_5(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) \\ & - (\sin \theta_t \sin \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1} - \cos \theta_t \cos \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1}) G_7(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}), \end{aligned} \quad (\text{G.33})$$

$$\begin{aligned} \Gamma_{\tilde{t}_2 \tilde{b}_2} = & -(\sin \theta_t \cos \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1} + \cos \theta_t \sin \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1}) G_6(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) \\ & - (\sin \theta_t \sin \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1} + \cos \theta_t \cos \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1}) G_4(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) \\ & - (\sin \theta_t \sin \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1} + \cos \theta_t \cos \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1}) G_5(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}) \\ & + (\sin \theta_t \cos \theta_b \beta_{\tilde{W}_i}^{\tilde{b}_1} \beta_{\tilde{W}_i}^{\tilde{t}_1} + \cos \theta_t \sin \theta_b \alpha_{\tilde{W}_i}^{\tilde{b}_1} \alpha_{\tilde{W}_i}^{\tilde{t}_1}) G_7(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{W}_i}). \end{aligned} \quad (\text{G.34})$$

The integrals  $G_1$  to  $G_8$  are given by the following, where  $s_t = m_{\tilde{g}}^2 + m_{\tilde{t}}^2 - 2E_t m_{\tilde{g}}$  and  $s_b = m_{\tilde{g}}^2 + m_{\tilde{b}}^2 - 2E_b m_{\tilde{g}}$ :

$$G_1(m_{\tilde{g}}, m_{\tilde{t}_k}, m_{\tilde{W}_i}) = m_{\tilde{g}} \int \frac{dE_t p_t E_t (s_t - m_{\tilde{W}_i}^2)^2}{(s_t - m_{\tilde{t}_k}^2)^2 s_t}, \quad (\text{G.35})$$

$$G_2(m_{\tilde{g}}, m_{\tilde{b}_k}, m_{\tilde{W}_i}) = m_{\tilde{g}} \int dE_b E_b^2 \lambda^{\frac{1}{2}}(s_b, m_{\tilde{W}_i}^2, m_{\tilde{t}}^2) \frac{s_b - m_{\tilde{t}}^2 - m_{\tilde{W}_i}^2}{(s_b^2 - m_{\tilde{b}_k}^2)^2 s_b}, \quad (\text{G.36})$$

$$G_3(m_{\tilde{g}}, m_{\tilde{b}_k}, m_{\tilde{W}_i}) = \int dE_b E_b^2 \lambda^{\frac{1}{2}}(s_b, m_{\tilde{W}_i}^2, m_{\tilde{t}}^2) \frac{4m_{\tilde{g}} m_{\tilde{W}_i} m_{\tilde{t}}}{(s_b^2 - m_{\tilde{b}_k}^2)^2 s_b}, \quad (\text{G.37})$$

$$G_4(m_{\tilde{g}}, m_{\tilde{t}_j}, m_{\tilde{b}_k}, m_{\tilde{W}_i}) = m_{\tilde{g}} m_{\tilde{W}_i} \int \frac{dE_t}{s_t - m_{\tilde{t}_j}^2} \left[ E_b(\max) - E_b(\min) - \frac{m_{\tilde{b}_j}^2 + m_{\tilde{t}}^2 - 2E_t m_{\tilde{g}} - m_{\tilde{W}_i}^2}{2m_{\tilde{g}}} \log X \right], \quad (\text{G.38})$$

$$G_5(m_{\tilde{g}}, m_{\tilde{t}_j}, m_{\tilde{b}_k}, m_{\tilde{W}_i}) = \frac{m_t}{2} \int dE_t \frac{s_t - m_{\tilde{W}_i}^2}{s_t - m_{\tilde{t}_j}^2} \log X, \quad (\text{G.39})$$

$$G_6(m_{\tilde{g}}, m_{\tilde{t}_j}, m_{\tilde{b}_k}, m_{\tilde{W}_i}) = \frac{1}{2} \int \frac{dE_t}{s_t - m_{\tilde{t}_j}^2} \left\{ [m_{\tilde{g}}(s_t - m_{\tilde{W}_i}^2) + \frac{m_{\tilde{b}_k}^2 - m_{\tilde{g}}^2}{m_{\tilde{g}}} s_t] \log X - 2s_t [E_b(\max) - E_b(\min)] \right\}, \quad (\text{G.40})$$

$$G_7(m_{\tilde{g}}, m_{\tilde{t}_j}, m_{\tilde{b}_k}, m_{\tilde{W}_i}) = \frac{1}{2} m_{\tilde{W}_i} m_t \int \frac{dE_t}{s_t - m_{\tilde{t}_j}^2} \left\{ 2[E_b(\max) - E_b(\min)] - \frac{m_{\tilde{b}_k}^2 - m_{\tilde{g}}^2}{m_{\tilde{g}}} \log X \right\}, \quad (\text{G.41})$$

$$G_8(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{W}_i}) = m_{\tilde{g}} m_t \int dE_t \frac{(s_t - m_{\tilde{W}_i}^2)[E_b(\max) - E_b(\min)]}{(s_t - m_{\tilde{t}_1}^2)(s_t - m_{\tilde{t}_2}^2)}. \quad (\text{G.42})$$

The limits of integration here are  $m_t$  to  $(m_{\tilde{g}}^2 + m_{\tilde{t}_i}^2 - (m_{\tilde{W}_i} + m_b)^2)/(2m_{\tilde{g}})$  for the  $E_t$  integrals, and  $m_b$  to  $(m_{\tilde{g}}^2 - (m_t + m_{\tilde{W}_i})^2)/(2m_{\tilde{g}})$  for the  $E_b$  integrals. Here  $p_t = \sqrt{E_t^2 - m_{\tilde{t}_i}^2}$ ,  $E_b(\max/\min)$  and  $X$  are given by:

$$E_b(\max/\min) = \frac{(s_t + m_b^2 - m_{\tilde{W}_i}^2)(m_{\tilde{g}} - E_t) \pm \lambda^{\frac{1}{2}}(s_t, m_b^2, m_{\tilde{W}_i}^2)}{2s_t}, \quad (\text{G.43})$$

$$X = \frac{m_{\tilde{b}_j}^2 + 2E_b(\max)m_{\tilde{g}} - m_{\tilde{g}}^2}{m_{\tilde{b}_j}^2 + 2E_b(\min)m_{\tilde{g}} - m_{\tilde{g}}^2}. \quad (\text{G.44})$$

The formulae for the first and second generation quarks can be obtained from those for the third generation straightforwardly, in fact they are simpler as the Yukawa coupling can often be neglected.

## Appendix G.2. Neutralino $1 \rightarrow 3$ Decays

$$\tilde{Z}_i \rightarrow \tilde{Z}_j f \bar{f}$$

For the 3 body decay of a neutralino into a lighter neutralino and a fermion-antifermion pair there are three types of contribution, as illustrated in the Feynman diagrams in Fig G.8;  $Z$  boson exchange, neutral Higgs boson exchange and sfermion exchange. The effects of Yukawa couplings, sfermion mixing and finite non-zero quark masses in the final state have been included. Similarly, the effects of negative neutralino masses in initial or final states are included. However, we have taken the approach of Baer et al in references [30, 39]; whilst we have included the effects of the quark mass in the phase space, the quark mass has been approximated as zero in the squared matrix element. This approximation is justified on the basis that the fermion-antifermion pair may not be a  $t\bar{t}$  pair as the decay calculator only evaluates the 3 body decays when 2 body decays are absent, given the dominance of 2 body modes over 3 body modes in branching ratios. Whenever the 3 body mode  $\tilde{Z}_i \rightarrow \tilde{Z}_j t\bar{t}$  is available then so are the 2 body modes  $\tilde{Z}_i \rightarrow \tilde{Z}_j Z$  and  $\tilde{Z}_i \rightarrow \tilde{Z}_j h$ , which will make the 3 body modes negligible. It is however crucial to include the effects of the non-zero quark masses in the phase space, as has been done, as often the phase space available to these decays is limited (e.g. for compressed spectra) and so the reduction in phase space caused by the finite quark masses is important. Similarly, with non-zero quark masses the Higgs intermediate contributions are allowed. Nonetheless the effect of the approximation is just to remove the Higgs boson -  $Z$  interferences and CP even - CP odd Higgs boson interferences, which are generally necessarily small compared to other contributions. The formulae for the included contributions themselves are all taken from sPHENO, as for the other 3 body modes. It should also be noted that the calculation in sPHENO was done in the Feynman gauge so a goldstone contribution, corresponding to the longitudinal component of the  $Z$  boson, is required. The included contributions are therefore the squared  $Z$  (including goldstone),  $\phi$  and  $\tilde{f}$  contributions as well as  $hH$ ,  $Z\tilde{f}$ ,  $\phi\tilde{f}$  and  $ZA$  interferences.<sup>3</sup>

<sup>3</sup>Calculations of the differential decay rates for this mode are also available, using a different approach to that used here, in reference [29].

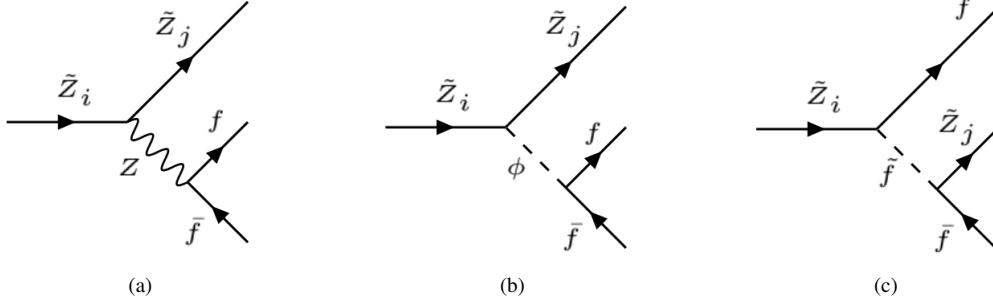


Figure G.8: Feynman diagrams for the 3 body decay of a neutralino into a lighter neutralino and a fermion antifermion pair, as mediated by Z bosons, Higgs bosons  $\phi = h, H, A$ , or sfermions  $\tilde{f}_{1/2}$ .  $i > j$  and  $i, j \in 1, 2, 3, 4$ .

$$\begin{aligned} \Gamma(\tilde{Z}_i \rightarrow \tilde{Z}_j f \bar{f}) = \frac{N_c}{512\pi^3 |m_{\tilde{Z}_i}|^3} & (\Gamma_Z + \Gamma_h + \Gamma_H + \Gamma_A + \Gamma_{hH} + \Gamma_{\tilde{f}} - 4\Gamma_{h\tilde{f}_1} - 4\Gamma_{h\tilde{f}_2} - 4\Gamma_{H\tilde{f}_1} \\ & - 4\Gamma_{H\tilde{f}_2} - 4\Gamma_{A\tilde{f}_1} - 4\Gamma_{A\tilde{f}_2} + 4\Gamma_{Z\tilde{f}_1} - 4\Gamma_{Z\tilde{f}_2} - 4\Gamma_{ZA} + \Gamma_G \\ & + 2\Gamma_{GA} - 4\Gamma_{ZG} - 4\Gamma_{G\tilde{f}_1} - 4\Gamma_{G\tilde{f}_2}). \end{aligned} \quad (\text{G.45})$$

$G$  indicates the goldstone contribution. In the following contributions we again account for negative neutralino masses via factors of  $-1$  corresponding to the effects of absorbing factors of  $-i\gamma_5$  into the couplings for negative mass neutralinos:

$$(-1)^{\theta_i} = \begin{cases} 1, & \text{for } m_{\tilde{Z}_i} > 0, \\ -1, & \text{for } m_{\tilde{Z}_i} < 0. \end{cases} \quad (\text{G.46})$$

The contributions are as follows:

$$\Gamma_h = 2(X_{ij}^h + X_{ji}^h)^2 f_q^2 t_{\alpha_h}^2 \left[ I_4^h - 2m_f^2 I_3^h + 2(-1)^{\theta_j} |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}| I_2^h - 4(-1)^{\theta_j} |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}| m_f^2 I_1^h \right], \quad (\text{G.47})$$

where

$$s = m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2 - 2|m_{\tilde{Z}_i}|E, \quad (\text{G.48})$$

$$E_{\max} = \frac{m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2 - 4m_f^2}{2|m_{\tilde{Z}_i}|}, \quad (\text{G.49})$$

$$I_1^h = \int_{|m_{\tilde{Z}_j}|}^{E_{\max}} dE \frac{2|m_{\tilde{Z}_i}| \sqrt{s - 4m_f^2} \sqrt{E^2 - m_{\tilde{Z}_j}^2} 2|m_{\tilde{Z}_i}|}{\sqrt{s}(s - m_h^2)^2}, \quad (\text{G.50})$$

$$I_2^h = \int_{|m_{\tilde{Z}_j}|}^{E_{\max}} dE \frac{2|m_{\tilde{Z}_i}| \sqrt{s - 4m_f^2} \sqrt{E^2 - m_{\tilde{Z}_j}^2} 2|m_{\tilde{Z}_i}|(s - 2m_f^2)}{\sqrt{s}(s - m_h^2)^2}, \quad (\text{G.51})$$

$$I_3^h = \int_{|m_{\tilde{Z}_j}|}^{E_{\max}} dE \frac{2|m_{\tilde{Z}_i}| \sqrt{s - 4m_f^2} \sqrt{E^2 - m_{\tilde{Z}_j}^2} 2|m_{\tilde{Z}_i}| 2|m_{\tilde{Z}_i}|E}{\sqrt{s}(s - m_h^2)^2}, \quad (\text{G.52})$$

$$I_4^h = \int_{|m_{\tilde{Z}_j}|}^{E_{\max}} dE \frac{2|m_{\tilde{Z}_i}| \sqrt{s - 4m_f^2} \sqrt{E^2 - m_{\tilde{Z}_j}^2} 2|m_{\tilde{Z}_i}|(s - 2m_f^2) 2|m_{\tilde{Z}_i}|E}{\sqrt{s}(s - m_h^2)^2}. \quad (\text{G.53})$$

$$\Gamma_H = 2(X_{ij}^H + X_{ji}^H)^2 f_q^2 t_{\alpha_H}^2 \left[ I_4^H - 2m_f^2 I_3^H + 2(-1)^{\theta_j} |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}| I_2^H - 4(-1)^{\theta_j} |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}| m_f^2 I_1^H \right]. \quad (\text{G.54})$$



where the  $I_{1,2,3,4}^H$  are exactly the same as the  $I_{1,2,3,4}^h$  with the change  $m_h \rightarrow m_H$ .

$$\Gamma_Z = 64g^2 \sin^2 \theta_W W_{ij}^2 \left[ 4(-1)^{\theta_j} |m_{\bar{Z}_i}| |m_{\bar{Z}_j}| m_f^2 (\alpha_f^2 - \beta_f^2) I_4^Z + m_f^2 (\alpha_f^2 - \beta_f^2) I_3^Z \right. \\ \left. + (-1)^{\theta_i} (-1)^{\theta_j} |m_{\bar{Z}_i}| |m_{\bar{Z}_j}| (\alpha_f^2 + \beta_f^2) I_2^Z + \frac{1}{2} (\alpha_f^2 + \beta_f^2) I_1^Z \right], \quad (\text{G.55})$$

where the integrals  $I_i^Z$  are:

$$I_1^Z = \int_{4m_f^2}^{(|m_{\bar{Z}_i}| - |m_{\bar{Z}_j}|)^2} ds \left[ \frac{1}{3s^2(s - m_Z^2)^2} \{-2s^4 + (m_{\bar{Z}_i}^2 + m_{\bar{Z}_j}^2 + 2m_f^2)s^3 + [(m_{\bar{Z}_i}^2 - m_{\bar{Z}_j}^2)^2 \right. \\ \left. - 2(m_{\bar{Z}_i}^2 + m_{\bar{Z}_j}^2)2m_f^2]s^2 + 2m_f^2(m_{\bar{Z}_i}^2 - m_{\bar{Z}_j}^2)^2 s\} \frac{1}{s} \right. \\ \left. \times \sqrt{(s - (|m_{\bar{Z}_i}| - |m_{\bar{Z}_j}|)^2)((s - (|m_{\bar{Z}_i}| + |m_{\bar{Z}_j}|)^2))} \sqrt{s(s - 4m_f^2)} \right], \quad (\text{G.56})$$

$$I_2^Z = \int_{4m_f^2}^{(|m_{\bar{Z}_i}| - |m_{\bar{Z}_j}|)^2} ds \left[ \frac{1}{s(s - m_Z^2)^2} (s - 2m_f^2) \sqrt{(s - (|m_{\bar{Z}_i}| - |m_{\bar{Z}_j}|)^2)((s - (|m_{\bar{Z}_i}| + |m_{\bar{Z}_j}|)^2))} \right. \\ \left. \times \sqrt{s(s - 4m_f^2)} \right], \quad (\text{G.57})$$

$$I_3^Z = \int_{4m_f^2}^{(|m_{\bar{Z}_i}| - |m_{\bar{Z}_j}|)^2} ds \left[ \frac{1}{s(s - m_Z^2)^2} (m_{\bar{Z}_i}^2 + m_{\bar{Z}_j}^2 - s) \sqrt{(s - (|m_{\bar{Z}_i}| - |m_{\bar{Z}_j}|)^2)((s - (|m_{\bar{Z}_i}| + |m_{\bar{Z}_j}|)^2))} \right. \\ \left. \times \sqrt{s(s - 4m_f^2)} \right], \quad (\text{G.58})$$

$$I_4^Z = \int_{4m_f^2}^{(|m_{\bar{Z}_i}| - |m_{\bar{Z}_j}|)^2} ds \left[ \frac{1}{s(s - m_Z^2)^2} \sqrt{(s - (|m_{\bar{Z}_i}| - |m_{\bar{Z}_j}|)^2)((s - (|m_{\bar{Z}_i}| + |m_{\bar{Z}_j}|)^2))} \sqrt{s(s - 4m_f^2)} \right]. \quad (\text{G.59})$$

$$\Gamma_A = (X_{ij}^A + X_{ji}^A)^2 A_q^2 \left[ I_4^A + 2m_f^2 I_3^A - 2(-1)^{\theta_i} (-1)^{\theta_j} |m_{\bar{Z}_i}| |m_{\bar{Z}_j}| I_2^A - 4(-1)^{\theta_i} (-1)^{\theta_j} m_f^2 |m_{\bar{Z}_i}| |m_{\bar{Z}_j}| I_1^A \right], \quad (\text{G.60})$$

where the integrals  $I_i^A$  are:

$$I_1^A = \int_{|m_{\bar{Z}_j}|}^{E_{\max}} dE \sqrt{E^2 - m_{\bar{Z}_j}^2} \sqrt{s - 4m_f^2} \frac{4m_{\bar{Z}_i}^2}{\sqrt{s(s - m_A^2)^2}}, \quad (\text{G.61})$$

$$I_2^A = \int_{|m_{\bar{Z}_j}|}^{E_{\max}} dE \sqrt{E^2 - m_{\bar{Z}_j}^2} \sqrt{s - 4m_f^2} \frac{4m_{\bar{Z}_i}^2 (s - 2m_f^2)}{\sqrt{s(s - m_A^2)^2}}, \quad (\text{G.62})$$

$$I_3^A = \int_{|m_{\bar{Z}_j}|}^{E_{\max}} dE \sqrt{E^2 - m_{\bar{Z}_j}^2} \sqrt{s - 4m_f^2} \frac{4m_{\bar{Z}_i}^2 2|m_{\bar{Z}_i}|E}{\sqrt{s(s - m_A^2)^2}}, \quad (\text{G.63})$$

$$I_4^A = \int_{|m_{\bar{Z}_j}|}^{E_{\max}} dE \sqrt{E^2 - m_{\bar{Z}_j}^2} \sqrt{s - 4m_f^2} \frac{4m_{\bar{Z}_i}^2 2|m_{\bar{Z}_i}|E(s - 2m_f^2)}{\sqrt{s(s - m_A^2)^2}}, \quad (\text{G.64})$$

$$\Gamma_{hH} = 4(X_{ij}^h + X_{ji}^h)(X_{ij}^H + X_{ji}^H) f_q^2 t_{\alpha_h} t_{\alpha_H} \left[ I_4^{hH} - 2m_f^2 I_3^{hH} + 2(-1)^{\theta_i} (-1)^{\theta_j} |m_{\bar{Z}_i}| |m_{\bar{Z}_j}| I_2^{hH} \right. \\ \left. - 4(-1)^{\theta_i} (-1)^{\theta_j} |m_{\bar{Z}_i}| |m_{\bar{Z}_j}| m_f^2 I_1^{hH} \right], \quad (\text{G.65})$$

where:

$$I_1^{hH} = \int_{|m_{\bar{Z}_j}|}^{E_{\max}} dE \left[ 2|m_{\bar{Z}_i}| \sqrt{s - 4m_f^2} \sqrt{E^2 - m_{\bar{Z}_j}^2} \frac{2|m_{\bar{Z}_i}|}{\sqrt{s(s - m_h^2)(s - m_H^2)}} \right], \quad (\text{G.66})$$

$$I_2^{hH} = \int_{|m_{Z_j}|}^{E_{\max}} dE \left[ 2|m_{\tilde{Z}_i}| \sqrt{s - 4m_f^2} \sqrt{E^2 - m_{\tilde{Z}_j}^2} \frac{2|m_{\tilde{Z}_i}|(s - 2m_f^2)}{\sqrt{s}(s - m_h^2)(s - m_H^2)} \right], \quad (G.67)$$

$$I_3^{hH} = \int_{|m_{Z_j}|}^{E_{\max}} dE \left[ 2|m_{\tilde{Z}_i}| \sqrt{s - 4m_f^2} \sqrt{E^2 - m_{\tilde{Z}_j}^2} \frac{4m_{\tilde{Z}_i}^2 E}{\sqrt{s}(s - m_h^2)(s - m_H^2)} \right], \quad (G.68)$$

$$I_4^{hH} = \int_{|m_{Z_j}|}^{E_{\max}} dE \left[ 2|m_{\tilde{Z}_i}| \sqrt{s - 4m_f^2} \sqrt{E^2 - m_{\tilde{Z}_j}^2} \frac{4m_{\tilde{Z}_i}^2 E(s - 2m_f^2)}{\sqrt{s}(s - m_h^2)(s - m_H^2)} \right]. \quad (G.69)$$

$$\Gamma_{\tilde{f}} = 2\Gamma_{\tilde{f}_1\tilde{f}_1}^{diag} + 2\Gamma_{\tilde{f}_2\tilde{f}_2}^{diag} + 2\Gamma_{\tilde{f}\tilde{f}}^{nondiag} + \Gamma_{\tilde{f}_1\tilde{f}_1}^{tu} + \Gamma_{\tilde{f}_2\tilde{f}_2}^{tu} + 2\Gamma_{\tilde{f}_1\tilde{f}_2}^{tu}, \quad (G.70)$$

There are many sfermion contributions included here, they are as follows:

$$\begin{aligned} \Gamma_{\tilde{f}_1\tilde{f}_1}^{diag} = & \frac{8m_{\tilde{Z}_i}^2}{\pi^2} (\alpha_{\tilde{f}_1}^{\tilde{Z}_i^2} + \beta_{\tilde{f}_1}^{\tilde{Z}_i^2}) (\alpha_{\tilde{f}_1}^{\tilde{Z}_j^2} + \beta_{\tilde{f}_1}^{\tilde{Z}_j^2}) \tilde{\psi}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \\ & + \frac{32m_{\tilde{Z}_i}^2}{\pi^2} (-1)^{\theta_j} (\alpha_{\tilde{f}_1}^{\tilde{Z}_i^2} + \beta_{\tilde{f}_1}^{\tilde{Z}_i^2}) \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j} m_f |m_{\tilde{Z}_j}| \tilde{\chi}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \\ & - \frac{32m_{\tilde{Z}_i}^2}{\pi^2} (-1)^{\theta_i} (\alpha_{\tilde{f}_1}^{\tilde{Z}_i^2} + \beta_{\tilde{f}_1}^{\tilde{Z}_i^2}) \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i} m_f |m_{\tilde{Z}_i}| X(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \\ & - \frac{64m_{\tilde{Z}_i}^2}{\pi^2} (-1)^{\theta_i} (-1)^{\theta_j} \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j} |m_{\tilde{Z}_i}| m_f^2 |m_{\tilde{Z}_j}| \zeta(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j), \end{aligned} \quad (G.71)$$

$$\begin{aligned} \Gamma_{\tilde{f}_2\tilde{f}_2}^{diag} = & \frac{8m_{\tilde{Z}_i}^2}{\pi^2} (\alpha_{\tilde{f}_2}^{\tilde{Z}_i^2} + \beta_{\tilde{f}_2}^{\tilde{Z}_i^2}) (\alpha_{\tilde{f}_2}^{\tilde{Z}_j^2} + \beta_{\tilde{f}_2}^{\tilde{Z}_j^2}) \tilde{\psi}(\tilde{Z}_i, \tilde{f}_2, \tilde{f}_2, \tilde{Z}_j) \\ & + \frac{32m_{\tilde{Z}_i}^2}{\pi^2} (-1)^{\theta_j} (\alpha_{\tilde{f}_2}^{\tilde{Z}_i^2} + \beta_{\tilde{f}_2}^{\tilde{Z}_i^2}) \alpha_{\tilde{f}_2}^{\tilde{Z}_j} \beta_{\tilde{f}_2}^{\tilde{Z}_j} m_f |m_{\tilde{Z}_j}| \tilde{\chi}(\tilde{Z}_i, \tilde{f}_2, \tilde{f}_2, \tilde{Z}_j) \\ & - \frac{32m_{\tilde{Z}_i}^2}{\pi^2} (-1)^{\theta_i} (\alpha_{\tilde{f}_2}^{\tilde{Z}_i^2} + \beta_{\tilde{f}_2}^{\tilde{Z}_i^2}) \alpha_{\tilde{f}_2}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} m_f |m_{\tilde{Z}_i}| X(\tilde{Z}_i, \tilde{f}_2, \tilde{f}_2, \tilde{Z}_j) \\ & - \frac{64m_{\tilde{Z}_i}^2}{\pi^2} (-1)^{\theta_i} (-1)^{\theta_j} \alpha_{\tilde{f}_2}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} \alpha_{\tilde{f}_2}^{\tilde{Z}_j} \beta_{\tilde{f}_2}^{\tilde{Z}_j} |m_{\tilde{Z}_i}| m_f^2 |m_{\tilde{Z}_j}| \zeta(\tilde{Z}_i, \tilde{f}_2, \tilde{f}_2, \tilde{Z}_j), \end{aligned} \quad (G.72)$$

where the  $\tilde{\psi}$ ,  $\tilde{\chi}$ ,  $X$ ,  $\zeta$  integrals have been used before for the gluino 3 body decays and are:

$$E_f^{max} = \frac{m_{\tilde{Z}_i}^2 - 2m_f |m_{\tilde{Z}_j}| - m_{\tilde{Z}_j}^2}{2|m_{\tilde{Z}_i}|}, \quad (G.73)$$

$$\tilde{\psi}(m_{\tilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\tilde{Z}_j}) = \pi^2 m_{\tilde{Z}_i} \int_{m_f}^{E_f^{max}} dE_f \sqrt{E_f^2 - m_f^2} E_f \frac{\lambda^{\frac{1}{2}}(s, m_f^2, m_{\tilde{Z}_j}^2)}{s} \frac{m_{\tilde{Z}_i}^2 - m_{\tilde{Z}_j}^2 - 2m_{\tilde{Z}_i} E_f}{(s - m_{\tilde{f}_1}^2)(s - m_{\tilde{f}_2}^2)}, \quad (G.74)$$

$$\tilde{\chi}(m_{\tilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\tilde{Z}_j}) = \pi^2 m_{\tilde{Z}_i} \int_{m_f}^{E_f^{max}} dE_f \sqrt{E_f^2 - m_f^2} E_f \frac{\lambda^{\frac{1}{2}}(s, m_f^2, m_{\tilde{Z}_j}^2)}{s} \frac{1}{(s - m_{\tilde{f}_1}^2)(s - m_{\tilde{f}_2}^2)}, \quad (G.75)$$

$$X(m_{\tilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\tilde{Z}_j}) = \frac{\pi^2}{2} \int_{m_f}^{E_f^{max}} dE_f \sqrt{E_f^2 - m_f^2} \frac{m_{\tilde{Z}_i}^2 - m_{\tilde{Z}_j}^2 - 2m_{\tilde{Z}_i} E_f}{m_{\tilde{Z}_i}^2 + m_f^2 - 2m_{\tilde{Z}_i} E_f} \frac{\lambda^{\frac{1}{2}}(s, m_f^2, m_{\tilde{Z}_j}^2)}{(s - m_{\tilde{f}_1}^2)(s - m_{\tilde{f}_2}^2)}, \quad (G.76)$$

$$\zeta(m_{\tilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\tilde{Z}_j}) = \pi^2 \int_{m_f}^{E_f^{max}} dE_f \frac{[E_f(max) - E_f(min)]}{(s - m_{\tilde{f}_1}^2)(s - m_{\tilde{f}_2}^2)}. \quad (G.77)$$

Later the following integrals will also be required:

$$\tilde{\phi}(m_{\tilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\tilde{Z}_j}) = \frac{1}{2} \pi^2 |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}| \int_{m_f}^{E_f^{\max}} dE_f \frac{1}{s - m_{\tilde{f}_1}^2} \left[ - [E_{\tilde{f}}(\max) - E_{\tilde{f}}(\min)] - \frac{m_{\tilde{Z}_j}^2 - m_f^2 + 2|m_{\tilde{Z}_i}|E_f - m_{\tilde{f}_2}^2}{2|m_{\tilde{Z}_i}|} \log Z(m_{\tilde{f}_2}) \right], \quad (\text{G.78})$$

$$\text{where } Z(m_{\tilde{f}_2}) = \frac{m_{\tilde{Z}_i}^2 + m_f^2 - 2|m_{\tilde{Z}_i}|E_f(\max) - m_{\tilde{f}_2}^2}{m_{\tilde{Z}_i}^2 + m_f^2 - 2|m_{\tilde{Z}_i}|E_f(\min) - m_{\tilde{f}_2}^2}.$$

$$\xi(m_{\tilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\tilde{Z}_j}) = \frac{1}{2} \pi^2 \int_{m_f}^{E_f^{\max}} dE_f \frac{1}{s - m_{\tilde{f}_1}^2} \left[ [E_{\tilde{f}}(\max) - E_{\tilde{f}}(\min)] - \frac{m_{\tilde{Z}_i}^2 - m_f^2 - 2|m_{\tilde{Z}_i}|E_f + m_{\tilde{f}_2}^2}{2|m_{\tilde{Z}_i}|} \log Z(m_{\tilde{f}_2}) \right], \quad (\text{G.79})$$

$$\tilde{\rho}(m_{\tilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\tilde{Z}_j}) = -\frac{\pi^2}{2|m_{\tilde{Z}_i}|} \int_{m_f}^{E_f^{\max}} dE_f \frac{1}{s - m_{\tilde{f}_1}^2} \log Z(m_{\tilde{f}_2}), \quad (\text{G.80})$$

$$Y(m_{\tilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\tilde{Z}_j}) = \frac{\pi^2}{2} \int_{m_f}^{E_f^{\max}} dE_f \frac{1}{s - m_{\tilde{f}_1}^2} \left[ [E_{\tilde{f}}(\max) - E_{\tilde{f}}(\min)] s + \frac{1}{2|m_{\tilde{Z}_i}|} (m_{\tilde{Z}_i}^2 m_{\tilde{Z}_j}^2 - m_{\tilde{Z}_i}^2 m_{\tilde{f}_2}^2 + m_f^4 + 2|m_{\tilde{Z}_i}|E_f m_{\tilde{f}_2}^2 - m_{\tilde{f}_2}^2 m_f^2) \times \log Z(m_{\tilde{f}_2}) \right], \quad (\text{G.81})$$

$$\chi'(m_{\tilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\tilde{Z}_j}) = \frac{-\pi^2}{2} \int_{m_f}^{E_f^{\max}} \frac{dE_f E_f}{s - m_{\tilde{f}_2}^2} \log Z(m_{\tilde{f}_1}), \quad (\text{G.82})$$

where here

$$E_{\tilde{f}}(\max/\min) = \frac{(s + m_f^2 - m_{\tilde{Z}_j}^2)(|m_{\tilde{Z}_i}| - E_f) \pm \sqrt{(E_f^2 - m_f^2)(s + m_f^2 - m_{\tilde{Z}_j}^2)^2 - 4(E_f^2 - m_f^2)m_{\tilde{f}_2}^2 s}}{2s}. \quad (\text{G.83})$$

$$\begin{aligned} \Gamma_{\tilde{f}\tilde{f}}^{\text{nondiag}} = & \frac{16m_{\tilde{Z}_i}^2}{\pi^2} \left[ (\beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_2}^{\tilde{Z}_i})(\alpha_{\tilde{f}_1}^{\tilde{Z}_j} \alpha_{\tilde{f}_2}^{\tilde{Z}_j} + \beta_{\tilde{f}_2}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j}) \tilde{\psi}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \right. \\ & + 2(-1)^{\theta_j} (\beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_2}^{\tilde{Z}_i})(\alpha_{\tilde{f}_2}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_2}^{\tilde{Z}_j}) m_f |m_{\tilde{Z}_j}| \tilde{\chi}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \\ & - 2(-1)^{\theta_i} (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} + \alpha_{\tilde{f}_2}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i})(\alpha_{\tilde{f}_1}^{\tilde{Z}_j} \alpha_{\tilde{f}_2}^{\tilde{Z}_j} + \beta_{\tilde{f}_2}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j}) |m_{\tilde{Z}_i}| m_f X(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \\ & \left. - 2(-1)^{\theta_i} (-1)^{\theta_j} (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} + \alpha_{\tilde{f}_2}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i})(\alpha_{\tilde{f}_2}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_2}^{\tilde{Z}_j}) m_f^2 |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}| \zeta(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \right], \quad (\text{G.84}) \end{aligned}$$

$$\begin{aligned}
\Gamma_{\tilde{f}_1 \tilde{f}_1}^{tu} = & -2 \left\{ 8(\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j}) m_{\tilde{Z}_i}^2 \pi^2 (-1)^{\theta_i} Y(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \right. \\
& - (\alpha_{\tilde{f}_1}^{\tilde{Z}_i^2} \alpha_{\tilde{f}_1}^{\tilde{Z}_j^2} + \beta_{\tilde{f}_1}^{\tilde{Z}_i^2} \beta_{\tilde{f}_1}^{\tilde{Z}_j^2}) 8 m_{\tilde{Z}_i}^2 \pi^2 (-1)^{\theta_i} (-1)^{\theta_j} \tilde{\phi}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \\
& + (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j}) m_f^2 \frac{8 m_{\tilde{Z}_i}^2}{\pi^2} (-1)^{\theta_i} \xi(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \\
& - \{ (\alpha_{\tilde{f}_1}^{\tilde{Z}_i^2} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i^2} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j}) |m_{\tilde{Z}_i}| m_f \} \left[ \frac{8 m_{\tilde{Z}_i}^2}{\pi^2} \xi(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \right. \\
& - \frac{4 m_{\tilde{Z}_i}^2}{\pi^2} (m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2) (-1)^{\theta_i} \tilde{\rho}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) + \frac{8 m_{\tilde{Z}_i}^2}{\pi^2} \chi'(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \Big] \\
& + |m_{\tilde{Z}_j}| (-1)^{\theta_j} m_f (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j^2} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j^2}) \left[ \frac{-8 m_{\tilde{Z}_i}^2}{\pi^2} \xi(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \right. \\
& + \frac{8 m_{\tilde{Z}_i}^4}{\pi^2} (-1)^{\theta_i} \tilde{\rho}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) - \frac{8 m_{\tilde{Z}_i}^2}{\pi^2} \chi'(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \Big] \\
& - (\beta_{\tilde{f}_1}^{\tilde{Z}_i^2} \beta_{\tilde{f}_1}^{\tilde{Z}_j} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i^2} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j}) |m_{\tilde{Z}_i}| (-1)^{\theta_i} m_f \left[ \frac{4 m_{\tilde{Z}_i}^2}{\pi^2} (m_{\tilde{Z}_i}^2 - m_{\tilde{Z}_j}^2) \tilde{\rho}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \right. \\
& - \frac{8 m_{\tilde{Z}_i}^2}{\pi^2} \chi'(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \Big] \\
& + (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j^2} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j^2}) m_f |m_{\tilde{Z}_j}| \frac{8 m_{\tilde{Z}_i}^2}{\pi^2} (-1)^{\theta_i} (-1)^{\theta_j} \chi'(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \\
& \left. - 2(\beta_{\tilde{f}_1}^{\tilde{Z}_i^2} \alpha_{\tilde{f}_1}^{\tilde{Z}_j^2} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i^2} \beta_{\tilde{f}_1}^{\tilde{Z}_j^2}) m_f^2 (-1)^{\theta_i} (-1)^{\theta_j} |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}| \frac{4 m_{\tilde{Z}_i}^2}{\pi^2} \tilde{\rho}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_1, \tilde{Z}_j) \right\}.
\end{aligned} \tag{G.85}$$

$\Gamma_{\tilde{f}_2 \tilde{f}_2}^{tu}$  is as above but with  $\tilde{f}_1 \rightarrow \tilde{f}_2$  everywhere including in the masses and integrals.

$$\begin{aligned}
\Gamma_{\tilde{f}_1 \tilde{f}_2}^{tu} = & -2 \left\{ 8(\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j} \alpha_{\tilde{f}_2}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_2}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_2}^{\tilde{Z}_j}) m_{\tilde{Z}_i}^2 \pi^2 Y(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \right. \\
& - (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_2}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \alpha_{\tilde{f}_2}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_2}^{\tilde{Z}_j}) 8 m_{\tilde{Z}_i}^2 \pi^2 (-1)^{\theta_i} (-1)^{\theta_j} \tilde{\phi}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \\
& + (\alpha_{\tilde{f}_2}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_2}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_2}^{\tilde{Z}_j}) m_f^2 \frac{8 m_{\tilde{Z}_i}^2}{\pi^2} (-1)^{\theta_i} \xi(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \\
& - \{ (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_2}^{\tilde{Z}_i} \alpha_{\tilde{f}_2}^{\tilde{Z}_j} \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_2}^{\tilde{Z}_j}) |m_{\tilde{Z}_i}| m_f \} \left[ \frac{8 m_{\tilde{Z}_i}^2}{\pi^2} \xi(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \right. \\
& - \frac{4 m_{\tilde{Z}_i}^2}{\pi^2} (m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2) (-1)^{\theta_i} \tilde{\rho}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) + \frac{8 m_{\tilde{Z}_i}^2}{\pi^2} \chi'(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \Big] \\
& + |m_{\tilde{Z}_j}| (-1)^{\theta_j} m_f (\alpha_{\tilde{f}_2}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \alpha_{\tilde{f}_2}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_2}^{\tilde{Z}_j}) \left[ \frac{-8 m_{\tilde{Z}_i}^2}{\pi^2} \xi(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \right. \\
& + \frac{8 m_{\tilde{Z}_i}^4}{\pi^2} (-1)^{\theta_i} \tilde{\rho}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) - \frac{8 m_{\tilde{Z}_i}^2}{\pi^2} \chi'(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \Big] \\
& - (\beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j} \alpha_{\tilde{f}_2}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_2}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_2}^{\tilde{Z}_j}) |m_{\tilde{Z}_i}| (-1)^{\theta_i} m_f \left[ \frac{4 m_{\tilde{Z}_i}^2}{\pi^2} (m_{\tilde{Z}_i}^2 - m_{\tilde{Z}_j}^2) \tilde{\rho}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \right. \\
& - \frac{8 m_{\tilde{Z}_i}^2}{\pi^2} \chi'(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \Big] \\
& + (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \alpha_{\tilde{f}_2}^{\tilde{Z}_j} + \alpha_{\tilde{f}_2}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_2}^{\tilde{Z}_j}) m_f (-1)^{\theta_j} |m_{\tilde{Z}_j}| \left[ \frac{8 m_{\tilde{Z}_i}^2}{\pi^2} \chi'(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \right. \\
& \left. \left. - 2(\beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_2}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} \alpha_{\tilde{f}_2}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_2}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j} \beta_{\tilde{f}_2}^{\tilde{Z}_j}) m_f^2 (-1)^{\theta_i} (-1)^{\theta_j} |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}| \frac{4 m_{\tilde{Z}_i}^2}{\pi^2} \tilde{\rho}(\tilde{Z}_i, \tilde{f}_1, \tilde{f}_2, \tilde{Z}_j) \right] \right\}. \tag{G.86}
\end{aligned}$$

As for the Z sfermion interferences:

$$\Gamma_{Z \tilde{f}_1} = (-1)^{\theta_i} (C_1^{Z \tilde{f}_1} I_1^{Z \tilde{f}_1} + C_2^{Z \tilde{f}_1} I_2^{Z \tilde{f}_1} + C_3^{Z \tilde{f}_1} I_3^{Z \tilde{f}_1} + C_4^{Z \tilde{f}_1} I_4^{Z \tilde{f}_1} + C_5^{Z \tilde{f}_1} I_5^{Z \tilde{f}_1} + C_6^{Z \tilde{f}_1} I_6^{Z \tilde{f}_1} + C_7^{Z \tilde{f}_1} I_7^{Z \tilde{f}_1} + C_8^{Z \tilde{f}_1} I_8^{Z \tilde{f}_1}), \tag{G.87}$$

where

$$C_1^{Z \tilde{f}_1} = -4 W_{ijg} \sin \theta_W [-\alpha_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f - \beta_f) \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f + \beta_f) \alpha_{\tilde{f}_1}^{\tilde{Z}_j}] m_f |m_{\tilde{Z}_i}|, \tag{G.88}$$

$$C_2^{Z \tilde{f}_1} = -4 (-1)^{\theta_i} (-1)^{\theta_j} W_{ijg} \sin \theta_W [-\alpha_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f + \beta_f) \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f - \beta_f) \alpha_{\tilde{f}_1}^{\tilde{Z}_j}] m_f |m_{\tilde{Z}_j}|, \tag{G.89}$$

$$C_3^{Z \tilde{f}_1} = -4 (-1)^{\theta_i} W_{ijg} \sin \theta_W [\beta_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f + \beta_f) \beta_{\tilde{f}_1}^{\tilde{Z}_j} - \alpha_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f - \beta_f) \alpha_{\tilde{f}_1}^{\tilde{Z}_j}], \tag{G.90}$$

$$C_4^{Z \tilde{f}_1} = -8 W_{ijg} \sin \theta_W [-\alpha_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f + \beta_f) \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f - \beta_f) \alpha_{\tilde{f}_1}^{\tilde{Z}_j}] |m_{\tilde{Z}_i}| m_f, \tag{G.91}$$

$$C_5^{Z \tilde{f}_1} = -8 (-1)^{\theta_i} (-1)^{\theta_j} W_{ijg} \sin \theta_W [-\alpha_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f - \beta_f) \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f + \beta_f) \alpha_{\tilde{f}_1}^{\tilde{Z}_j}] |m_{\tilde{Z}_j}| m_f, \tag{G.92}$$

$$C_6^{Z \tilde{f}_1} = -4 (-1)^{\theta_j} W_{ijg} \sin \theta_W [\beta_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f + \beta_f) \beta_{\tilde{f}_1}^{\tilde{Z}_j} - \alpha_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f - \beta_f) \alpha_{\tilde{f}_1}^{\tilde{Z}_j}] |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}|, \tag{G.93}$$

$$C_7^{Z \tilde{f}_1} = -4 (-1)^{\theta_i} W_{ijg} \sin \theta_W [\beta_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f - \beta_f) \beta_{\tilde{f}_1}^{\tilde{Z}_j} - \alpha_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f + \beta_f) \alpha_{\tilde{f}_1}^{\tilde{Z}_j}] m_f^2, \tag{G.94}$$

$$C_8^{Z \tilde{f}_1} = -16 (-1)^{\theta_j} W_{ijg} \sin \theta_W [\beta_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f - \beta_f) \beta_{\tilde{f}_1}^{\tilde{Z}_j} - \alpha_{\tilde{f}_1}^{\tilde{Z}_i} (\alpha_f + \beta_f) \alpha_{\tilde{f}_1}^{\tilde{Z}_j}] m_f^2 |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}|. \tag{G.95}$$

The upper limit for the integrals here is  $E_{upper} = \frac{(m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2 - 4m_f^2)}{2|m_{\tilde{Z}_i}|}$ . The argument of the logarithm in these integrals is as follows:

$$L = [|m_{\tilde{Z}_i}| (E_Q + Q') - \mu^2] / [|m_{\tilde{Z}_i}| (E_Q - Q') - \mu^2], \tag{G.96}$$

In these expressions,  $E_Q = \frac{s+m_{\tilde{Z}_i}^2-m_{\tilde{Z}_j}^2}{2|m_{\tilde{Z}_i}|}$ ,  $Q' = \sqrt{E_Q^2-s}\sqrt{1-4m_f^2/s}$  and  $\mu^2 = s + m_{\tilde{f}_1}^2 - m_{\tilde{Z}_j}^2 - m_q^2$ , where  $s = m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2 - 2|m_{\tilde{Z}_i}|E$ . The necessary integrals are given by:

$$I_1^{Z\tilde{f}_1} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s-m_Z^2} \left[ -2|m_{\tilde{Z}_i}| \sqrt{1-4m_q^2/s} \sqrt{E^2-m_{\tilde{Z}_j}^2} - (m_{\tilde{f}_1}^2 - m_f^2 + m_{\tilde{Z}_j}^2 - 2|m_{\tilde{Z}_i}|E) \log L \right], \quad (G.97)$$

$$I_2^{Z\tilde{f}_1} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s-m_Z^2} \left[ 2|m_{\tilde{Z}_i}| \sqrt{1-4m_q^2/s} \sqrt{E^2-m_{\tilde{Z}_j}^2} + (m_{\tilde{f}_1}^2 + m_{\tilde{Z}_i}^2 - 2|m_{\tilde{Z}_i}|E - m_f^2) \log L \right], \quad (G.98)$$

$$\begin{aligned} I_3^{Z\tilde{f}_1} = & 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s-m_Z^2} \left[ \{m_{\tilde{Z}_i}^2 + 2m_f^2 + m_{\tilde{Z}_j}^2 - \frac{3}{2}m_{\tilde{f}_1}^2 - \frac{1}{2}(m_f^2 + |m_{\tilde{Z}_i}|E \right. \\ & + |m_{\tilde{Z}_i}| \sqrt{1-4m_f^2/s} \sqrt{E^2-m_{\tilde{Z}_j}^2}) (m_f^2 + |m_{\tilde{Z}_i}|E + |m_{\tilde{Z}_i}| \sqrt{1-4m_f^2/s} \sqrt{E^2-m_{\tilde{Z}_j}^2} - m_{\tilde{f}_1}^2) \\ & - \{m_{\tilde{Z}_i}^2 + 2m_f^2 + m_{\tilde{Z}_j}^2 - \frac{3}{2}m_{\tilde{f}_1}^2 - \frac{1}{2}(m_f^2 + |m_{\tilde{Z}_i}|E - |m_{\tilde{Z}_i}| \sqrt{1-4m_f^2/s} \sqrt{E^2-m_{\tilde{Z}_j}^2}) \\ & \times (m_f^2 + |m_{\tilde{Z}_i}|E - |m_{\tilde{Z}_i}| \sqrt{1-4m_f^2/s} \sqrt{E^2-m_{\tilde{Z}_j}^2}) - m_{\tilde{f}_1}^2\} \\ & \left. + (m_{\tilde{Z}_i}^2 + m_f^2 - m_{\tilde{f}_1}^2)(m_{\tilde{f}_1}^2 - m_f^2 - m_{\tilde{Z}_j}^2) \log L \right], \end{aligned} \quad (G.99)$$

$$I_4^{Z\tilde{f}_1} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s-m_Z^2} \left[ 2|m_{\tilde{Z}_i}| \sqrt{1-4m_f^2/s} \sqrt{E^2-m_{\tilde{Z}_j}^2} + (m_{\tilde{f}_1}^2 - m_f^2 - m_{\tilde{Z}_j}^2) \log L \right], \quad (G.100)$$

$$I_5^{Z\tilde{f}_1} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{-1}{s-m_Z^2} \left[ 2|m_{\tilde{Z}_i}| \sqrt{1-4m_f^2/s} \sqrt{E^2-m_{\tilde{Z}_j}^2} + (m_{\tilde{f}_1}^2 - m_f^2 - m_{\tilde{Z}_i}^2) \log L \right], \quad (G.101)$$

$$I_6^{Z\tilde{f}_1} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s-m_Z^2} (s-2m_f^2) \log L, \quad (G.102)$$

$$I_7^{Z\tilde{f}_1} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s-m_Z^2} 2|m_{\tilde{Z}_i}|E \log L, \quad (G.103)$$

$$I_8^{Z\tilde{f}_1} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s-m_Z^2} \log L. \quad (G.104)$$

$\Gamma_{Z\tilde{f}_2}$  is exactly the same as  $\Gamma_{Z\tilde{f}_1}$  but with the change  $\tilde{f}_1 \rightarrow \tilde{f}_2$  throughout to get the couplings  $C_{1,\dots,8}^{Z\tilde{f}_2}$  and the integrals  $I_{1,\dots,8}^{Z\tilde{f}_2}$ .

$$\Gamma_{h\tilde{f}_1} = C_1^{h\tilde{f}_1} I_1^{h\tilde{f}_1} + C_2^{h\tilde{f}_1} I_2^{h\tilde{f}_1} + C_3^{h\tilde{f}_1} I_3^{h\tilde{f}_1} + C_4^{h\tilde{f}_1} I_4^{h\tilde{f}_1} + C_5^{h\tilde{f}_1} I_5^{h\tilde{f}_1} + C_6^{h\tilde{f}_1} I_6^{h\tilde{f}_1} + C_7^{h\tilde{f}_1} I_7^{h\tilde{f}_1} + C_8^{h\tilde{f}_1} I_8^{h\tilde{f}_1}, \quad (G.105)$$

where here the couplings are:

$$C_1^{h\tilde{f}_1} = -\frac{1}{2}(-1)^{\theta_i}(-1)^{\theta_j}(X_{ij}^h + X_{ji}^h)\frac{f_q}{\sqrt{2}}t_{\alpha_h}(\alpha_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (G.106)$$

$$C_2^{h\tilde{f}_1} = -(-1)^{\theta_j}(X_{ij}^h + X_{ji}^h)\frac{f_q}{\sqrt{2}}t_{\alpha_h}(\beta_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j})|m_{\tilde{Z}_i}|m_f, \quad (G.107)$$

$$C_3^{h\tilde{f}_1} = (-1)^{\theta_i}(X_{ij}^h + X_{ji}^h)\frac{f_q}{\sqrt{2}}t_{\alpha_h}(\beta_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j})|m_{\tilde{Z}_j}|m_f, \quad (G.108)$$

$$C_4^{h\tilde{f}_1} = -(-1)^{\theta_j}(X_{ij}^h + X_{ji}^h)\frac{f_q}{\sqrt{2}}t_{\alpha_h}(-\beta_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} - \alpha_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j})|m_{\tilde{Z}_i}|m_f, \quad (G.109)$$

$$C_5^{h\tilde{f}_1} = -(-1)^{\theta_i}(X_{ij}^h + X_{ji}^h)\frac{f_q}{\sqrt{2}}t_{\alpha_h}(\beta_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j})|m_{\tilde{Z}_j}|m_f, \quad (G.110)$$

$$C_6^{h\tilde{f}_1} = (X_{ij}^h + X_{ji}^h)\frac{f_q}{\sqrt{2}}t_{\alpha_h}(-\alpha_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} - \beta_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j})|m_{\tilde{Z}_i}||m_{\tilde{Z}_j}|, \quad (G.111)$$

$$C_7^{h\tilde{f}_1} = (-1)^{\theta_i}(-1)^{\theta_j}(X_{ij}^h + X_{ji}^h)\frac{f_q}{\sqrt{2}}t_{\alpha_h}(\alpha_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j})m_f^2, \quad (G.112)$$

$$C_8^{h\tilde{f}_1} = 2(X_{ij}^h + X_{ji}^h)\frac{f_q}{\sqrt{2}}t_{\alpha_h}(\alpha_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j})|m_{\tilde{Z}_i}|m_f^2|m_{\tilde{Z}_j}|. \quad (G.113)$$

The necessary integrals are:

$$I_1^{h\tilde{f}_1} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{2}{s - m_h^2} \left[ 2s|m_{\tilde{Z}_i}| \sqrt{E^2 - m_{\tilde{Z}_j}^2} \sqrt{1 - 4m_f^2/s} + \{m_{\tilde{f}_1}^2 s - m_f^2(m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2)\} \log L \right], \quad (G.114)$$

$$I_2^{h\tilde{f}_1} = -2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s - m_h^2} \left[ 2|m_{\tilde{Z}_i}| \sqrt{E^2 - m_{\tilde{Z}_j}^2} \sqrt{1 - 4m_f^2/s} + (m_{\tilde{f}_1}^2 - m_f^2 + m_{\tilde{Z}_j}^2 - 2m_{\tilde{Z}_i}E) \log L \right], \quad (G.115)$$

$$I_3^{h\tilde{f}_1} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s - m_h^2} \left[ 2|m_{\tilde{Z}_i}| \sqrt{E^2 - m_{\tilde{Z}_j}^2} \sqrt{1 - 4m_f^2/s} + (m_{\tilde{f}_1}^2 - m_f^2 + m_{\tilde{Z}_i}^2 - 2m_{\tilde{Z}_i}E) \log L \right], \quad (G.116)$$

$$I_4^{h\tilde{f}_1} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s - m_h^2} \left[ 2|m_{\tilde{Z}_i}| \sqrt{E^2 - m_{\tilde{Z}_j}^2} \sqrt{1 - 4m_f^2/s} + (m_{\tilde{f}_1}^2 - m_f^2 + m_{\tilde{Z}_j}^2) \log L \right], \quad (G.117)$$

$$I_5^{h\tilde{f}_1} = -2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s - m_h^2} \left[ 2|m_{\tilde{Z}_i}| \sqrt{E^2 - m_{\tilde{Z}_j}^2} \sqrt{1 - 4m_f^2/s} + (m_{\tilde{f}_1}^2 - m_f^2 + m_{\tilde{Z}_i}^2) \log L \right], \quad (G.118)$$

$$I_6^{h\tilde{f}_1} = -2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s - m_h^2} (s - 2m_f^2) \log L, \quad (G.119)$$

$$I_7^{h\tilde{f}_1} = -2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s - m_h^2} (2|m_{\tilde{Z}_i}|E) \log L, \quad (G.120)$$

$$I_8^{h\tilde{f}_1} = -2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{s - m_h^2} \log L. \quad (G.121)$$

Note  $\Gamma_{h\tilde{f}_2}$  is exactly the same as  $\Gamma_{h\tilde{f}_1}$  but with the replacement  $\tilde{f}_1 \rightarrow \tilde{f}_2$  throughout to get the couplings  $C_{1,\dots,8}^{h\tilde{f}_2}$  and the integrals  $I_{1,\dots,8}^{h\tilde{f}_2}$ . Similarly, one can obtain the  $\Gamma_{H\tilde{f}_1}$  from  $\Gamma_{h\tilde{f}_1}$  by replacing h by H throughout all the couplings,

masses and integrals; therefore the changes  $X_{ij}^h + X_{ji}^h \rightarrow X_{ij}^H + X_{ji}^H$  and  $t_{\alpha_h} \rightarrow t_{\alpha_H}$  are made. One can then obtain  $\Gamma_{H\tilde{f}_2}$  again by changing  $\tilde{f}_1 \rightarrow \tilde{f}_2$  throughout the couplings, masses and integrals.

$$\Gamma_{A\tilde{f}_1} = C_1^{A\tilde{f}_1} I_1^{A\tilde{f}_1} + C_2^{A\tilde{f}_1} I_2^{A\tilde{f}_1} + C_3^{A\tilde{f}_1} I_3^{A\tilde{f}_1} + C_4^{A\tilde{f}_1} I_4^{A\tilde{f}_1} + C_5^{A\tilde{f}_1} I_5^{A\tilde{f}_1} + C_6^{A\tilde{f}_1} I_6^{A\tilde{f}_1} + C_7^{A\tilde{f}_1} I_7^{A\tilde{f}_1} + C_8^{A\tilde{f}_1} I_8^{A\tilde{f}_1}, \quad (\text{G.122})$$

here we have:

$$C_1^{A\tilde{f}_1} = \frac{1}{2}(-1)^{\theta_i}(X_{ij}^A + X_{ji}^A)\frac{A_q}{2}(\alpha_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (\text{G.123})$$

$$C_2^{A\tilde{f}_1} = -(X_{ij}^A + X_{ji}^A)\frac{A_q}{2}|m_{\tilde{Z}_i}|m_f(\beta_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (\text{G.124})$$

$$C_3^{A\tilde{f}_1} = (-1)^{\theta_i}(-1)^{\theta_j}(X_{ij}^A + X_{ji}^A)\frac{A_q}{2}|m_{\tilde{Z}_j}|m_f(\beta_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (\text{G.125})$$

$$C_4^{A\tilde{f}_1} = -(X_{ij}^A + X_{ji}^A)\frac{A_q}{2}|m_{\tilde{Z}_i}|m_f(\beta_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (\text{G.126})$$

$$C_5^{A\tilde{f}_1} = (-1)^{\theta_i}(-1)^{\theta_j}(X_{ij}^A + X_{ji}^A)\frac{A_q}{2}|m_{\tilde{Z}_j}|m_f(\beta_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (\text{G.127})$$

$$C_6^{A\tilde{f}_1} = -(-1)^{\theta_j}(X_{ij}^A + X_{ji}^A)\frac{A_q}{2}|m_{\tilde{Z}_i}||m_{\tilde{Z}_j}|(\alpha_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (\text{G.128})$$

$$C_7^{A\tilde{f}_1} = (-1)^{\theta_i}(X_{ij}^A + X_{ji}^A)\frac{A_q}{2}m_f^2(\alpha_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (\text{G.129})$$

$$C_8^{A\tilde{f}_1} = -(-1)^{\theta_j}(X_{ij}^A + X_{ji}^A)A_qm_f^2|m_{\tilde{Z}_i}||m_{\tilde{Z}_j}|(\alpha_{\tilde{f}_1}^{\tilde{Z}_i}\beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i}\alpha_{\tilde{f}_1}^{\tilde{Z}_j}). \quad (\text{G.130})$$

The  $I_i^{A\tilde{f}_1}$  are exactly as the  $I_i^{h\tilde{f}_1}$  but with the change  $m_h \rightarrow m_A$ .

$$\Gamma_{ZA} = 2C_1^{ZA}I_1^{ZA} + 2C_2^{ZA}I_2^{ZA}, \quad (\text{G.131})$$

where

$$C_1^{ZA} = -4(-1)^{\theta_i}(-1)^{\theta_j}W_{ij}(X_{ij}^A + X_{ji}^A)A_qg \sin \theta_W\beta_f|m_{\tilde{Z}_j}|m_f, \quad (\text{G.132})$$

$$C_2^{ZA} = 4W_{ij}(X_{ij}^A + X_{ji}^A)A_qg \sin \theta_W\beta_f|m_{\tilde{Z}_i}|m_f. \quad (\text{G.133})$$

The integrals included here are:

$$I_1^{ZA} = 2|m_{\tilde{Z}_i}|\int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{(s-m_{\tilde{Z}_i}^2)(s-m_A^2)}2|m_{\tilde{Z}_i}|\sqrt{E^2-m_{\tilde{Z}_j}^2}\sqrt{1-4m_f^2/s}\{m_{\tilde{Z}_i}^2-|m_{\tilde{Z}_i}|E\}, \quad (\text{G.134})$$

$$I_2^{ZA} = -2|m_{\tilde{Z}_i}|\int_{|m_{\tilde{Z}_j}|}^{E_{upper}} dE \frac{1}{(s-m_{\tilde{Z}_i}^2)(s-m_A^2)}2|m_{\tilde{Z}_i}|\sqrt{E^2-m_{\tilde{Z}_j}^2}\sqrt{1-4m_f^2/s}\{m_{\tilde{Z}_j}^2-|m_{\tilde{Z}_i}|E\}. \quad (\text{G.135})$$

The goldstone contribution is:

$$\Gamma_G = 4c_{G\tilde{Z}\tilde{Z}_j}^2c_{Gff}^2\left[I_4^G + 2m_f^2I_3^G - 2(-1)^{\theta_i}(-1)^{\theta_j}|m_{\tilde{Z}_i}||m_{\tilde{Z}_j}|I_2^G - 4(-1)^{\theta_i}(-1)^{\theta_j}m_f^2|m_{\tilde{Z}_i}||m_{\tilde{Z}_j}|I_1^G\right]. \quad (\text{G.136})$$

The goldstone interferences with fermions are given by:

$$\Gamma_{G\tilde{f}_1} = C_1^{G\tilde{f}_1}I_1^{G\tilde{f}_1} + C_2^{G\tilde{f}_1}I_2^{G\tilde{f}_1} + C_3^{G\tilde{f}_1}I_3^{G\tilde{f}_1} + C_4^{G\tilde{f}_1}I_4^{G\tilde{f}_1} + C_5^{G\tilde{f}_1}I_5^{G\tilde{f}_1} + C_6^{G\tilde{f}_1}I_6^{G\tilde{f}_1} + C_7^{G\tilde{f}_1}I_7^{G\tilde{f}_1} + C_8^{G\tilde{f}_1}I_8^{G\tilde{f}_1}, \quad (\text{G.137})$$



where

$$C_1^{G\tilde{f}_1} = \frac{1}{2} c_{G\tilde{Z}_i\tilde{Z}_j} c_{Gff} (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (G.138)$$

$$C_2^{G\tilde{f}_1} = -(-1)^{\theta_i} m_f |m_{\tilde{Z}_i}| c_{G\tilde{Z}_i\tilde{Z}_j} c_{Gff} (\beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \alpha_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (G.139)$$

$$C_3^{G\tilde{f}_1} = (-1)^{\theta_j} m_f |m_{\tilde{Z}_j}| c_{G\tilde{Z}_i\tilde{Z}_j} c_{Gff} (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (G.140)$$

$$C_4^{G\tilde{f}_1} = -(-1)^{\theta_i} m_f |m_{\tilde{Z}_i}| c_{G\tilde{Z}_i\tilde{Z}_j} c_{Gff} (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (G.141)$$

$$C_5^{G\tilde{f}_1} = (-1)^{\theta_j} |m_{\tilde{Z}_i}| m_f c_{G\tilde{Z}_i\tilde{Z}_j} c_{Gff} (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (G.142)$$

$$C_6^{G\tilde{f}_1} = -(-1)^{\theta_i} (-1)^{\theta_j} |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}| c_{G\tilde{Z}_i\tilde{Z}_j} c_{Gff} (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (G.143)$$

$$C_7^{G\tilde{f}_1} = m_f^2 c_{G\tilde{Z}_i\tilde{Z}_j} c_{Gff} (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j}), \quad (G.144)$$

$$C_8^{G\tilde{f}_1} = -2(-1)^{\theta_i} (-1)^{\theta_j} |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}| c_{G\tilde{Z}_i\tilde{Z}_j} c_{Gff} (\alpha_{\tilde{f}_1}^{\tilde{Z}_i} \beta_{\tilde{f}_1}^{\tilde{Z}_j} + \beta_{\tilde{f}_1}^{\tilde{Z}_i} \alpha_{\tilde{f}_1}^{\tilde{Z}_j}). \quad (G.145)$$

The  $I_{1,\dots,8}^{G\tilde{f}_1}$  integrals are the same as the  $I_{1,\dots,8}^{h\tilde{f}_1}$  but with the replacement  $m_h \rightarrow m_Z$  as the mass of the goldstone is the  $Z$  mass as it represents the longitudinal component of the  $Z$  boson. Similar changes apply to the  $I_{1,\dots,8}^{G\tilde{f}_2}$ , whilst in the couplings we apply the replacement  $\tilde{f}_1 \rightarrow \tilde{f}_2$  throughout. The  $Z$ -goldstone interference contribution is:

$$\Gamma_{ZG} = 2C_1^{ZG} I_1^{ZG} + 2C_2^{ZG} I_2^{ZG}, \quad (G.146)$$

where

$$C_1^{ZG} = -8W_{ij}(-1)^{\theta_j} m_f |m_{\tilde{Z}_j}| c_{Gff} c_{G\tilde{Z}_i\tilde{Z}_j} g \sin \theta_w \beta_f, \quad (G.147)$$

$$C_2^{ZG} = 8W_{ij}(-1)^{\theta_i} m_f |m_{\tilde{Z}_i}| c_{Gff} c_{G\tilde{Z}_i\tilde{Z}_j} g \sin \theta_w \beta_f. \quad (G.148)$$

The  $I_{1,2}^{ZG}$  are the same as the  $I_{1,2}^{ZA}$  but with the expected change  $m_A \rightarrow m_Z$ .

$$\Gamma_{GA} = C_1^{GA} C_3^{GA} I_4^{GA} - 2C_1^{GA} C_4^{GA} m_f^2 I_3^{GA} + 2C_2^{GA} C_3^{GA} |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}| I_2^{GA} - 4C_2^{GA} C_4^{GA} m_f^2 |m_{\tilde{Z}_i}| |m_{\tilde{Z}_j}| I_1^{GA}, \quad (G.149)$$

where here

$$C_1^{GA} = -2(-1)^{\theta_j} c_{G\tilde{Z}_i\tilde{Z}_j} (X_{ij}^A + X_{ji}^A), \quad (G.150)$$

$$C_2^{GA} = 2(-1)^{\theta_j} c_{G\tilde{Z}_i\tilde{Z}_j} (X_{ij}^A + X_{ji}^A), \quad (G.151)$$

$$C_3^{GA} = -A_q c_{Gff}, \quad (G.152)$$

$$C_4^{GA} = A_q c_{Gff}. \quad (G.153)$$

The integrals are:

$$I_1^{GA} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{\max}} dE \frac{2|m_{\tilde{Z}_i}| \sqrt{E^2 - m_{\tilde{Z}_j}^2} \sqrt{1 - 4m_f^2/s}}{(s - m_{\tilde{Z}_j}^2)(s - m_A^2)}, \quad (G.154)$$

$$I_2^{GA} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{\max}} dE \frac{2|m_{\tilde{Z}_i}|(s - 2m_f^2) \sqrt{E^2 - m_{\tilde{Z}_j}^2} \sqrt{1 - 4m_f^2/s}}{(s - m_{\tilde{Z}_j}^2)(s - m_A^2)}, \quad (G.155)$$

$$I_3^{GA} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{\max}} dE \frac{4|m_{\tilde{Z}_i}|^2 E \sqrt{E^2 - m_{\tilde{Z}_j}^2} \sqrt{1 - 4m_f^2/s}}{(s - m_{\tilde{Z}_j}^2)(s - m_A^2)}, \quad (G.156)$$

$$I_4^{GA} = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{Z}_j}|}^{E_{\max}} dE \frac{4|m_{\tilde{Z}_i}|^2 E(s - 2m_f^2) \sqrt{E^2 - m_{\tilde{Z}_j}^2} \sqrt{1 - 4m_f^2/s}}{(s - m_{\tilde{Z}_j}^2)(s - m_A^2)}. \quad (G.157)$$

Now the list of the couplings used is:

$$W_{ij} = 0.25 \sqrt{g^2 + g'^2} (N_{4i} N_{4j} - N_{3i} N_{3j}). \quad (\text{G.158})$$

The  $X_{ij}^\phi$  couplings are:

$$X_{ij}^h = -\frac{1}{2} (-1)^{\theta_i} (-1)^{\theta_j} [-N_{3i} \sin \alpha - N_{4i} \cos \alpha] (-g N_{2j} + g' N_{1j}), \quad (\text{G.159})$$

$$X_{ij}^H = -\frac{1}{2} (-1)^{\theta_i} (-1)^{\theta_j} [N_{3i} \cos \alpha - N_{4i} \sin \alpha] (-g N_{2j} + g' N_{1j}), \quad (\text{G.160})$$

$$X_{ij}^A = \frac{1}{2} (-1)^{\theta_i} (-1)^{\theta_j} [N_{3i} \sin \beta - N_{4i} \cos \beta] (-g N_{2j} + g' N_{1j}). \quad (\text{G.161})$$

$$f_q = \begin{cases} \frac{gm_q^{\text{run}}}{\sqrt{2}m_W \sin \beta}, & \text{for } u\text{-type quarks,} \\ \frac{gm_q^{\text{run}}}{\sqrt{2}m_W \cos \beta}, & \text{for } d\text{-type quarks,} \\ 0, & \text{for neutrinos } \nu, \\ \frac{gm_l^{\text{run}}}{\sqrt{2}m_W \cos \beta}, & \text{for charged leptons.} \end{cases} \quad (\text{G.162})$$

$$A_q = \begin{cases} \frac{gm_q^{\text{run}}}{m_W \tan \beta}, & \text{for } u\text{-type quarks,} \\ \frac{gm_q^{\text{run}} \tan \beta}{m_W}, & \text{for } d\text{-type quarks,} \\ 0, & \text{for neutrinos } \nu, \\ \frac{gm_l^{\text{run}} \tan \beta}{m_W}, & \text{for charged leptons.} \end{cases} \quad (\text{G.163})$$

$$t_{\alpha_h} = \begin{cases} \cos \alpha, & \text{for } u\text{-type quarks,} \\ -\sin \alpha, & \text{for } d\text{-type quarks,} \\ \cos \alpha, & \text{for neutrinos } \nu, \\ -\sin \alpha, & \text{for charged leptons.} \end{cases} \quad (\text{G.164})$$

$$t_{\alpha_H} = \begin{cases} \sin \alpha, & \text{for } u\text{-type quarks,} \\ \cos \alpha, & \text{for } d\text{-type quarks,} \\ \sin \alpha, & \text{for neutrinos } \nu, \\ \cos \alpha, & \text{for charged leptons.} \end{cases} \quad (\text{G.165})$$

$$\alpha_f = \begin{cases} -\frac{5g'p}{12g} + \frac{g}{4g'}, & \text{for } u\text{-type quarks,} \\ \frac{g'p}{12g} - \frac{g}{4g'}, & \text{for } d\text{-type quarks,} \\ \frac{1}{4} \left( \frac{g'}{g} + \frac{g}{g'} \right), & \text{for neutrinos } \nu, \\ \frac{3}{4} \frac{g'}{g} - \frac{g}{4g'}, & \text{for charged leptons.} \end{cases} \quad (\text{G.166})$$

$$\beta_f = \begin{cases} -\frac{1}{4} \left( \frac{g'}{g} + \frac{g}{g'} \right), & \text{for } u\text{-type quarks,} \\ \frac{1}{4} \left( \frac{g'}{g} + \frac{g}{g'} \right), & \text{for } d\text{-type quarks,} \\ -\frac{1}{4} \left( \frac{g'}{g} + \frac{g}{g'} \right), & \text{for neutrinos } \nu, \\ \frac{1}{4} \left( \frac{g'}{g} + \frac{g}{g'} \right), & \text{for charged leptons.} \end{cases} \quad (\text{G.167})$$

$$\alpha_{\tilde{f}_i}^{\tilde{Z}_i} = \begin{cases} A_{\tilde{Z}_i} \cos \theta_q - f_q N_{4i} \sin \theta_q, & \text{for } u\text{-type quarks,} \\ A_{\tilde{Z}_i} \cos \theta_q - f_q N_{3i} \sin \theta_q, & \text{for } d\text{-type quarks,} \\ A_{\tilde{Z}_i} \cos \theta_q, & \text{for neutrinos } \nu, \\ A_{\tilde{Z}_i} \sin \theta_q + f_q N_{3i} \cos \theta_q, & \text{for charged leptons.} \end{cases} \quad (\text{G.168})$$

$$\alpha_{\tilde{f}_2}^{\tilde{Z}_i} = \begin{cases} A_{\tilde{Z}_i} \sin \theta_q + f_q N_{4i} \cos \theta_q, & \text{for } u\text{-type quarks,} \\ A_{\tilde{Z}_i} \sin \theta_q + f_q N_{3i} \cos \theta_q, & \text{for } d\text{-type quarks,} \\ A_{\tilde{Z}_i} \sin \theta_q, & \text{for neutrinos } \nu, \\ -A_{\tilde{Z}_i} \cos \theta_q + f_q N_{3i} \sin \theta_q, & \text{for charged leptons.} \end{cases} \quad (\text{G.169})$$

$$\beta_{\tilde{f}_1}^{\tilde{Z}_i} = \begin{cases} f_q N_{4i} \cos \theta_q + B_{\tilde{Z}_i} \sin \theta_q, & \text{for } u\text{-type quarks,} \\ f_q N_{3i} \cos \theta_q + B_{\tilde{Z}_i} \sin \theta_q, & \text{for } d\text{-type quarks,} \\ 0, & \text{for neutrinos } \nu, \\ f_q N_{3i} \sin \theta_q - B_{\tilde{Z}_i} \cos \theta_q, & \text{for charged leptons.} \end{cases} \quad (\text{G.170})$$

$$\beta_{\tilde{f}_2}^{\tilde{Z}_i} = \begin{cases} f_q N_{4i} \sin \theta_q - B_{\tilde{Z}_i} \cos \theta_q, & \text{for } u\text{-type quarks,} \\ f_q N_{3i} \sin \theta_q - B_{\tilde{Z}_i} \cos \theta_q, & \text{for } d\text{-type quarks,} \\ 0, & \text{for neutrinos } \nu, \\ -f_q N_{3i} \cos \theta_q - B_{\tilde{Z}_i} \sin \theta_q, & \text{for charged leptons.} \end{cases} \quad (\text{G.171})$$

$$A_{\tilde{Z}_i} = \begin{cases} -\frac{g}{\sqrt{2}} N_{2i} - \frac{g'}{3\sqrt{2}} N_{1i}, & \text{for } u\text{-type quarks,} \\ \frac{g}{\sqrt{2}} N_{2i} - \frac{g'}{3\sqrt{2}} N_{1i}, & \text{for } d\text{-type quarks,} \\ -\frac{g}{\sqrt{2}} N_{2i} + \frac{g'}{3\sqrt{2}} N_{1i}, & \text{for neutrinos } \nu, \\ \frac{g}{\sqrt{2}} N_{2i} + \frac{g'}{3\sqrt{2}} N_{1i}, & \text{for charged leptons.} \end{cases} \quad (\text{G.172})$$

$$B_{\tilde{Z}_i} = \begin{cases} -\frac{4g'}{3\sqrt{2}} N_{1i}, & \text{for } u\text{-type quarks,} \\ \frac{2g'}{3\sqrt{2}} N_{1i}, & \text{for } d\text{-type quarks,} \\ 0, & \text{for neutrinos } \nu, \\ \sqrt{2}g' N_{1i}, & \text{for charged leptons.} \end{cases} \quad (\text{G.173})$$

$$c_{Gff} = \begin{cases} \frac{-f_q \sin \beta}{\sqrt{2}}, & \text{for } u\text{-type quarks,} \\ \frac{f_q \cos \beta}{\sqrt{2}}, & \text{for } d\text{-type quarks,} \\ 0, & \text{for neutrinos } \nu, \\ \frac{f_q \cos \beta}{\sqrt{2}}, & \text{for charged leptons.} \end{cases} \quad (\text{G.174})$$

$$c_{G\tilde{Z}_i\tilde{Z}_j} = \frac{1}{2} \left[ (g' N_{1i} - g N_{2i})(N_{3j} \cos \beta + N_{4j} \sin \beta) + (g' N_{1j} - g N_{2j})(N_{3i} \cos \beta + N_{4i} \sin \beta) \right]. \quad (\text{G.175})$$

$$\tilde{Z}_i \rightarrow \tilde{W}_j f' \bar{f}$$

We turn now onto the 3 body decays of a neutralino into a chargino, fermion and antifermion. As for all the other 3 body modes included, this mode is only calculated if no 2 body modes are kinematically accessible. There are 4 main contributions to these decays, with  $W$  boson,  $H^\pm$ ,  $\tilde{f}'_k$  and  $\tilde{f}_k$  intermediates, the Feynman diagrams for these are shown in Figure G.9. Therefore there are nominally 6 squared contributions and 15 interferences; however, as the calculation is again done in Feynman gauge, the goldstone boson corresponding to the longitudinal components of the  $W$  boson must be added, adding a further squared contribution and its 6 interferences.

For this decay mode, and the “reverse” decay mode  $\tilde{W}_j \rightarrow \tilde{Z}_i f' \bar{f}$ , the formulae used are extracted from the `sPHENO` code, based on the work in references [30, 39]. Note that  $f'$ ,  $f$  are fermions with third components of weak isospin  $\frac{1}{2}$  and  $-\frac{1}{2}$  respectively. A difference relative to these references is that, following the formulae of `sPHENO`, the expressions given do not neglect  $m_f$  in the Dirac algebra of the squared matrix element (whereas in [30, 39] it is neglected here, but of course included in the phase space). As a result there is also  $WH^\pm$  interference which is not present if  $m_f$  is neglected in the Dirac algebra. The possibilities of positive and negative neutralino and chargino masses are included via  $(-1)^{\theta_i}$  and  $(-1)^{\theta_j}$  factors. Similarly the fermion Yukawa couplings are included and the formulae themselves allow for mixing of the fermions. However in our program, mixing is only considered for the third generation of sfermions and here this 3 body mode  $\tilde{Z}_i \rightarrow \tilde{W}_j t \bar{b}$  is not calculated as the 2 body modes  $\tilde{Z}_i \rightarrow W \tilde{W}_j$

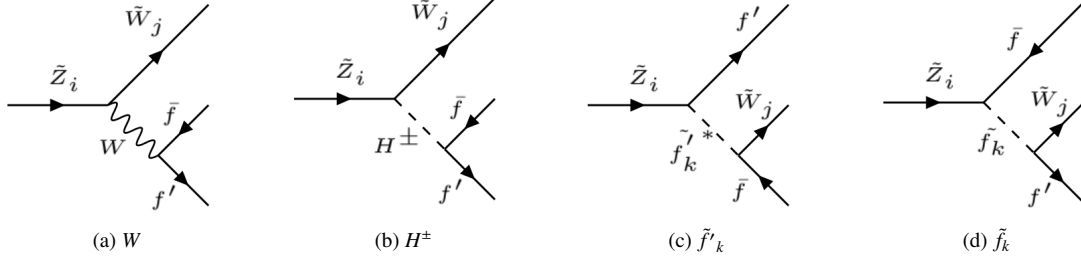


Figure G.9:  $W, H^\pm, \tilde{f}'_k, \tilde{f}_k$  contributions to the  $\tilde{Z}_i \rightarrow \tilde{W}_j f' \bar{f}$  decay.  $i = 1, 2, 3, 4, j = 1, 2, k = 1, 2$ . There are then also interferences between all these contributions.

and  $\tilde{Z}_i \rightarrow h \tilde{W}_j$  are then kinematically available and will dominate the branching ratios. The overall expression for the partial width is therefore given by:

$$\Gamma = \frac{N_c}{512\pi^3 |m_{Z_i}|^3} \left[ \Gamma_W + \Gamma_{\tilde{f}_1} + \Gamma_{\tilde{f}_2} + \Gamma_{\tilde{f}'_1} + \Gamma_{\tilde{f}'_2} - 2\Gamma_{\tilde{f}'_1 \tilde{f}_1} - 2\Gamma_{\tilde{f}'_1 \tilde{f}_2} - 2\Gamma_{\tilde{f}'_2 \tilde{f}_1} - 2\Gamma_{\tilde{f}'_2 \tilde{f}_2} + 2\Gamma_{WH^\pm} \right. \\ \left. + 2\Gamma_{WG} + \Gamma_{H^\pm} + \Gamma_G - 2\Gamma_{W\tilde{f}'_1} - 2\Gamma_{W\tilde{f}'_2} - 2\Gamma_{W\tilde{f}_1} - 2\Gamma_{W\tilde{f}_2} + 2\Gamma_{H^\pm G} - 2\Gamma_{G\tilde{f}'_1} \right. \\ \left. - 2\Gamma_{G\tilde{f}'_2} - 2\Gamma_{G\tilde{f}_1} - 2\Gamma_{G\tilde{f}_2} - 2\Gamma_{H^\pm \tilde{f}'_1} - 2\Gamma_{H^\pm \tilde{f}'_2} - 2\Gamma_{H^\pm \tilde{f}_1} - 2\Gamma_{H^\pm \tilde{f}_2} + 2\Gamma_{\tilde{f}'_1 \tilde{f}'_2} + 2\Gamma_{\tilde{f}_1 \tilde{f}_2} \right]. \quad (\text{G.176})$$

Note here  $G$  refers to the goldstone contribution which is the longitudinal component of the  $W$  and so has mass equal to the  $W$  boson mass. Here the following variables and couplings are used:

$$N_c = \begin{cases} 3, & \text{for } f' \bar{f} \text{ quarks,} \\ 1, & \text{for } f' \bar{f} \text{ charged leptons or neutrinos.} \end{cases} \quad (\text{G.177})$$

There are several factors of  $(-1)$  depending on whether the neutralino or chargino have negative masses, and also there are factors of  $(-1)$  if the decay chargino  $\rightarrow$  neutralino  $f' \bar{f}$  is being considered rather than neutralino  $\rightarrow$  chargino  $f' \bar{f}$ .

$$(-1)^{\theta_i} = \begin{cases} 1, & \text{for } m_{\tilde{Z}_i} > 0, \\ -1, & \text{for } m_{\tilde{Z}_i} < 0. \end{cases} \quad (\text{G.178})$$

$$(-1)^{\theta_j} = \begin{cases} 1, & \text{for } m_{\tilde{W}_j} > 0, \\ -1, & \text{for } m_{\tilde{W}_j} < 0. \end{cases} \quad (\text{G.179})$$

$$(-1)^{\theta_c} = \begin{cases} 1, & \text{for neutralino decaying to chargino,} \\ -1, & \text{for chargino decaying to neutralino.} \end{cases} \quad (\text{G.180})$$

The following couplings are used:

$$f_u = \frac{gm_{q'}^{run}}{\sqrt{2} \sin \beta m_W}, \quad (\text{G.181})$$

$$f_d = \frac{gm_q^{run}}{\sqrt{2} \cos \beta m_W}. \quad (\text{G.182})$$

For  $\tilde{W}_1^+$ , i.e. the lightest chargino ( $j = 1$ ), and where  $i$  is the index of the neutralino:

$$C_{\tilde{W} \tilde{Z} W}^L = g \sin \theta_L N_{2i} + \frac{\cos \theta_L N_{3i}}{\sqrt{2}}, \quad (\text{G.183})$$

$$C_{\tilde{W} \tilde{Z} W}^R = g \sin \theta_R N_{2i} - \frac{\cos \theta_R N_{4i}}{\sqrt{2}}, \quad (\text{G.184})$$

$$C_{\tilde{W}\tilde{Z}H^+}^L = (g \sin \theta_R N_{4i} + \frac{\cos \theta_R}{\sqrt{2}})(g' N_{1i} + g N_{2i}), \quad (\text{G.185})$$

$$C_{\tilde{W}\tilde{Z}H^+}^R = (g \sin \theta_L N_{3i} - \frac{\cos \theta_L}{\sqrt{2}})(g' N_{1i} + g N_{2i}), \quad (\text{G.186})$$

$$\alpha_{\tilde{f}'_1}^{\tilde{W}_1} = -g \sin \theta_R \cos \theta_{q'} + f_u \cos \theta_R \sin \theta_{q'}, \quad (\text{G.187})$$

$$\beta_{\tilde{f}'_1}^{\tilde{W}_1} = -f_d \cos \theta_L \cos \theta_{q'} (-1)^{\theta_c}, \quad (\text{G.188})$$

$$\alpha_{\tilde{f}_1}^{\tilde{W}_1} = -g \sin \theta_L \cos \theta_q + f_d \cos \theta_L \sin \theta_q (-1)^{\theta_c}, \quad (\text{G.189})$$

$$\beta_{\tilde{f}_1}^{\tilde{W}_1} = -f_u \cos \theta_R \cos \theta_q, \quad (\text{G.190})$$

$$\alpha_{\tilde{f}'_2}^{\tilde{W}_1} = g \sin \theta_R \sin \theta_{q'} (-1)^{\theta_c} - f_u \cos \theta_R \cos \theta_{q'}, \quad (\text{G.191})$$

$$\beta_{\tilde{f}'_2}^{\tilde{W}_1} = -f_d \cos \theta_L \sin \theta_{q'} (-1)^{\theta_c}, \quad (\text{G.192})$$

$$\alpha_{\tilde{f}_2}^{\tilde{W}_1} = -f_d \cos \theta_L \cos \theta_q + g \sin \theta_L \sin \theta_q, \quad (\text{G.193})$$

$$\beta_{\tilde{f}_2}^{\tilde{W}_1} = -f_u \cos \theta_R \sin \theta_q. \quad (\text{G.194})$$

Note that because of the conventions adopted, if the fermions considered are  $\tau$  and  $\nu_\tau$ , so that the intermediates are  $\tilde{\tau}_1$  and  $\tilde{\tau}_2$ , then the mixing angles in the formulae for this 3 body decay must be rotated so that one must take  $\cos \theta_\tau \rightarrow \sin \theta_\tau$  and  $\sin \theta_\tau \rightarrow -\cos \theta_\tau$  in the formulae listed for the  $\tilde{Z}_i \rightarrow \tilde{W}_j f' \bar{f}$  and for the reverse decay  $\tilde{W}_j \rightarrow \tilde{Z}_i f' \bar{f}$ . Note that in this case where the fermions are  $\tau$  and  $\nu_\tau$ , then  $\theta_q$  would be the mixing angle for the  $\tilde{\tau}$ , whilst  $\theta_{q'} = 0$  as there is no mixing for  $\tilde{\nu}_\tau$ .

For  $\tilde{W}_2^+$ , i.e. the heaviest chargino ( $j = 2$ ), where  $i$  is the index of the neutralino:

$$C_{\tilde{W}\tilde{Z}W}^L = g \cos \theta_L N_{2i} - \frac{\sin \theta_L N_{3i}}{\sqrt{2}}, \quad (\text{G.195})$$

$$C_{\tilde{W}\tilde{Z}W}^R = g \cos \theta_R N_{2i} + \frac{\sin \theta_R N_{4i}}{\sqrt{2}}, \quad (\text{G.196})$$

$$C_{\tilde{W}\tilde{Z}H^+}^L = \left[ g \cos \theta_R N_{4i} - \frac{\sin \theta_R}{\sqrt{2}} \right] (g' N_{1i} + g N_{2i}), \quad (\text{G.197})$$

$$C_{\tilde{W}\tilde{Z}H^+}^R = \left[ g \cos \theta_L N_{3i} + \frac{\sin \theta_L}{\sqrt{2}} \right] (g' N_{1i} + g N_{2i}), \quad (\text{G.198})$$

$$\alpha_{\tilde{f}'_1}^{\tilde{W}_2} = -g \cos \theta_R \cos \theta_{q'} - f_u \sin \theta_R \sin \theta_{q'}, \quad (\text{G.199})$$

$$\beta_{\tilde{f}'_1}^{\tilde{W}_2} = f_d \sin \theta_L \cos \theta_{q'} (-1)^{\theta_c}, \quad (\text{G.200})$$

$$\alpha_{\tilde{f}_1}^{\tilde{W}_2} = -g \cos \theta_L \cos \theta_q - f_d \sin \theta_L \sin \theta_q (-1)^{\theta_c}, \quad (\text{G.201})$$

$$\beta_{\tilde{f}_1}^{\tilde{W}_2} = f_u \sin \theta_R \cos \theta_q, \quad (\text{G.202})$$

$$\alpha_{\tilde{f}'_2}^{\tilde{W}_2} = g \cos \theta_R \sin \theta_{q'} (-1)^{\theta_c} + f_u \sin \theta_R \cos \theta_{q'}, \quad (\text{G.203})$$

$$\beta_{\tilde{f}'_2}^{\tilde{W}_2} = f_d \sin \theta_L \sin \theta_{q'} (-1)^{\theta_c}, \quad (\text{G.204})$$

$$\alpha_{\tilde{f}_2}^{\tilde{W}_2} = f_d \sin \theta_L \cos \theta_q + g \cos \theta_L \sin \theta_q, \quad (\text{G.205})$$

$$\beta_{\tilde{f}_2}^{\tilde{W}_2} = f_u \sin \theta_R \sin \theta_q. \quad (\text{G.206})$$

There are also the neutralino couplings to the  $f' \bar{f}$  pair and these depend upon whether they are quarks(q) or leptons(l) and whether they are “ $u$ -type” or “ $d$ -type” (i.e. third component of weak isospin being  $+\frac{1}{2}$  or  $-\frac{1}{2}$  respectively). For quarks:

$$A_{Z_i}^u = -\frac{g}{\sqrt{2}}N_{2i} - \frac{g'}{3\sqrt{2}}N_{1i}, \quad (\text{G.207})$$

$$B_{Z_i}^u = -\frac{4g'}{3\sqrt{2}}N_{1i}, \quad (\text{G.208})$$

$$\alpha_{Z_i \tilde{f}_1}^u = -A_{Z_i}^u \cos \theta_{q'} (-1)^{\theta_j} (-1)^{\theta_i} (-1)^{\theta_c} - f_u N_{4i} \sin \theta_{q'}, \quad (\text{G.209})$$

$$\beta_{Z_i \tilde{f}_1}^u = f_u N_{4i} \cos \theta_{q'} (-1)^{\theta_c} - B_{Z_i}^u \sin \theta_{q'}, \quad (\text{G.210})$$

$$\alpha_{Z_i \tilde{f}_2}^u = -f_u N_{4i} \cos \theta_{q'} (-1)^{\theta_i} + A_{Z_i}^u \sin \theta_{q'} (-1)^{\theta_c}, \quad (\text{G.211})$$

$$\beta_{Z_i \tilde{f}_2}^u = B_{Z_i}^u \cos \theta_{q'} (-1)^{\theta_j} (-1)^{\theta_i} (-1)^{\theta_c} + f_u N_{4i} \sin \theta_{q'}. \quad (\text{G.212})$$

$$A_{Z_i}^d = \frac{g}{\sqrt{2}}N_{2i} - \frac{g'}{3\sqrt{2}}N_{1i}, \quad (\text{G.213})$$

$$B_{Z_i}^d = \frac{2g'}{3\sqrt{2}}N_{1i}, \quad (\text{G.214})$$

$$\alpha_{Z_i \tilde{f}_1}^d = -A_{Z_i}^d \cos \theta_q (-1)^{\theta_j} (-1)^{\theta_i} (-1)^{\theta_c} - f_d N_{4i} \sin \theta_q, \quad (\text{G.215})$$

$$\beta_{Z_i \tilde{f}_1}^d = f_d N_{3i} \cos \theta_q (-1)^{\theta_c} - B_{Z_i}^d \sin \theta_q (-1)^{\theta_i}, \quad (\text{G.216})$$

$$\alpha_{Z_i \tilde{f}_2}^d = f_d N_{3i} \cos \theta_q (-1)^{\theta_c} - (-1)^{\theta_c} (-1)^{\theta_i} A_{Z_i}^d \sin \theta_q, \quad (\text{G.217})$$

$$\beta_{Z_i \tilde{f}_2}^d = B_{Z_i}^d \cos \theta_q (-1)^{\theta_j} (-1)^{\theta_i} (-1)^{\theta_c} + f_d N_{4i} \sin \theta_q. \quad (\text{G.218})$$

Again, remember for the case of  $\tau$  and  $\nu_\tau$  as  $f$  and  $f'$  respectively then one must take  $\cos \theta_\tau \rightarrow \sin \theta_\tau$  and  $\sin \theta_\tau \rightarrow -\cos \theta_\tau$  in the formulae listed for this decay mode.

For leptons instead the neutralino couplings are:

$$A_{Z_i}^u = -\frac{g}{\sqrt{2}}N_{2i} + \frac{g'}{\sqrt{2}}N_{1i}, \quad (\text{G.219})$$

$$B_{Z_i}^u = 0, \quad (\text{G.220})$$

$$A_{Z_i}^d = \frac{g}{\sqrt{2}}N_{2i} + \frac{g'}{\sqrt{2}}N_{1i}, \quad (\text{G.221})$$

$$B_{Z_i}^d = \sqrt{2}g'N_{1i}. \quad (\text{G.222})$$

The  $\alpha$  and  $\beta$  couplings are as before except  $\alpha_{Z_i \tilde{f}_2}^u = 0$  and  $\beta_{Z_i \tilde{f}_2}^u = 0$  as there are no RH sneutrinos. Note in SOFTSUSY we use the same function for a neutralino decaying to a chargino as a chargino decaying to a neutralino, in general the changes required are  $m_{Z_i} \leftrightarrow m_{\tilde{W}_j}$ ,  $m_{f'} \leftrightarrow m_f$  and  $m_{\tilde{f}} \leftrightarrow m_f$ , in some places there are further effects on the integrals or couplings, where this occurs it's listed in the following formulae.

Contribution by contribution, the couplings are:

$\frac{\Gamma_W}{\Gamma}$

The upper limit of integration here is:

$$\mathcal{T} = \frac{1}{2|m_{Z_i}|} (m_{Z_i}^2 + m_{\tilde{W}_j}^2 - m_f^2 - m_{f'}^2 - 2m_f m_{f'}), \quad (\text{G.223})$$

We also use  $s$  and  $\lambda$  given by:

$$s = m_{Z_i}^2 + m_{Z_j}^2 - 2|m_{Z_i}|E, \quad (\text{G.224})$$

$$\lambda = \sqrt{(s - (m_f + m_{f'})^2)(s - (m_f - m_{f'})^2)}. \quad (\text{G.225})$$

The necessary integrals are:

$$\begin{aligned} I_W^1 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{\mathcal{T}} dE \frac{2|m_{\tilde{Z}_i}|}{s} \lambda \sqrt{E^2 - m_{\tilde{W}_j}^2} & \left[ -2s^4 + ((m_{\tilde{Z}_i}^2 + m_{\tilde{W}_j}^2 + m_f^2 + m_{f'}^2))s^3 \right. \\ & + ((m_{\tilde{Z}_i}^2 - m_{\tilde{W}_j}^2)^2 + (m_f^2 - m_{f'}^2)^2 - 2(m_{\tilde{Z}_i}^2 + m_{\tilde{W}_j}^2)(m_f^2 + m_{f'}^2))s^2 \\ & + ((m_{\tilde{Z}_i}^2 + m_{\tilde{W}_j}^2)(m_f^2 - m_{f'}^2)^2 + (m_f^2 + m_{f'}^2)(m_{\tilde{Z}_i}^2 - m_{\tilde{W}_j}^2)^2)s \\ & \left. - 2(m_{\tilde{Z}_i}^2 - m_{\tilde{W}_j}^2)^2(m_f^2 + m_{f'}^2)^2 \right] \frac{1}{3s^2} \frac{1}{(s - m_{\tilde{W}}^2)^2}, \end{aligned} \quad (\text{G.226})$$

$$I_W^2 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{\mathcal{T}} dE \frac{2|m_{\tilde{Z}_i}|}{s} \lambda \sqrt{E^2 - m_{\tilde{W}_j}^2} (s - m_f^2 - m_{f'}^2) \frac{1}{(s - m_{\tilde{W}}^2)^2}. \quad (\text{G.227})$$

Then

$$\Gamma_W = -8C_{\tilde{W}\tilde{Z}W}^L C_{\tilde{W}\tilde{Z}W}^R \frac{g^2}{2} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| I_W^2 (-1)^{\theta_i} (-1)^{\theta_j} + 2(C_{\tilde{W}\tilde{Z}W}^L)^2 + C_{\tilde{W}\tilde{Z}W}^R)^2 \frac{g^2}{2} I_W^1. \quad (\text{G.228})$$

$\underline{\Gamma_{H^\pm}}$

$$\omega_{H^+ \tilde{W}^+ \tilde{Z}}^L = C_{\tilde{W}\tilde{Z}H^+}^L \cos \beta, \quad (\text{G.229})$$

$$\omega_{H^+ \tilde{W}^+ \tilde{Z}}^R = C_{\tilde{W}\tilde{Z}H^+}^R \sin \beta, \quad (\text{G.230})$$

$$C_{H^+ ff'}^u = f_u \cos \beta, \quad (\text{G.231})$$

$$C_{H^+ ff'}^d = f_d \sin \beta, \quad (\text{G.232})$$

The relevant combinations of these couplings for this contribution are:

$$\mathcal{V}_{H^+}^{(1)} = \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L{}^2 + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R{}^2, \quad (\text{G.233})$$

$$\mathcal{V}_{H^+}^{(2)} = \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R (-1)^{\theta_i}, \quad (\text{G.234})$$

$$\mathcal{V}_{H^+}^{(3)} = C_{H^+ ff'}^u{}^2 + C_{H^+ ff'}^d{}^2, \quad (\text{G.235})$$

$$\mathcal{V}_{H^+}^{(4)} = C_{H^+ ff'}^u C_{H^+ ff'}^d. \quad (\text{G.236})$$

The integrals are:

$$I_{H^\pm}^1 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{\mathcal{T}} dE \frac{2|m_{\tilde{Z}_i}|}{s} \sqrt{(s - (m_{f'} + m_f)^2)(s - (m_{f'} - m_f)^2)} \sqrt{E^2 - m_{\tilde{W}_j}^2} \frac{1}{(s - m_{H^\pm}^2)^2}, \quad (\text{G.237})$$

$$\begin{aligned} I_{H^\pm}^2 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{\mathcal{T}} dE & \left[ \frac{2|m_{\tilde{Z}_i}|}{s} \sqrt{(s - (m_{f'} + m_f)^2)(s - (m_{f'} - m_f)^2)} \sqrt{E^2 - m_{\tilde{W}_j}^2} \right. \\ & \left. \times (s - m_f^2 - m_{f'}^2) \frac{1}{(s - m_{H^\pm}^2)^2} \right], \end{aligned} \quad (\text{G.238})$$

$$\begin{aligned} I_{H^\pm}^3 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{\mathcal{T}} dE & \left[ \frac{2|m_{\tilde{Z}_i}|}{s} \sqrt{(s - (m_{f'} + m_f)^2)(s - (m_{f'} - m_f)^2)} \sqrt{E^2 - m_{\tilde{W}_j}^2} \right. \\ & \left. \times 2|m_{\tilde{Z}_i}| E \frac{1}{(s - m_{H^\pm}^2)^2} \right], \end{aligned} \quad (\text{G.239})$$

$$I_{H^\pm}^4 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{\mathcal{T}} dE \left[ \frac{2|m_{\tilde{Z}_i}|}{s} \sqrt{(s - (m_{f'} + m_f)^2)(s - (m_{f'} - m_f)^2)} \sqrt{E^2 - m_{\tilde{W}_j}^2} \right. \\ \left. \times 2|m_{\tilde{Z}_i}| E (s - m_f^2 - m_{f'}^2) \frac{1}{(s - m_{H^\pm}^2)^2} \right] \quad (\text{G.240})$$

The overall contribution is then:

$$\Gamma_{H^\pm} = \mathcal{V}_{H^+}^{(1)} \mathcal{V}_{H^+}^{(3)} I_{H^\pm}^4 - 4\mathcal{V}_{H^+}^{(1)} \mathcal{V}_{H^+}^{(4)} I_{H^\pm}^3 m_f m_{f'} + 4\mathcal{V}_{H^+}^{(2)} \mathcal{V}_{H^+}^{(3)} I_{H^\pm}^2 |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_j} \\ - 16\mathcal{V}_{H^+}^{(2)} \mathcal{V}_{H^+}^{(4)} I_{H^\pm}^1 m_f m_{f'} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_j}. \quad (\text{G.241})$$

$\frac{\Gamma_G}{\Gamma_G}$

Note here  $G$  refers to the goldstone contribution which is the longitudinal component of the  $W$  and so has mass equal to the  $W$  boson mass. The couplings used are:

$$\omega_{G\tilde{W}\tilde{Z}}^L = C_{\tilde{W}\tilde{Z}H^+}^L \sin\beta, \quad (\text{G.242})$$

$$\omega_{G\tilde{W}\tilde{Z}}^R = -C_{\tilde{W}\tilde{Z}H^+}^R \cos\beta, \quad (\text{G.243})$$

$$C_{Gff'}^u = f_u \sin\beta, \quad (\text{G.244})$$

$$C_{Gff'}^d = -f_d \cos\beta, \quad (\text{G.245})$$

$$\mathcal{V}_G^{(1)} = \omega_{G\tilde{W}\tilde{Z}}^L{}^2 + \omega_{G\tilde{W}\tilde{Z}}^R{}^2, \quad (\text{G.246})$$

$$\mathcal{V}_G^{(2)} = \omega_{G\tilde{W}\tilde{Z}}^L \omega_{G\tilde{W}\tilde{Z}}^R (-1)^{\theta_i}, \quad (\text{G.247})$$

$$\mathcal{V}_G^{(3)} = C_{Gff'}^u{}^2 + C_{Gff'}^d{}^2, \quad (\text{G.248})$$

$$\mathcal{V}_G^{(4)} = C_{Gff'}^u C_{Gff'}^d. \quad (\text{G.249})$$

The integrals here  $I_G^1$  etc are exactly the same as those for  $H^\pm$  but with the change  $m_{H^\pm} \rightarrow m_W$ .

$$\Gamma_G = \mathcal{V}_G^{(1)} \mathcal{V}_G^{(3)} I_G^4 - 4\mathcal{V}_G^{(1)} \mathcal{V}_G^{(4)} I_G^3 m_f m_{f'} + 4\mathcal{V}_G^{(2)} \mathcal{V}_G^{(3)} I_G^2 |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_j} \\ - 16\mathcal{V}_G^{(2)} \mathcal{V}_G^{(4)} I_G^1 m_f m_{f'} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_j}. \quad (\text{G.250})$$

$\frac{\Gamma_{\tilde{f}'_1}}{\Gamma_{\tilde{f}'_1}}$

$$\mathcal{V}_{\tilde{f}'_1}^{(1)} = \alpha_{\tilde{Z}_i \tilde{f}'_1}^u{}^2 + \beta_{\tilde{Z}_i \tilde{f}'_1}^u{}^2, \quad (\text{G.251})$$

$$\mathcal{V}_{\tilde{f}'_1}^{(2)} = -\alpha_{\tilde{Z}_i \tilde{f}'_1}^u{}^2 \beta_{\tilde{Z}_i \tilde{f}'_1}^u{}^2 (-1)^{\theta_i}, \quad (\text{G.252})$$

$$\mathcal{V}_{\tilde{f}'_1}^{(3)} = \alpha_{\tilde{f}'_1}^{\tilde{W}}{}^2 + \beta_{\tilde{f}'_1}^{\tilde{W}}{}^2, \quad (\text{G.253})$$

$$\mathcal{V}_{\tilde{f}'_1}^{(4)} = -\alpha_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}'_1}^{\tilde{W}}. \quad (\text{G.254})$$

Now the integrals  $I_{\tilde{f}'_1}^{1,2,3,4}$  are exactly as the  $I_{H^\pm}^{1,2,3,4}$  integrals in (G.237) to (G.240) but with lower limit  $m_{f'}$ , upper limit of integration  $E_{upper} = \frac{1}{2|m_{\tilde{Z}_1}|}(m_{\tilde{Z}_i}^2 + m_{f'}^2 - m_f^2 - m_{\tilde{Z}_j}^2 - 2m_f|m_{\tilde{W}_j}|)$  and the replacements  $m_{H^\pm} \rightarrow m_{\tilde{f}'_1}$ ,  $|m_{\tilde{W}_j}| \rightarrow m_{f'}$  and  $m_{f'} \rightarrow |m_{\tilde{W}_j}|$ . Then:

$$\Gamma_{\tilde{f}'_1} = \mathcal{V}_{\tilde{f}'_1}^{(1)} \mathcal{V}_{\tilde{f}'_1}^{(3)} I_{\tilde{f}'_1}^4 - 4\mathcal{V}_{\tilde{f}'_1}^{(1)} \mathcal{V}_{\tilde{f}'_1}^{(4)} m_f |m_{\tilde{W}_j}| I_{\tilde{f}'_1}^3 + 4\mathcal{V}_{\tilde{f}'_1}^{(2)} \mathcal{V}_{\tilde{f}'_1}^{(3)} |m_{\tilde{Z}_i}| m_{f'} I_{\tilde{f}'_1}^2 \\ - 16\mathcal{V}_{\tilde{f}'_1}^{(2)} \mathcal{V}_{\tilde{f}'_1}^{(4)} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_{f'} m_f I_{\tilde{f}'_1}^1. \quad (\text{G.255})$$

$\frac{\Gamma_{\tilde{f}'_2}}{\Gamma_{\tilde{f}'_2}}$



Everything for  $\Gamma_{\tilde{f}'_2}$  is exactly as for  $\Gamma_{\tilde{f}'_1}$  but with the change  $m_{\tilde{f}'_1} \rightarrow m_{\tilde{f}'_2}$  and the coupling combinations:

$$\mathcal{V}_{\tilde{f}'_2}^{(1)} = \alpha_{\tilde{Z}_i \tilde{f}'_2}^u{}^2 + \beta_{\tilde{Z}_i \tilde{f}'_2}^u{}^2, \quad (\text{G.256})$$

$$\mathcal{V}_{\tilde{f}'_2}^{(2)} = \alpha_{\tilde{Z}_i \tilde{f}'_2}^u{}^2 \beta_{\tilde{Z}_i \tilde{f}'_2}^u{}^2, \quad (\text{G.257})$$

$$\mathcal{V}_{\tilde{f}'_2}^{(3)} = \alpha_{\tilde{f}'_2}^{\tilde{W}}{}^2 + \beta_{\tilde{f}'_2}^{\tilde{W}}{}^2, \quad (\text{G.258})$$

$$\mathcal{V}_{\tilde{f}'_2}^{(4)} = \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}'_2}^{\tilde{W}}. \quad (\text{G.259})$$

The contribution is then:

$$\begin{aligned} \Gamma_{\tilde{f}'_2} = & \mathcal{V}_{\tilde{f}'_2}^{(1)} \mathcal{V}_{\tilde{f}'_2}^{(3)} I_{\tilde{f}'_2}^4 - 4 \mathcal{V}_{\tilde{f}'_2}^{(1)} \mathcal{V}_{\tilde{f}'_2}^{(4)} m_f |m_{\tilde{W}_j}| I_{\tilde{f}'_2}^3 (-1)^{\theta_c} + 4 \mathcal{V}_{\tilde{f}'_2}^{(2)} \mathcal{V}_{\tilde{f}'_2}^{(3)} |m_{\tilde{Z}_i}| m_{f'} I_{\tilde{f}'_2}^2 (-1)^{\theta_c} \\ & - 16 \mathcal{V}_{\tilde{f}'_2}^{(2)} \mathcal{V}_{\tilde{f}'_2}^{(4)} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_{f'} m_f I_{\tilde{f}'_2}^1. \end{aligned} \quad (\text{G.260})$$

$\Gamma_{\tilde{f}_1}$

The coupling combinations are now:

$$\mathcal{V}_{\tilde{f}_1}^{(1)} = \alpha_{\tilde{Z}_i \tilde{f}_1}^d{}^2 + \beta_{\tilde{Z}_i \tilde{f}_1}^d{}^2, \quad (\text{G.261})$$

$$\mathcal{V}_{\tilde{f}_1}^{(2)} = -(-1)^{\theta_i} \alpha_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{Z}_i \tilde{f}_1}^d, \quad (\text{G.262})$$

$$\mathcal{V}_{\tilde{f}_1}^{(3)} = \alpha_{\tilde{f}_1}^{\tilde{W}}{}^2 + \beta_{\tilde{f}_1}^{\tilde{W}}{}^2, \quad (\text{G.263})$$

$$\mathcal{V}_{\tilde{f}_1}^{(4)} = -\alpha_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} (-1)^{\theta_j}. \quad (\text{G.264})$$

The integrals are exactly as the  $I_{H^\pm}^{1,2,3,4}$  integrals in (G.237) to (G.240) but the lower limit is now  $m_f$ , the upper limit is  $E_{upper2} = \frac{1}{2|m_{\tilde{Z}_i}|} [m_{\tilde{Z}_i}^2 + m_f^2 - m_{\tilde{W}_j}^2 - m_{f'}^2 - 2m_{f'} |m_{\tilde{W}_j}|]$  and in general relative to the  $H^\pm$  integrals we must make the changes  $m_{H^\pm} \rightarrow m_{\tilde{f}_1}$ ,  $|m_{\tilde{W}_j}| \rightarrow m_f$  and  $m_f \rightarrow |m_{\tilde{W}_j}|$ .

$$\begin{aligned} \Gamma_{\tilde{f}_1} = & \mathcal{V}_{\tilde{f}_1}^{(1)} \mathcal{V}_{\tilde{f}_1}^{(3)} I_{\tilde{f}_1}^4 - 4 \mathcal{V}_{\tilde{f}_1}^{(1)} \mathcal{V}_{\tilde{f}_1}^{(4)} m_{f'} |m_{\tilde{W}_j}| I_{\tilde{f}_1}^3 (-1)^{\theta_c} (-1)^{\theta_j} + 4 \mathcal{V}_{\tilde{f}_1}^{(2)} \mathcal{V}_{\tilde{f}_1}^{(3)} |m_{\tilde{Z}_i}| m_f I_{\tilde{f}_1}^2 (-1)^{\theta_c} \\ & - 16 \mathcal{V}_{\tilde{f}_1}^{(2)} \mathcal{V}_{\tilde{f}_1}^{(4)} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_{f'} m_{f'} (-1)^{\theta_j} I_{\tilde{f}_1}^1. \end{aligned} \quad (\text{G.265})$$

$\Gamma_{\tilde{f}_2}$

Nominally  $\Gamma_{\tilde{f}_2}$  has the same expression as  $\Gamma_{\tilde{f}_1}$  with the replacement  $\tilde{f}_1 \rightarrow \tilde{f}_2$ , however differences in expressions for couplings mean we have slight differences; the coupling combinations are now:

$$\mathcal{V}_{\tilde{f}_2}^{(1)} = \alpha_{\tilde{Z}_i \tilde{f}_2}^d{}^2 + \beta_{\tilde{Z}_i \tilde{f}_2}^d{}^2, \quad (\text{G.266})$$

$$\mathcal{V}_{\tilde{f}_2}^{(2)} = \alpha_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{Z}_i \tilde{f}_2}^d, \quad (\text{G.267})$$

$$\mathcal{V}_{\tilde{f}_2}^{(3)} = \alpha_{\tilde{f}_2}^{\tilde{W}}{}^2 + \beta_{\tilde{f}_2}^{\tilde{W}}{}^2, \quad (\text{G.268})$$

$$\mathcal{V}_{\tilde{f}_2}^{(4)} = \alpha_{\tilde{f}_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} (-1)^{\theta_c}. \quad (\text{G.269})$$

Therefore the contribution is given by:

$$\begin{aligned} \Gamma_{\tilde{f}_2} = & \mathcal{V}_{\tilde{f}_2}^{(1)} \mathcal{V}_{\tilde{f}_2}^{(3)} I_{\tilde{f}_2}^4 - 4 \mathcal{V}_{\tilde{f}_2}^{(1)} \mathcal{V}_{\tilde{f}_2}^{(4)} m_{f'} |m_{\tilde{W}_j}| I_{\tilde{f}_2}^3 (-1)^{\theta_c} + 4 \mathcal{V}_{\tilde{f}_2}^{(2)} \mathcal{V}_{\tilde{f}_2}^{(3)} |m_{\tilde{Z}_i}| m_f I_{\tilde{f}_2}^2 (-1)^{\theta_c} \\ & - 16 \mathcal{V}_{\tilde{f}_2}^{(2)} \mathcal{V}_{\tilde{f}_2}^{(4)} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_{f'} m_{f'} I_{\tilde{f}_2}^1. \end{aligned} \quad (\text{G.270})$$

$\Gamma_{\tilde{f}'_1 \tilde{f}_1}$

Here the coupling combinations differ significantly depending on which way around the decay is being considered, i.e. neutralino to chargino or chargino to neutralino. The following are fixed regardless of this:

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(1)} = \frac{1}{2} \left[ \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} (-1)^{\theta_i} + \beta_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right] (-1)^{\theta_j}, \quad (\text{G.271})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(2)} = -|m_{\tilde{Z}_i}| m_{\tilde{W}_j} \left[ (-1)^{\theta_i} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + \beta_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right] (-1)^{\theta_j}, \quad (\text{G.272})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(3)} = -m_f m_{f'} \left[ \beta_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + (-1)^{\theta_i} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right] (-1)^{\theta_j} (-1)^{\theta_c}, \quad (\text{G.273})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(8)} = -2|m_{\tilde{Z}_i}| m_{\tilde{W}_j} |m_f m_{f'}| \left[ \beta_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + (-1)^{\theta_i} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right] (-1)^{\theta_j} (-1)^{\theta_c}. \quad (\text{G.274})$$

Meanwhile, if the decay is neutralino to chargino:

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(4)} = -|m_{\tilde{Z}_i}| m_f \left[ (-1)^{\theta_i} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + \beta_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right], \quad (\text{G.275})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(5)} = m_{f'} |m_{\tilde{W}_j}| \left[ \beta_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + (-1)^{\theta_i} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right] (-1)^{\theta_j}, \quad (\text{G.276})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(6)} = -|m_{\tilde{Z}_i}| m_{f'} \left[ \beta_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + (-1)^{\theta_i} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right], \quad (\text{G.277})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(7)} = |m_{\tilde{W}_j}| m_f \left[ (-1)^{\theta_i} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + \beta_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right] (-1)^{\theta_j}. \quad (\text{G.278})$$

Whilst if the decay is instead chargino to neutralino:

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(4)} = |m_{\tilde{Z}_i}| m_f \left[ \alpha_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d + \beta_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \right] (-1)^{\theta_j}, \quad (\text{G.279})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(5)} = -|m_{\tilde{W}_j}| m_{f'} \left[ \alpha_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d + \alpha_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \right] (-1)^{\theta_j}, \quad (\text{G.280})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(6)} = -|m_{\tilde{Z}_i}| m_{f'} \left[ \beta_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d + \alpha_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \right] (-1)^{\theta_j}, \quad (\text{G.281})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(7)} = |m_{\tilde{W}_j}| m_f \left[ \alpha_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d + \beta_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \right] (-1)^{\theta_j}. \quad (\text{G.282})$$

We also need the following integrals, note  $s = m_{\tilde{Z}_i}^2 + m_{f'}^2 - 2|m_{\tilde{Z}_i}|E$ ,  $\lambda = \sqrt{(s - (m_f + m_{\tilde{W}})^2)(s - (m_f - m_{\tilde{W}})^2)}$ ,  $A = [m_f^2 + m_{\tilde{W}_j}^2 + 2|m_{\tilde{Z}_i}|E + (m_{\tilde{Z}_i}^2 - m_{f'}^2)(m_{\tilde{W}}^2 - m_{f'}^2)\frac{1}{s} + 2|m_{\tilde{Z}_i}|\lambda\frac{1}{s}\sqrt{E^2 - m_{f'}^2 - 2m_{\tilde{Z}_i}^2}]$  and  $B = [m_f^2 + m_{\tilde{W}_j}^2 + 2|m_{\tilde{Z}_i}|E + (m_{\tilde{Z}_i}^2 - m_{f'}^2)(m_{\tilde{W}}^2 - m_{f'}^2)\frac{1}{s} - 2|m_{\tilde{Z}_i}|\lambda\frac{1}{s}\sqrt{E^2 - m_{f'}^2 - 2m_{\tilde{Z}_i}^2}]$ :

$$I_{\tilde{f}'_1 \tilde{f}_1}^1 = 4|m_{\tilde{Z}_i}| \int_{m_{f'}}^{E_{upper}} dE \frac{[2|m_{\tilde{Z}_i}|\lambda\sqrt{E^2 - m_{f'}^2} + (m_{\tilde{Z}_i}^2 s - m_{\tilde{Z}_i}^2 m_{\tilde{W}_j}^2 - m_{f'}^2 m_{f'}^2) \log(A/B)]}{s - m_{\tilde{f}'_1}^2}, \quad (\text{G.283})$$

$$I_{\tilde{f}'_1 \tilde{f}_1}^2 = -2|m_{\tilde{Z}_i}| \int_{m_{f'}}^{E_{upper}} dE \frac{[2|m_{\tilde{Z}_i}|\lambda\frac{1}{s}\sqrt{E^2 - m_{f'}^2} + (m_{\tilde{Z}_i}^2 - 2|m_{\tilde{Z}_i}|E + m_{f'}^2 - m_{\tilde{W}_j}^2) \log(A/B)]}{s - m_{\tilde{f}'_1}^2}, \quad (\text{G.284})$$

$$I_{\tilde{f}'_1 \tilde{f}_1}^3 = 2|m_{\tilde{Z}_i}| \int_{m_{f'}}^{E_{upper}} dE \frac{[2|m_{\tilde{Z}_i}|\lambda\frac{1}{s}\sqrt{E^2 - m_{f'}^2} + (m_{\tilde{Z}_i}^2 - 2|m_{\tilde{Z}_i}|E + m_{\tilde{Z}_i}^2 - m_{f'}^2) \log(A/B)]}{s - m_{\tilde{f}'_1}^2}, \quad (\text{G.285})$$

$$I_{\tilde{f}'_1 \tilde{f}_1}^4 = 2|m_{\tilde{Z}_i}| \int_{m_{f'}}^{E_{upper}} dE \frac{[2|m_{\tilde{Z}_i}|\lambda\frac{1}{s}\sqrt{E^2 - m_{f'}^2} + (m_{\tilde{Z}_i}^2 - m_{\tilde{W}_j}^2 - m_{f'}^2) \log(A/B)]}{s - m_{\tilde{f}'_1}^2}, \quad (\text{G.286})$$

$$I_{\tilde{f}'_1 \tilde{f}_1}^5 = 2|m_{\tilde{Z}_i}| \int_{m_{f'}}^{E_{upper}} dE \frac{[2|m_{\tilde{Z}_i}| \lambda_s^{\frac{1}{2}} \sqrt{E^2 - m_{f'}^2} + (m_{\tilde{f}_1}^2 - m_{\tilde{Z}_i}^2 - m_f^2) \log(A/B)]}{s - m_{\tilde{f}'_1}^2}, \quad (\text{G.287})$$

$$I_{\tilde{f}'_1 \tilde{f}_1}^6 = 2|m_{\tilde{Z}_i}| \int_{m_{f'}}^{E_{upper}} dE \frac{\log(A/B)(s - m_{f'}^2 - m_{\tilde{W}_j}^2)}{s - m_{\tilde{f}'_1}^2}, \quad (\text{G.288})$$

$$I_{\tilde{f}'_1 \tilde{f}_1}^7 = 2|m_{\tilde{Z}_i}| \int_{m_{f'}}^{E_{upper}} dE \frac{\log(A/B)2|m_{\tilde{Z}_i}|E}{s - m_{\tilde{f}'_1}^2}, \quad (\text{G.289})$$

$$I_{\tilde{f}'_1 \tilde{f}_1}^8 = 2|m_{\tilde{Z}_i}| \int_{m_{f'}}^{E_{upper}} dE \frac{\log(A/B)}{s - m_{\tilde{f}'_1}^2}. \quad (\text{G.290})$$

The  $\Gamma_{\tilde{f}'_1 \tilde{f}_1}$  contribution is then given by:

$$\begin{aligned} \Gamma_{\tilde{f}'_1 \tilde{f}_1} = & \mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(1)} I_{\tilde{f}'_1 \tilde{f}_1}^1 + \mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(2)} I_{\tilde{f}'_1 \tilde{f}_1}^2 + \mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(3)} I_{\tilde{f}'_1 \tilde{f}_1}^3 + \mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(4)} I_{\tilde{f}'_1 \tilde{f}_1}^4 \\ & + \mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(5)} I_{\tilde{f}'_1 \tilde{f}_1}^5 + \mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(6)} I_{\tilde{f}'_1 \tilde{f}_1}^6 + \mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(7)} I_{\tilde{f}'_1 \tilde{f}_1}^7 + \mathcal{V}_{\tilde{f}'_1 \tilde{f}_1}^{(8)} I_{\tilde{f}'_1 \tilde{f}_1}^8. \end{aligned} \quad (\text{G.291})$$

The  $\Gamma_{\tilde{f}'_1 \tilde{f}_2}$ ,  $\Gamma_{\tilde{f}'_2 \tilde{f}_1}$  and  $\Gamma_{\tilde{f}'_2 \tilde{f}_2}$  contributions follow analogously, they are given here as slight differences in the expressions for couplings complicated the expressions.

$\Gamma_{\tilde{f}'_1 \tilde{f}_2}$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(1)} = -\frac{1}{2} \left[ \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} + \beta_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right] (-1)^{\theta_i}, \quad (\text{G.292})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(2)} = |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| \left[ (-1)^{\theta_i} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} + \beta_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.293})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(3)} = (-1)^{\theta_i} m_f m_{f'} \left[ \beta_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} - (-1)^{\theta_c} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.294})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(8)} = 2|m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_f m_{f'} \left[ (-1)^{\theta_i} \beta_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} + \alpha_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right]. \quad (\text{G.295})$$

If the decay is neutralino to chargino:

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(4)} = (-1)^{\theta_i} |m_{\tilde{Z}_i}| m_f \left[ \alpha_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} - \beta_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.296})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(5)} = m_{f'} |m_{\tilde{W}_j}| \left[ (-1)^{\theta_i} \beta_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} + \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right] (-1)^{\theta_j}, \quad (\text{G.297})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(6)} = -(-1)^{\theta_i} |m_{\tilde{Z}_i}| m_{f'} \left[ -\beta_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} + \alpha_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.298})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(7)} = |m_{\tilde{W}_j}| m_f \left[ (-1)^{\theta_i} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} + \beta_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right] (-1)^{\theta_j}. \quad (\text{G.299})$$

Whilst if the decay is instead chargino to neutralino:

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(4)} = |m_{\tilde{Z}_i}| m_f \left[ (-1)^{\theta_j} \alpha_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d - \beta_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \right], \quad (\text{G.300})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(5)} = -|m_{\tilde{W}_j}| m_{f'} \left[ \alpha_{\tilde{f}_2}^{\tilde{W}} \beta_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d - (-1)^{\theta_j} \alpha_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \right], \quad (\text{G.301})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(6)} = |m_{\tilde{Z}_i}| m_{f'} \left[ -\beta_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d - \alpha_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \right], \quad (\text{G.302})$$

$$\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(7)} = |m_{\tilde{W}_j}| m_f \left[ -\alpha_{\tilde{f}'_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_1}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d + (-1)^{\theta_j} \beta_{\tilde{f}'_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_1}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \right]. \quad (\text{G.303})$$

The integrals are as in the  $\tilde{f}'_1 \tilde{f}_1$  case, with the appropriate mass replacements. Similarly,  $\Gamma_{\tilde{f}'_1 \tilde{f}_2}$  is just the product of each coupling combination  $\mathcal{V}_{\tilde{f}'_1 \tilde{f}_2}^{(k)}$  with each corresponding integral  $I_{\tilde{f}'_1 \tilde{f}_2}^k$ .

$$\underline{\Gamma_{\tilde{f}'_2 \tilde{f}_1}}$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(8)} = |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_f m_{f'} (-1)^{\theta_i} (-1)^{\theta_c} \left[ \beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} - \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right]. \quad (\text{G.304})$$

If the decay is neutralino to chargino:

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(1)} = \frac{1}{2} (-1)^{\theta_i} \left[ -\alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right], \quad (\text{G.305})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(2)} = -|m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| \left[ \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} - \beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right], \quad (\text{G.306})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(3)} = (-1)^{\theta_i} m_f m_{f'} \left[ -\beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right], \quad (\text{G.307})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(4)} = |m_{\tilde{Z}_i}| m_f \left[ \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + (-1)^{\theta_i} \beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right], \quad (\text{G.308})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(5)} = (-1)^{\theta_i} m_{f'} |m_{\tilde{W}_j}| \left[ \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} - \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right] (-1)^{\theta_j}, \quad (\text{G.309})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(6)} = -(-1)^{\theta_i} |m_{\tilde{Z}_i}| m_{f'} \left[ -\beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + (-1)^{\theta_i} \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right], \quad (\text{G.310})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(7)} = -|m_{\tilde{W}_j}| m_f \left[ \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + (-1)^{\theta_i} \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right] (-1)^{\theta_j}. \quad (\text{G.311})$$

$$(\text{G.312})$$

Whilst if the decay is instead chargino to neutralino:

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(1)} = \frac{1}{2} \left[ \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right] (-1)^{\theta_i}, \quad (\text{G.313})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(2)} = |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_i} \left[ \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + \beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right], \quad (\text{G.314})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(3)} = -m_f m_{f'} \left[ \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} + \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \right], \quad (\text{G.315})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(4)} = -|m_{\tilde{Z}_i}| m_f \left[ \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d + \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \right], \quad (\text{G.316})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(5)} = |m_{\tilde{W}_j}| m_{f'} \left[ \alpha_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d + \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \right], \quad (\text{G.317})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(6)} = |m_{\tilde{Z}_i}| m_{f'} \left[ \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d + \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \right], \quad (\text{G.318})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(7)} = -|m_{\tilde{W}_j}| m_f \left[ \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_1}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d + \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d \right]. \quad (\text{G.319})$$

Again, the integrals are as in the  $\tilde{f}'_1 \tilde{f}_1$  case with the obvious mass replacements.  $\Gamma_{\tilde{f}'_2 \tilde{f}_1}$  is just the product of each coupling combination  $\mathcal{V}_{\tilde{f}'_2 \tilde{f}_1}^{(k)}$  with each corresponding integral  $I_{\tilde{f}'_2 \tilde{f}_1}^k$ .

$$\underline{\Gamma_{\tilde{f}'_2 \tilde{f}_2}}$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(8)} = -|m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_f m_{f'} (-1)^{\theta_i} (-1)^{\theta_j} \left[ \beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} - \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right]. \quad (\text{G.320})$$

If the decay is neutralino to chargino:

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(1)} = \frac{1}{2}(-1)^{\theta_j} \left[ -\alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} + (-1)^{\theta_i} \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.321})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(2)} = |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_j} \left[ \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} - \beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.322})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(3)} = -m_f m_{f'} \left[ -(-1)^{\theta_i} \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} + \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.323})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(4)} = -|m_{\tilde{Z}_i}| m_f (-1)^{\theta_j} \left[ \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} - (-1)^{\theta_i} \beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.324})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(5)} = -m_{f'} |m_{\tilde{W}_j}| \left[ (-1)^{\theta_i} (-1)^{\theta_j} \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} + \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.325})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(6)} = |m_{\tilde{Z}_i}| m_{f'} \left[ (-1)^{\theta_i} \beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} - \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.326})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(7)} = |m_{\tilde{W}_j}| m_f \left[ \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} - \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right] (-1)^{\theta_j}. \quad (\text{G.327})$$

Whilst if the decay is instead chargino to neutralino:

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(1)} = -\frac{1}{2} \left[ (-1)^{\theta_j} \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} - \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.328})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(2)} = -|m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_i} \left[ (-1)^{\theta_j} \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} - \beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.329})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(3)} = m_f m_{f'} \left[ -(-1)^{\theta_j} \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} - \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \right], \quad (\text{G.330})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(4)} = |m_{\tilde{Z}_i}| m_f (-1)^{\theta_j} \left[ \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d + \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \right], \quad (\text{G.331})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(5)} = |m_{\tilde{W}_j}| m_{f'} \left[ \alpha_{\tilde{f}_2}^{\tilde{W}} \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d - \alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \right], \quad (\text{G.332})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(6)} = (-1)^{\theta_j} |m_{\tilde{Z}_i}| m_{f'} \left[ \beta_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d + \alpha_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \right], \quad (\text{G.333})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(7)} = |m_{\tilde{W}_j}| m_f \left[ -\alpha_{\tilde{f}'_2}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} \alpha_{\tilde{Z}_i \tilde{f}_2}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^d + \beta_{\tilde{f}'_2}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}} \beta_{\tilde{Z}_i \tilde{f}_2}^u \beta_{\tilde{Z}_i \tilde{f}_2}^d \right]. \quad (\text{G.334})$$

The integrals are again as in the  $\tilde{f}'_1 \tilde{f}_1$  case with the obvious mass replacements and  $\Gamma_{\tilde{f}'_2 \tilde{f}_2}$  is just the product of each coupling combination  $\mathcal{V}_{\tilde{f}'_2 \tilde{f}_2}^{(k)}$  with each corresponding integral  $I_{\tilde{f}'_2 \tilde{f}_2}^k$ .

$\Gamma_{WH^\pm}$

For this contribution the relevant coupling combinations are:

$$\mathcal{V}_{WH^\pm}^{(1)} = -(C_{\tilde{W}\tilde{Z}W}^R \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R + C_{\tilde{W}\tilde{Z}W}^L \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L) \frac{g}{\sqrt{2}} C_{H^+ f f'}^u |m_{\tilde{W}_j}| m_{f'} (-1)^{\theta_c}, \quad (\text{G.335})$$

$$\mathcal{V}_{WH^\pm}^{(2)} = (C_{\tilde{W}\tilde{Z}W}^L \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R + C_{\tilde{W}\tilde{Z}W}^R \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L) \frac{g}{\sqrt{2}} C_{H^+ f f'}^d |m_{\tilde{Z}_i}| m_f (-1)^{\theta_i} (-1)^{\theta_j} (-1)^{\theta_c}, \quad (\text{G.336})$$

$$\mathcal{V}_{WH^\pm}^{(3)} = (C_{\tilde{W}\tilde{Z}W}^R \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R + C_{\tilde{W}\tilde{Z}W}^L \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L) \frac{g}{\sqrt{2}} C_{H^+ f f'}^d |m_{\tilde{W}_j}| m_{f'} (-1)^{\theta_c}, \quad (\text{G.337})$$

$$\mathcal{V}_{WH^\pm}^{(4)} = -(C_{\tilde{W}\tilde{Z}W}^L \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R + C_{\tilde{W}\tilde{Z}W}^R \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L) \frac{g}{\sqrt{2}} C_{H^+ f f'}^u |m_{\tilde{Z}_i}| m_{f'} (-1)^{\theta_i} (-1)^{\theta_j} (-1)^{\theta_c}. \quad (\text{G.338})$$

The integrals are as follows, with upper limit  $E_{upper3} = \frac{1}{2|m_{\tilde{Z}_i}|} (m_{\tilde{Z}_i}^2 + m_{\tilde{W}_j}^2 - m_f^2 - m_{f'}^2 - 2m_f m_{f'})$ ,  $s = m_{\tilde{Z}_i}^2 + m_{\tilde{W}_j}^2 - 2|m_{\tilde{Z}_i}|E$ ,

$$\lambda = \sqrt{(s - (m_f + m_{f'})^2)(s - (m_f - m_{f'})^2)}, \mathcal{A} = 2|m_{\tilde{Z}_i}|E + m_f^2 + m_{f'}^2 - (m_{\tilde{Z}_i}^2 - m_{\tilde{W}_j}^2)(m_f^2 - m_{f'}^2)/s \text{ and } \mathcal{B} = \frac{2|m_{\tilde{Z}_i}|}{s}\lambda\sqrt{E^2 - m_{\tilde{W}_j}^2}.$$

$$I_{WH^\pm}^1 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{-\frac{1}{2}\mathcal{A}\mathcal{B} + (m_{\tilde{Z}_i}^2 + m_f^2)\mathcal{B}}{(s - m_W^2)(s - m_{H^\pm}^2)}, \quad (\text{G.339})$$

$$I_{WH^\pm}^2 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{\frac{1}{2}\mathcal{A}\mathcal{B} - (m_{\tilde{W}_j}^2 + m_{f'}^2)\mathcal{B}}{(s - m_W^2)(s - m_{H^\pm}^2)}, \quad (\text{G.340})$$

$$I_{WH^\pm}^3 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{-\frac{1}{2}\mathcal{A}\mathcal{B} + (m_{\tilde{Z}_i}^2 - 2|m_{\tilde{Z}_i}|E - m_f^2)\mathcal{B}}{(s - m_W^2)(s - m_{H^\pm}^2)}, \quad (\text{G.341})$$

$$I_{WH^\pm}^4 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{-\frac{1}{2}\mathcal{A}\mathcal{B} - (m_{\tilde{Z}_i}^2 - 2|m_{\tilde{Z}_i}| - m_{f'}^2)\mathcal{B}}{(s - m_W^2)(s - m_{H^\pm}^2)}, \quad (\text{G.342})$$

So overall:

$$\Gamma_{WH^\pm} = \mathcal{V}_{WH^\pm}^{(1)} I_{WH^\pm}^1 (-1)^{\theta_c} (-1)^{\theta_j} + \mathcal{V}_{WH^\pm}^{(2)} I_{WH^\pm}^2 (-1)^{\theta_c} (-1)^{\theta_j} + \mathcal{V}_{WH^\pm}^{(3)} I_{WH^\pm}^3 + \mathcal{V}_{WH^\pm}^{(4)} I_{WH^\pm}^4. \quad (\text{G.343})$$

$\Gamma_{WG}$

Here everything is as above but in the coupling combinations we must make the appropriate replacements  $\omega_{H^\pm \tilde{W}^\pm \tilde{Z}}^{L/R} \rightarrow \omega_{G\tilde{W}\tilde{Z}}^{L/R}$ , whilst in the integrals we make the change  $m_{H^\pm} \rightarrow m_{goldstone} = m_W$ . However because of subtle differences in the definitions of the couplings, the overall contribution here is given by:

$$\Gamma_{WG} = \mathcal{V}_{WG}^{(1)} I_{WG}^1 + \mathcal{V}_{WG}^{(2)} I_{WG}^2 + \mathcal{V}_{WG}^{(3)} I_{WG}^3 + \mathcal{V}_{WG}^{(4)} I_{WG}^4. \quad (\text{G.344})$$

$\Gamma_{W\tilde{f}'_1}$

The coupling combinations are:

$$\mathcal{V}_{W\tilde{f}'_1}^{(1)} = -2C_{\tilde{W}\tilde{Z}W}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u \frac{g}{\sqrt{2}} \beta_{\tilde{f}'_1}^{\tilde{W}_j} |m_{\tilde{Z}_i}| m_f (-1)^{\theta_j}, \quad (\text{G.345})$$

$$\mathcal{V}_{W\tilde{f}'_1}^{(2)} = -2C_{\tilde{W}\tilde{Z}W}^L \beta_{\tilde{Z}_i\tilde{f}_1} \frac{g}{\sqrt{2}} \alpha_{\tilde{f}_1}^{\tilde{W}_j} m_{f'} |m_{\tilde{W}_j}| (-1)^{\theta_c}, \quad (\text{G.346})$$

$$\mathcal{V}_{W\tilde{f}'_1}^{(3)} = 2C_{\tilde{W}\tilde{Z}W}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u \frac{g}{\sqrt{2}} \alpha_{\tilde{f}'_1}^{\tilde{W}_j} (-1)^{\theta_i} (-1)^{\theta_j} (-1)^{\theta_c}, \quad (\text{G.347})$$

$$\mathcal{V}_{W\tilde{f}'_1}^{(4)} = (-1)^{\theta_i} 4C_{\tilde{W}\tilde{Z}W}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u \frac{g}{\sqrt{2}} \alpha_{\tilde{f}'_1}^{\tilde{W}_j} |m_{\tilde{Z}_i}| m_{f'} (-1)^{\theta_c}, \quad (\text{G.348})$$

$$\mathcal{V}_{W\tilde{f}'_1}^{(5)} = 4C_{\tilde{W}\tilde{Z}W}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u \frac{g}{\sqrt{2}} \beta_{\tilde{f}'_1}^{\tilde{W}_j} m_f |m_{\tilde{W}_j}| (-1)^{\theta_i}, \quad (\text{G.349})$$

$$\mathcal{V}_{W\tilde{f}'_1}^{(6)} = -2C_{\tilde{W}\tilde{Z}W}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u \frac{g}{\sqrt{2}} \alpha_{\tilde{f}'_1}^{\tilde{W}_j} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_c}, \quad (\text{G.350})$$

$$\mathcal{V}_{W\tilde{f}'_1}^{(7)} = -2C_{\tilde{W}\tilde{Z}W}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u \frac{g}{\sqrt{2}} \beta_{\tilde{f}'_1}^{\tilde{W}_j} m_{f'} m_f, \quad (\text{G.351})$$

$$\mathcal{V}_{W\tilde{f}'_1}^{(8)} = 8C_{\tilde{W}\tilde{Z}W}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u \frac{g}{\sqrt{2}} \beta_{\tilde{f}'_1}^{\tilde{W}_j} |m_{\tilde{Z}_i}| m_{f'} m_f |m_{\tilde{W}_j}| (-1)^{\theta_i} (-1)^{\theta_j}. \quad (\text{G.352})$$

$$(\text{G.353})$$

The integrals are as follows with  $s = m_{\tilde{Z}_i}^2 + m_{\tilde{W}_j}^2 - 2|m_{\tilde{Z}_i}|E$ ,  $\lambda = \sqrt{(s - (m_{f'} + m_f)^2)(s - (m_{f'} - m_f)^2)}$ ,  $A = m_{f'}^2 + m_f^2 + 2|m_{\tilde{Z}_i}|E + (m_{\tilde{Z}_i}^2 - m_{\tilde{W}_j}^2)(m_f^2 - m_{f'}^2)/s$ ,  $B = 2|m_{\tilde{Z}_i}|\lambda\sqrt{E^2 - m_{\tilde{W}_j}^2}$ ,  $Z = \frac{A+B-2m_{f'}^2}{A-B-2m_{f'}^2}$ :

$$I_{W\tilde{f}'_1}^1 = -2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{B + (m_{\tilde{f}'_1}^2 + m_{\tilde{W}_j}^2 - 2|m_{\tilde{Z}_i}|E - m_f^2) \log(Z)}{s - m_W^2}, \quad (\text{G.354})$$

$$I_{W\tilde{f}'_1}^2 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{B + (m_{\tilde{f}'_1}^2 + m_{\tilde{Z}_i}^2 - 2|m_{\tilde{Z}_i}|E - m_{f'}^2) \log(Z)}{s - m_W^2}, \quad (G.355)$$

$$\begin{aligned} I_{W\tilde{f}'_1}^3 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE & \left[ \{m_{\tilde{Z}_i}^2 + m_f^2 + m_{f'}^2 + m_{\tilde{W}_j}^2 - 1.5m_{\tilde{f}'_1}^2 - 0.25(A+B)\} \left( \frac{1}{2}(A+B) - m_{\tilde{f}'_1}^2 \right. \right. \\ & - (m_{\tilde{Z}_i}^2 + m_f^2 + m_{f'}^2 + m_{\tilde{W}_j}^2 - 1.5m_{\tilde{f}'_1}^2 - 0.25(A-B)) \left( \frac{1}{2}(A-B) - m_{\tilde{f}'_1}^2 \right) \\ & \left. \left. + (m_{\tilde{Z}_i}^2 + m_{f'}^2 - m_{\tilde{f}'_1}^2)(m_{\tilde{f}'_1}^2 - m_f^2 - m_{\tilde{W}_j}^2) \log(Z) \right] \frac{1}{s - m_W^2}, \end{aligned} \quad (G.356)$$

$$I_{W\tilde{f}'_1}^4 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{B + (m_{\tilde{f}'_1}^2 - m_{\tilde{Z}_i}^2 - m_{\tilde{W}_j}^2) \log(Z)}{s - m_W^2}, \quad (G.357)$$

$$I_{W\tilde{f}'_1}^5 = -2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{B + (m_{\tilde{f}'_1}^2 - m_{\tilde{Z}_i}^2 - m_{f'}^2) \log(Z)}{s - m_W^2}, \quad (G.358)$$

$$I_{W\tilde{f}'_1}^6 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{(s - m_{f'}^2 - m_{\tilde{f}'_1}^2) \log(Z)}{s - m_W^2}, \quad (G.359)$$

$$I_{W\tilde{f}'_1}^7 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{2|m_{\tilde{Z}_i}|E \log(Z)}{s - m_W^2}, \quad (G.360)$$

$$I_{W\tilde{f}'_1}^8 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{\log(Z)}{s - m_W^2}. \quad (G.361)$$

Therefore the overall contribution is:

$$\begin{aligned} \Gamma_{W\tilde{f}'_1} = & \mathcal{V}_{W\tilde{f}'_1}^{(1)} I_{W\tilde{f}'_1}^1 + \mathcal{V}_{W\tilde{f}'_1}^{(2)} I_{W\tilde{f}'_1}^2 + \mathcal{V}_{W\tilde{f}'_1}^{(3)} I_{W\tilde{f}'_1}^3 + \mathcal{V}_{W\tilde{f}'_1}^{(4)} I_{W\tilde{f}'_1}^4 \\ & + \mathcal{V}_{W\tilde{f}'_1}^{(5)} I_{W\tilde{f}'_1}^5 + \mathcal{V}_{W\tilde{f}'_1}^{(6)} I_{W\tilde{f}'_1}^6 + \mathcal{V}_{W\tilde{f}'_1}^{(7)} I_{W\tilde{f}'_1}^7 + \mathcal{V}_{W\tilde{f}'_1}^{(8)} I_{W\tilde{f}'_1}^8. \end{aligned} \quad (G.362)$$

$\Gamma_{W\tilde{f}'_2}$

The coupling combinations here are:

$$\mathcal{V}_{W\tilde{f}'_2}^{(1)} = 2C_{\tilde{W}\tilde{Z}W}^L \alpha_{\tilde{Z}_i\tilde{f}_2}^u \frac{g}{\sqrt{2}} \beta_{\tilde{f}'_2}^{\tilde{W}_j} |m_{\tilde{Z}_i}| m_f (-1)^{\theta_j} (-1)^{\theta_c}, \quad (G.363)$$

$$\mathcal{V}_{W\tilde{f}'_2}^{(2)} = 2C_{\tilde{W}\tilde{Z}W}^L \beta_{\tilde{Z}_i\tilde{f}_2}^u \frac{g}{\sqrt{2}} \alpha_{\tilde{f}'_2}^{\tilde{W}_j} m_{f'} |m_{\tilde{W}_j}| (-1)^{\theta_i} (-1)^{\theta_j}, \quad (G.364)$$

$$\mathcal{V}_{W\tilde{f}'_2}^{(3)} = 2C_{\tilde{W}\tilde{Z}W}^R \alpha_{\tilde{Z}_i\tilde{f}_2}^u \frac{g}{\sqrt{2}} \alpha_{\tilde{f}'_2}^{\tilde{W}_j} (-1)^{\theta_i}, \quad (G.365)$$

$$\mathcal{V}_{W\tilde{f}'_2}^{(4)} = -4C_{\tilde{W}\tilde{Z}W}^R \beta_{\tilde{Z}_i\tilde{f}_2}^u \frac{g}{\sqrt{2}} \alpha_{\tilde{f}'_2}^{\tilde{W}_j} |m_{\tilde{Z}_i}| m_{f'}, \quad (G.366)$$

$$\mathcal{V}_{W\tilde{f}'_2}^{(5)} = -4C_{\tilde{W}\tilde{Z}W}^R \alpha_{\tilde{Z}_i\tilde{f}_2}^u \frac{g}{\sqrt{2}} \beta_{\tilde{f}'_2}^{\tilde{W}_j} m_f |m_{\tilde{W}_j}| (-1)^{\theta_i} (-1)^{\theta_c}, \quad (G.367)$$

$$\mathcal{V}_{W\tilde{f}'_2}^{(6)} = -2C_{\tilde{W}\tilde{Z}W}^L \alpha_{\tilde{Z}_i\tilde{f}_2}^u \frac{g}{\sqrt{2}} \alpha_{\tilde{f}'_2}^{\tilde{W}_j} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_j}, \quad (G.368)$$

$$\mathcal{V}_{W\tilde{f}'_2}^{(7)} = -2C_{\tilde{W}\tilde{Z}W}^L \beta_{\tilde{Z}_i\tilde{f}_2}^u \frac{g}{\sqrt{2}} \beta_{\tilde{f}'_2}^{\tilde{W}_j} m_{f'} m_f (-1)^{\theta_i} (-1)^{\theta_j} (-1)^{\theta_c}, \quad (G.369)$$

$$\mathcal{V}_{W\tilde{f}'_2}^{(8)} = 8C_{\tilde{W}\tilde{Z}W}^R \beta_{\tilde{Z}_i\tilde{f}_2}^u \frac{g}{\sqrt{2}} \beta_{\tilde{f}'_2}^{\tilde{W}_j} |m_{\tilde{Z}_i}| m_{f'} m_f |m_{\tilde{W}_j}| (-1)^{\theta_c}. \quad (G.370)$$

The integrals here are exactly as for  $W\tilde{f}'_1$  with the change  $m_{\tilde{f}'_1} \rightarrow m_{\tilde{f}'_2}$ . As above  $\Gamma_{W\tilde{f}'_2}$  is then the sum of the products of coupling combinations,  $\mathcal{V}^{(i)}$  and integrals,  $I_{W\tilde{f}'_2}^i$ .

$\Gamma_{W\tilde{f}_1}$

Here the coupling combinations required are:

$$\mathcal{V}_{W\tilde{f}_1}^{(6)} = -2C_{\tilde{W}\tilde{Z}W}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^d \frac{g}{\sqrt{2}} \alpha_{\tilde{f}_1}^{\tilde{W}_j} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_c}, \quad (\text{G.371})$$

$$\mathcal{V}_{W\tilde{f}_1}^{(7)} = -2C_{\tilde{W}\tilde{Z}W}^R \beta_{\tilde{Z}_i\tilde{f}_1}^d \frac{g}{\sqrt{2}} \beta_{\tilde{f}_1}^{\tilde{W}_j} m_f m_{f'}, \quad (\text{G.372})$$

$$\mathcal{V}_{W\tilde{f}_1}^{(8)} = 8C_{\tilde{W}\tilde{Z}W}^L \beta_{\tilde{Z}_i\tilde{f}_1}^d \frac{g}{\sqrt{2}} \beta_{\tilde{f}_1}^{\tilde{W}_j} |m_{\tilde{Z}_i}| m_f m_{f'} |m_{\tilde{W}_j}| (-1)^{\theta_i} (-1)^{\theta_j}. \quad (\text{G.373})$$

The other coupling combinations depend upon if it is a neutralino decaying into a chargino or a chargino decaying into a neutralino, for a neutralino decaying:

$$\mathcal{V}_{W\tilde{f}_1}^{(1)} = -2C_{\tilde{W}\tilde{Z}W}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^d \frac{g}{\sqrt{2}} \beta_{\tilde{f}_1}^{\tilde{W}_j} |m_{\tilde{Z}_i}| m_{f'}, \quad (\text{G.374})$$

$$\mathcal{V}_{W\tilde{f}_1}^{(2)} = -2C_{\tilde{W}\tilde{Z}W}^R \beta_{\tilde{Z}_i\tilde{f}_1}^d \frac{g}{\sqrt{2}} \alpha_{\tilde{f}_1}^{\tilde{W}_j} m_f |m_{\tilde{W}_j}| (-1)^{\theta_j}, \quad (\text{G.375})$$

$$\mathcal{V}_{W\tilde{f}_1}^{(3)} = 2C_{\tilde{W}\tilde{Z}W}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^d \frac{g}{\sqrt{2}} \alpha_{\tilde{f}_1}^{\tilde{W}_j}, \quad (\text{G.376})$$

$$\mathcal{V}_{W\tilde{f}_1}^{(4)} = 4C_{\tilde{W}\tilde{Z}W}^L \beta_{\tilde{Z}_i\tilde{f}_1}^d \frac{g}{\sqrt{2}} \alpha_{\tilde{f}_1}^{\tilde{W}_j} |m_{\tilde{Z}_i}| m_f (-1)^{\theta_i}, \quad (\text{G.377})$$

$$\mathcal{V}_{W\tilde{f}_1}^{(5)} = 4C_{\tilde{W}\tilde{Z}W}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^d \frac{g}{\sqrt{2}} \beta_{\tilde{f}_1}^{\tilde{W}_j} m_{f'} |m_{\tilde{W}_j}| (-1)^{\theta_i} (-1)^{\theta_j}, \quad (\text{G.378})$$

whilst if it's a chargino decaying:

$$\mathcal{V}_{W\tilde{f}_1}^{(1)} = 2C_{\tilde{W}\tilde{Z}W}^R \beta_{\tilde{Z}_i\tilde{f}_1}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} \frac{g}{\sqrt{2}} |m_{\tilde{Z}_i}| m_{f'} (-1)^{\theta_i}, \quad (\text{G.379})$$

$$\mathcal{V}_{W\tilde{f}_1}^{(2)} = -2C_{\tilde{W}\tilde{Z}W}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} \frac{g}{\sqrt{2}} m_f |m_{\tilde{W}_j}|, \quad (\text{G.380})$$

$$\mathcal{V}_{W\tilde{f}_1}^{(3)} = -2C_{\tilde{W}\tilde{Z}W}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} \frac{g}{\sqrt{2}} (-1)^{\theta_i} (-1)^{\theta_j}, \quad (\text{G.381})$$

$$\mathcal{V}_{W\tilde{f}_1}^{(4)} = 4C_{\tilde{W}\tilde{Z}W}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} \frac{g}{\sqrt{2}} |m_{\tilde{Z}_i}| m_f (-1)^{\theta_i} (-1)^{\theta_j}, \quad (\text{G.382})$$

$$\mathcal{V}_{W\tilde{f}_1}^{(5)} = -4C_{\tilde{W}\tilde{Z}W}^L \beta_{\tilde{Z}_i\tilde{f}_1}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} \frac{g}{\sqrt{2}} m_{f'} |m_{\tilde{W}_j}| (-1)^{\theta_j}. \quad (\text{G.383})$$

Then the integrals are exactly as for  $W\tilde{f}'_1$  but with the changes  $m_{f'} \leftrightarrow m_f$ ,  $m_{\tilde{f}'_1} \rightarrow m_{\tilde{f}_1}$ .  $\Gamma_{W\tilde{f}_1}$  is, as above, just the sum of the products of coupling combinations and corresponding integrals.

$\Gamma_{W\tilde{f}_2}$

The coupling combinations now are:

$$\mathcal{V}_{W\tilde{f}_2}^{(6)} = 2C_{\tilde{W}\tilde{Z}W}^R \alpha_{\tilde{Z}_i\tilde{f}_2}^d \frac{g}{\sqrt{2}} \alpha_{\tilde{f}_2}^{\tilde{W}_j} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_j} (-1)^{\theta_c}, \quad (\text{G.384})$$

$$\mathcal{V}_{W\tilde{f}_2}^{(7)} = -2C_{\tilde{W}\tilde{Z}W}^R \beta_{\tilde{Z}_i\tilde{f}_2}^d \frac{g}{\sqrt{2}} \beta_{\tilde{f}_2}^{\tilde{W}_j} m_f m_{f'} (-1)^{\theta_c}, \quad (\text{G.385})$$

$$\mathcal{V}_{W\tilde{f}_2}^{(8)} = 8C_{\tilde{W}\tilde{Z}W}^L \beta_{\tilde{Z}_i\tilde{f}_2}^d \frac{g}{\sqrt{2}} \beta_{\tilde{f}_2}^{\tilde{W}_j} |m_{\tilde{Z}_i}| m_f m_{f'} |m_{\tilde{W}_j}| (-1)^{\theta_i} (-1)^{\theta_j} (-1)^{\theta_c}. \quad (\text{G.386})$$



Again, here some of the coupling combinations depend upon which way around the decay occurs, i.e. neutralino to chargino or chargino to neutralino, for neutralino decaying:

$$\mathcal{V}_{W\tilde{f}_2}^{(1)} = -2C_{\tilde{W}\tilde{Z}W}^R \alpha_{\tilde{Z}_i\tilde{f}_2}^d \frac{g}{\sqrt{2}} \beta_{\tilde{f}_2}^{\tilde{W}_j} |m_{\tilde{Z}_i}| m_{f'}, \quad (\text{G.387})$$

$$\mathcal{V}_{W\tilde{f}_2}^{(2)} = 2C_{\tilde{W}\tilde{Z}W}^R \beta_{\tilde{Z}_i\tilde{f}_2}^d \frac{g}{\sqrt{2}} \alpha_{\tilde{f}_2}^{\tilde{W}_j} m_f |m_{\tilde{W}_j}| (-1)^{\theta_j}, \quad (\text{G.388})$$

$$\mathcal{V}_{W\tilde{f}_2}^{(3)} = -2C_{\tilde{W}\tilde{Z}W}^R \alpha_{\tilde{Z}_i\tilde{f}_2}^d \frac{g}{\sqrt{2}} \alpha_{\tilde{f}_2}^{\tilde{W}_j}, \quad (\text{G.389})$$

$$\mathcal{V}_{W\tilde{f}_2}^{(4)} = -4C_{\tilde{W}\tilde{Z}W}^L \beta_{\tilde{Z}_i\tilde{f}_2}^d \frac{g}{\sqrt{2}} \alpha_{\tilde{f}_2}^{\tilde{W}_j} |m_{\tilde{Z}_i}| m_f (-1)^{\theta_i}, \quad (\text{G.390})$$

$$\mathcal{V}_{W\tilde{f}_2}^{(5)} = 4C_{\tilde{W}\tilde{Z}W}^L \alpha_{\tilde{Z}_i\tilde{f}_2}^d \frac{g}{\sqrt{2}} \beta_{\tilde{f}_2}^{\tilde{W}_j} m_{f'} |m_{\tilde{W}_j}| (-1)^{\theta_i} (-1)^{\theta_j}, \quad (\text{G.391})$$

whilst if it's a chargino decaying:

$$\mathcal{V}_{W\tilde{f}_2}^{(1)} = -2C_{\tilde{W}\tilde{Z}W}^R \beta_{\tilde{Z}_i\tilde{f}_2}^d \alpha_{\tilde{f}_2}^{\tilde{W}_j} \frac{g}{\sqrt{2}} |m_{\tilde{Z}_i}| m_{f'} (-1)^{\theta_i} (-1)^{\theta_j} (-1)^{\theta_c}, \quad (\text{G.392})$$

$$\mathcal{V}_{W\tilde{f}_2}^{(2)} = 2C_{\tilde{W}\tilde{Z}W}^R \alpha_{\tilde{Z}_i\tilde{f}_2}^d \beta_{\tilde{f}_2}^{\theta W_j} \frac{g}{\sqrt{2}} m_f |m_{\tilde{W}_j}| (-1)^{\theta_c}, \quad (\text{G.393})$$

$$\mathcal{V}_{W\tilde{f}_2}^{(3)} = -2C_{\tilde{W}\tilde{Z}W}^L \alpha_{\tilde{Z}_i\tilde{f}_2}^d \alpha_{\tilde{f}_2}^{\tilde{W}_j} \frac{g}{\sqrt{2}} (-1)^{\theta_i} (-1)^{\theta_c}, \quad (\text{G.394})$$

$$\mathcal{V}_{W\tilde{f}_2}^{(4)} = -4C_{\tilde{W}\tilde{Z}W}^L \alpha_{\tilde{Z}_i\tilde{f}_2}^d \beta_{\tilde{f}_2}^{\tilde{W}_j} \frac{g}{\sqrt{2}} |m_{\tilde{Z}_i}| m_f (-1)^{\theta_i} (-1)^{\theta_j} (-1)^{\theta_c}, \quad (\text{G.395})$$

$$\mathcal{V}_{W\tilde{f}_2}^{(5)} = 4C_{\tilde{W}\tilde{Z}W}^L \beta_{\tilde{Z}_i\tilde{f}_2}^d \alpha_{\tilde{f}_2}^{\tilde{W}_j} \frac{g}{\sqrt{2}} m_{f'} |m_{\tilde{W}_j}| (-1)^{\theta_c}. \quad (\text{G.396})$$

Then the integrals, and indeed the overall expression for  $\Gamma_{W\tilde{f}_2}$ , are just like that for  $W\tilde{f}_1$  but with the expected replacement  $m_{\tilde{f}_1} \rightarrow m_{\tilde{f}_2}$ .

$\Gamma_{H^\pm G}$

The coupling combinations are:

$$\mathcal{V}_{H^\pm G}^{(1)} = \omega_{G\tilde{W}\tilde{Z}}^L \omega_{H^+\tilde{W}^+\tilde{Z}}^L + \omega_{G\tilde{W}\tilde{Z}}^R \omega_{H^+\tilde{W}^+\tilde{Z}}^R, \quad (\text{G.397})$$

$$\mathcal{V}_{H^\pm G}^{(2)} = (\omega_{G\tilde{W}\tilde{Z}}^R \omega_{H^+\tilde{W}^+\tilde{Z}}^L + \omega_{G\tilde{W}\tilde{Z}}^L \omega_{H^+\tilde{W}^+\tilde{Z}}^R) (-1)^{\theta_i} (-1)^{\theta_j}, \quad (\text{G.398})$$

$$\mathcal{V}_{H^\pm G}^{(3)} = C_{Gff'}^u C_{H^+ff'}^u + C_{Gff'}^d C_{H^+ff'}^d, \quad (\text{G.399})$$

$$\mathcal{V}_{H^\pm G}^{(4)} = C_{Gff'}^d C_{H^+ff'}^u + C_{Gff'}^u C_{H^+ff'}^d, \quad (\text{G.400})$$

and the integrals with  $s = m_{\tilde{Z}_i}^2 + m_{\tilde{W}_j}^2 - 2|m_{\tilde{Z}_i}|E$ ,  $\lambda = \sqrt{(s - (m_f + m_{f'})^2)(s - (m_f - m_{f'})^2)}$  are:

$$I_{H^\pm G}^1 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{\lambda \sqrt{E^2 - m_{\tilde{W}_j}^2}}{s(s - m_W^2)(s - m_{H^\pm}^2)}, \quad (\text{G.401})$$

$$I_{H^\pm G}^2 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{2|m_{\tilde{Z}_i}| \lambda \sqrt{E^2 - m_{\tilde{W}_j}^2} (s - m_f^2 - m_{f'}^2)}{s(s - m_W^2)(s - m_{H^\pm}^2)}, \quad (\text{G.402})$$

$$I_{H^\pm G}^3 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{2|m_{\tilde{Z}_i}| \lambda \sqrt{E^2 - m_{\tilde{W}_j}^2} 2|m_{\tilde{Z}_i}| E}{s(s - m_W^2)(s - m_{H^\pm}^2)}, \quad (\text{G.403})$$

$$I_{H^\pm G}^4 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{2|m_{\tilde{Z}_i}|\lambda \sqrt{E^2 - m_{\tilde{W}_j}^2} (s - m_f^2 - m_{f'}^2) 2|m_{\tilde{Z}_i}|E}{s(s - m_{\tilde{W}}^2)(s - m_{H^\pm}^2)}. \quad (G.404)$$

The overall contribution is then:

$$\begin{aligned} \Gamma_{H^\pm G} &= \mathcal{V}_{H^\pm G}^{(1)} \mathcal{V}_{H^\pm G}^{(3)} I_{H^\pm G}^4 - 2\mathcal{V}_{H^\pm G}^{(1)} \mathcal{V}_{H^\pm G}^{(4)} m_f m_{f'} I_{H^\pm G}^3 + 2\mathcal{V}_{H^\pm G}^{(2)} \mathcal{V}_{H^\pm G}^{(3)} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| I_{H^\pm G}^2 \\ &\quad - 4\mathcal{V}_{H^\pm G}^{(2)} \mathcal{V}_{H^\pm G}^{(4)} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_f m_{f'} I_{H^\pm G}^1 (-1)^{\theta_j}. \end{aligned} \quad (G.405)$$

$\Gamma_{G\tilde{f}_1}$

Here the required coupling combinations are dependent on whether it's neutralino to chargino or chargino to neutralino. For a neutralino decaying:

$$\mathcal{V}_{G\tilde{f}_1}^{(1)} = \frac{1}{2}(\omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^u \alpha_{\tilde{f}_1}^{\tilde{W}_j}), \quad (G.406)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(2)} = -(\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j}) m_f |m_{\tilde{Z}_i}|, \quad (G.407)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(3)} = (\omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j}) (-1)^{\theta_i} m_{f'} |m_{\tilde{W}_j}|, \quad (G.408)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(4)} = (\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j}) m_{f'} |m_{\tilde{Z}_i}|, \quad (G.409)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(5)} = (\omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} - (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j}) m_f |m_{\tilde{W}_j}|, \quad (G.410)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(6)} = (-1)^{\theta_i} (\omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j}) |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}|, \quad (G.411)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(7)} = -(\omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j}) m_{f'} m_f, \quad (G.412)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(8)} = 2(\omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j}) |m_{\tilde{Z}_i}| m_{f'} m_f |m_{\tilde{W}_j}|. \quad (G.413)$$

For a chargino decaying:

$$\mathcal{V}_{G\tilde{f}_1}^{(1)} = -\frac{1}{2}(\omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + (-1)^{\theta_f} \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j}), \quad (G.414)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(2)} = (\omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j}) m_{f'} |m_{\tilde{Z}_i}|, \quad (G.415)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(3)} = -(\omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j}) (-1)^{\theta_j} m_f |m_{\tilde{W}_j}|, \quad (G.416)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(4)} = -(\omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j}) m_f |m_{\tilde{Z}_i}|, \quad (G.417)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(5)} = ((-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j}) m_{f'} |m_{\tilde{W}_j}|, \quad (G.418)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(6)} = -((-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j}) |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}|, \quad (G.419)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(7)} = (\omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j}) m_{f'} m_f, \quad (G.420)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(8)} = 2((-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j}) |m_{\tilde{Z}_i}| m_{f'} m_f |m_{\tilde{W}_j}|. \quad (G.421)$$

The integrals necessary are, for neutralino decaying with

$$s = m_{\tilde{Z}_i}^2 + m_{\tilde{W}_j}^2 - 2|m_{\tilde{Z}_i}|E, \lambda = \sqrt{(s - (m_{f'} + m_f)^2)(s - (m_{f'} - m_f)^2)}, A = m_f^2 + m_{f'}^2 + 2|m_{\tilde{Z}_i}|E + (m_{\tilde{Z}_i}^2 - m_{\tilde{W}_j}^2)(m_f^2 - m_{f'}^2)/s,$$

$B = 2|m_{\tilde{Z}_i}|\lambda/s \sqrt{E^2 - m_{\tilde{W}_j}^2}$ ,  $Z = \frac{(\frac{1}{2}(A+B)-m_{\tilde{f}'_1}^2)}{(\frac{1}{2}(A-B)-m_{\tilde{f}'_1}^2)}$ , given by:

$$I_{G\tilde{f}'_1}^1 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{2[sB + (m_{\tilde{f}'_1}^2 s - m_{\tilde{Z}_i}^2 m_f^2 - m_{\tilde{f}'_1}^2 m_{\tilde{W}_j}^2) \log(Z)]}{s - m_{\tilde{W}}^2}, \quad (G.422)$$

$$I_{G\tilde{f}'_1}^2 = -2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{[B + (m_{\tilde{f}'_1}^2 + m_{\tilde{W}_j}^2 - 2|m_{\tilde{Z}_i}|E - m_f^2) \log(Z)]}{s - m_{\tilde{W}}^2}, \quad (G.423)$$

$$I_{G\tilde{f}'_1}^3 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{[B + (m_{\tilde{f}'_1}^2 + m_{\tilde{Z}_i}^2 - 2|m_{\tilde{Z}_i}|E - m_{\tilde{f}'_1}^2) \log(Z)]}{s - m_{\tilde{W}}^2}, \quad (G.424)$$

$$I_{G\tilde{f}'_1}^4 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{[B + (m_{\tilde{f}'_1}^2 - m_f^2 - m_{\tilde{W}_j}^2) \log(Z)]}{s - m_{\tilde{W}}^2}, \quad (G.425)$$

$$I_{G\tilde{f}'_1}^5 = -2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{[B + (m_{\tilde{f}'_1}^2 - m_{\tilde{Z}_i}^2 - m_{\tilde{f}'_1}^2) \log(Z)]}{s - m_{\tilde{W}}^2}, \quad (G.426)$$

$$I_{G\tilde{f}'_1}^6 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{(s - m_{\tilde{f}'_1}^2 - m_f^2) \log(Z)}{s - m_{\tilde{W}}^2}, \quad (G.427)$$

$$I_{G\tilde{f}'_1}^7 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{2|m_{\tilde{Z}_i}|E \log(Z)}{s - m_{\tilde{W}}^2}, \quad (G.428)$$

$$I_{G\tilde{f}'_1}^8 = 2|m_{\tilde{Z}_i}| \int_{|m_{\tilde{W}_j}|}^{E_{upper3}} dE \frac{\log(Z)}{s - m_{\tilde{W}}^2}. \quad (G.429)$$

For a chargino decaying the integrals have the same expressions but one must swap integrals 2 and 4 and integrals 3 and 5. The overall contribution is the product of each coupling combination with the corresponding integral:

$$\Gamma_{G\tilde{f}'_1} = \mathcal{V}_{G\tilde{f}'_1}^{(1)} I_{G\tilde{f}'_1}^1 + \mathcal{V}_{G\tilde{f}'_1}^{(2)} I_{G\tilde{f}'_1}^2 + \mathcal{V}_{G\tilde{f}'_1}^{(3)} I_{G\tilde{f}'_1}^3 + \mathcal{V}_{G\tilde{f}'_1}^{(4)} I_{G\tilde{f}'_1}^4 + \mathcal{V}_{G\tilde{f}'_1}^{(5)} I_{G\tilde{f}'_1}^5 + \mathcal{V}_{G\tilde{f}'_1}^{(6)} I_{G\tilde{f}'_1}^6 + \mathcal{V}_{G\tilde{f}'_1}^{(7)} I_{G\tilde{f}'_1}^7 + \mathcal{V}_{G\tilde{f}'_1}^{(8)} I_{G\tilde{f}'_1}^8. \quad (G.430)$$

$\frac{\Gamma_{G\tilde{f}'_2}}{\Gamma_{G\tilde{f}'_1}}$

Again here the coupling combinations depend upon if we are considering neutralino to chargino or chargino to neutralino, for neutralino decaying:

$$\mathcal{V}_{G\tilde{f}'_2}^{(1)} = -\frac{1}{2}(-1)^{\theta_i} [(-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^d \beta_{\tilde{f}'_2}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^u \alpha_{\tilde{f}'_2}^{\tilde{W}_j}], \quad (G.431)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(2)} = (-1)^{\theta_i} (\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^u \beta_{\tilde{f}'_2}^{\tilde{W}_j} - \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^d \alpha_{\tilde{f}'_2}^{\tilde{W}_j}) |m_{\tilde{Z}_i}| m_f, \quad (G.432)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(3)} = -(\omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^d \beta_{\tilde{f}'_2}^{\tilde{W}_j} - \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^u \alpha_{\tilde{f}'_2}^{\tilde{W}_j}) m_{f'} |m_{\tilde{W}_j}| (-1)^{\theta_j}, \quad (G.433)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(4)} = -(\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^d \beta_{\tilde{f}'_2}^{\tilde{W}_j} - (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^u \alpha_{\tilde{f}'_2}^{\tilde{W}_j}) |m_{\tilde{Z}_i}| m_{f'}, \quad (G.434)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(5)} = (\omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^u \beta_{\tilde{f}'_2}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^d \alpha_{\tilde{f}'_2}^{\tilde{W}_j}) m_f |m_{\tilde{W}_j}| (-1)^{\theta_j}, \quad (G.435)$$

$$\mathcal{V}_{G\tilde{f}'_1}^{(6)} = [\omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^d \beta_{\tilde{f}'_2}^{\tilde{W}_j} - (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^u \alpha_{\tilde{f}'_2}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}|, \quad (G.436)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(7)} = [(-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^u \beta_{\tilde{f}'_2}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^d \alpha_{\tilde{f}'_2}^{\tilde{W}_j}] m_{f'} m_f, \quad (G.437)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(8)} = 2[-\omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^u \beta_{\tilde{f}'_2}^{\tilde{W}_j} + (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_2}^u C_{Gff'}^d \alpha_{\tilde{f}'_2}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| m_{f'} m_f |m_{\tilde{W}_j}|, \quad (G.438)$$

whilst if it's a chargino decaying into a neutralino:

$$\mathcal{V}_{G\tilde{f}'_2}^{(1)} = \frac{1}{2}[\omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^u \alpha_{\tilde{Z}_i\tilde{f}_2}^u (-1)^{\theta_j} + \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^d \beta_{\tilde{Z}_i\tilde{f}_2}^u], \quad (G.439)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(2)} = -((-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^d \alpha_{\tilde{Z}_i\tilde{f}_2}^u + \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^u \beta_{\tilde{Z}_i\tilde{f}_2}^u) |m_{\tilde{Z}_i}| m_{f'}, \quad (G.440)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(3)} = (\omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^u \alpha_{\tilde{Z}_i\tilde{f}_2}^u + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^d \beta_{\tilde{Z}_i\tilde{f}_2}^u) m_f |m_{\tilde{W}_j}|, \quad (G.441)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(4)} = ((-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^u \alpha_{\tilde{Z}_i\tilde{f}_2}^u + \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^d \beta_{\tilde{Z}_i\tilde{f}_2}^u) |m_{\tilde{Z}_i}| m_f, \quad (G.442)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(5)} = -(\omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^d \alpha_{\tilde{Z}_i\tilde{f}_2}^u + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^u \beta_{\tilde{Z}_i\tilde{f}_2}^u) m_{f'} |m_{\tilde{W}_j}|, \quad (G.443)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(6)} = [\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^u \alpha_{\tilde{Z}_i\tilde{f}_2}^u + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^d \beta_{\tilde{Z}_i\tilde{f}_2}^u] |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|, \quad (G.444)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(7)} = -[(-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^d \alpha_{\tilde{Z}_i\tilde{f}_2}^u + \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^u \beta_{\tilde{Z}_i\tilde{f}_2}^u] m_f m_{f'}, \quad (G.445)$$

$$\mathcal{V}_{G\tilde{f}'_2}^{(8)} = -2[\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^d \alpha_{\tilde{Z}_i\tilde{f}_2}^u + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{Gff'}^u \beta_{\tilde{Z}_i\tilde{f}_2}^u] m_f m_{f'} |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|. \quad (G.446)$$

The integrals are exactly as for  $G\tilde{f}'_1$  but with the change  $m_{\tilde{f}'_1} \rightarrow m_{\tilde{f}'_2}$ , and similar changes produce the overall expression for  $\Gamma_{G\tilde{f}'_2}$ .

$\Gamma_{H^\pm \tilde{f}'_1}$

Here the couplings required are dependent again on which particle is initial state and which final state, for the neutralino as the decaying (i.e. initial state) particle:

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(1)} = -\frac{1}{2}[(-1)^{\theta_i} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^d \beta_{\tilde{f}'_1}^{\tilde{W}_j} + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^u \alpha_{\tilde{f}'_1}^{\tilde{W}_j}], \quad (G.447)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(2)} = -(\omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^u \beta_{\tilde{f}'_1}^{\tilde{W}_j} + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^d \alpha_{\tilde{f}'_1}^{\tilde{W}_j}) |m_{\tilde{Z}_i}| m_f, \quad (G.448)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(3)} = (\omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^d \beta_{\tilde{f}'_1}^{\tilde{W}_j} + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^u \alpha_{\tilde{f}'_1}^{\tilde{W}_j}) m_{f'} |m_{\tilde{W}_j}| (-1)^{\theta_i}, \quad (G.449)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(4)} = -[(-1)^{\theta_i} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^d \beta_{\tilde{f}'_1}^{\tilde{W}_j} - \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^u \alpha_{\tilde{f}'_1}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| m_{f'}, \quad (G.450)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(5)} = -(\omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^u \beta_{\tilde{f}'_1}^{\tilde{W}_j} + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^d \alpha_{\tilde{f}'_1}^{\tilde{W}_j}) m_f |m_{\tilde{W}_j}| (-1)^{\theta_i}, \quad (G.451)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(6)} = (-1)^{\theta_i} [\omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^d \beta_{\tilde{f}'_1}^{\tilde{W}_j} + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^u \alpha_{\tilde{f}'_1}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}|, \quad (G.452)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(7)} = -(\omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^u \beta_{\tilde{f}'_1}^{\tilde{W}_j} + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^d \alpha_{\tilde{f}'_1}^{\tilde{W}_j}) m_{f'} m_f, \quad (G.453)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(8)} = -2(-1)^{\theta_i} [\omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^u \beta_{\tilde{f}'_1}^{\tilde{W}_j} + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{Z}_i\tilde{f}_1}^u C_{H^+ ff'}^d \alpha_{\tilde{f}'_1}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_{f'} m_f. \quad (G.454)$$

If the initial state is a chargino:

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(1)} = \frac{1}{2}(-1)^{\theta_j}[\omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^u + (-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^d \beta_{\tilde{Z}_i \tilde{f}_1}^u], \quad (G.455)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(2)} = [\omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^d \alpha_{\tilde{Z}_i \tilde{f}_1}^u + (-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^u \beta_{\tilde{Z}_i \tilde{f}_1}^u] |m_{\tilde{Z}_i}| m_{f'}, \quad (G.456)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(3)} = -[(-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^u + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^d \beta_{\tilde{Z}_i \tilde{f}_1}^u] m_f |m_{\tilde{W}_j}|, \quad (G.457)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(4)} = -[\omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^u + (-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^d \beta_{\tilde{Z}_i \tilde{f}_1}^u] |m_{\tilde{Z}_i}| m_f, \quad (G.458)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(5)} = [(-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^d \alpha_{\tilde{Z}_i \tilde{f}_1}^u + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^u \beta_{\tilde{Z}_i \tilde{f}_1}^u] m_{f'} |m_{\tilde{W}_j}|, \quad (G.459)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(6)} = [\omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^u + (-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^d \beta_{\tilde{Z}_i \tilde{f}_1}^u] |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|, \quad (G.460)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(7)} = -[(-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^d \alpha_{\tilde{Z}_i \tilde{f}_1}^u + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^u \beta_{\tilde{Z}_i \tilde{f}_1}^u] m_f m_{f'}, \quad (G.461)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_1}^{(8)} = -2[\omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^d \alpha_{\tilde{Z}_i \tilde{f}_1}^u + (-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{f}'_1}^{\tilde{W}_j} C_{H^+ f f'}^u \beta_{\tilde{Z}_i \tilde{f}_1}^u] m_f m_{f'} |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|. \quad (G.462)$$

The integrals required are exactly as in the  $G\tilde{f}'_1$  but with the expected change  $m_W \rightarrow m_{H^\pm}$ .  $\Gamma_{H^\pm \tilde{f}'_1}$  is then given exactly as  $\Gamma_{G\tilde{f}'_1}$ .

$\Gamma_{H^\pm \tilde{f}'_2}$

The coupling combinations now are, if it's a neutralino decaying:

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(1)} = -\frac{1}{2}[(-1)^{\theta_i} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^d \beta_{\tilde{f}'_2}^{\tilde{W}_j} + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^u \alpha_{\tilde{f}'_2}^{\tilde{W}_j}], \quad (G.463)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(2)} = (\omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^u \beta_{\tilde{f}'_2}^{\tilde{W}_j} - \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^d \alpha_{\tilde{f}'_2}^{\tilde{W}_j}) m_f |m_{\tilde{Z}_i}| (-1)^{\theta_i}, \quad (G.464)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(3)} = -[(-1)^{\theta_i} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^d \beta_{\tilde{f}'_2}^{\tilde{W}_j} + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^u \alpha_{\tilde{f}'_2}^{\tilde{W}_j}] m_{f'} |m_{\tilde{W}_j}|, \quad (G.465)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(4)} = -[(-1)^{\theta_i} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^d \beta_{\tilde{f}'_2}^{\tilde{W}_j} + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^u \alpha_{\tilde{f}'_2}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| m_{f'}, \quad (G.466)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(5)} = (\omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^u \beta_{\tilde{f}'_2}^{\tilde{W}_j} - \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^d \alpha_{\tilde{f}'_2}^{\tilde{W}_j}) m_{f'} |m_{\tilde{W}_j}| (-1)^{\theta_j}, \quad (G.467)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(6)} = [\omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^d \beta_{\tilde{f}'_2}^{\tilde{W}_j} - (-1)^{\theta_i} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^u \alpha_{\tilde{f}'_2}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}|, \quad (G.468)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(7)} = [-(-1)^{\theta_i} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^d \beta_{\tilde{f}'_2}^{\tilde{W}_j} + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^u \alpha_{\tilde{f}'_2}^{\tilde{W}_j}] m_{f'} m_f, \quad (G.469)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(8)} = 2(-1)^{\theta_i} [-\omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^d \beta_{\tilde{f}'_2}^{\tilde{W}_j} + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{Z}_i \tilde{f}_2}^u C_{H^+ f f'}^u \alpha_{\tilde{f}'_2}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_{f'} m_f. \quad (G.470)$$

Whilst if it's a chargino decaying:

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(1)} = \frac{1}{2}[(-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^u + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^d \beta_{\tilde{Z}_i \tilde{f}_2}^u], \quad (G.471)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(2)} = -[(-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^d \alpha_{\tilde{Z}_i \tilde{f}_2}^u + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^u \beta_{\tilde{Z}_i \tilde{f}_2}^u] |m_{\tilde{Z}_i}| m_{f'}, \quad (G.472)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(3)} = (\omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^u + (-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^d \beta_{\tilde{Z}_i \tilde{f}_2}^u) m_f |m_{\tilde{W}_j}|, \quad (G.473)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(4)} = [(-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^u + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^d \beta_{\tilde{Z}_i \tilde{f}_2}^u] |m_{\tilde{Z}_i}| m_f, \quad (G.474)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(5)} = -[\omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^d \alpha_{\tilde{Z}_i \tilde{f}_2}^u + (-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^u \beta_{\tilde{Z}_i \tilde{f}_2}^u] m_{f'} |m_{\tilde{W}_j}|, \quad (G.475)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(6)} = (\omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^u \alpha_{\tilde{Z}_i \tilde{f}_2}^u + (-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^d \beta_{\tilde{Z}_i \tilde{f}_2}^u) |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|, \quad (G.476)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(7)} = -[(-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^d \alpha_{\tilde{Z}_i \tilde{f}_2}^u + \omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^u \beta_{\tilde{Z}_i \tilde{f}_2}^u] m_f m_{f'}, \quad (G.477)$$

$$\mathcal{V}_{H^\pm \tilde{f}'_2}^{(8)} = -2[\omega_{H^+ \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^d \alpha_{\tilde{Z}_i \tilde{f}_2}^u + (-1)^{\theta_j} \omega_{H^+ \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{f}'_2}^{\tilde{W}_j} C_{H^+ f f'}^u \beta_{\tilde{Z}_i \tilde{f}_2}^u] m_f m_{f'} |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|. \quad (G.478)$$

The required integrals are as for  $G\tilde{f}'_2$  but with the mass change  $m_W \rightarrow m_{H^\pm}$ . Similarly,  $\Gamma_{H^\pm\tilde{f}'_2}$  is given analogously.

$\Gamma_{G\tilde{f}_1}$

The coupling combinations here again depend upon if it's a neutralino decaying or chargino decaying, if it's a neutralino decaying:

$$\mathcal{V}_{G\tilde{f}_1}^{(1)} = -\frac{1}{2}[\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} + (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^u \beta_{\tilde{f}_1}^{\tilde{W}_j}], \quad (G.479)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(2)} = -(\omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^u \alpha_{\tilde{f}_1}^{\tilde{W}_j} + (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j}) |m_{\tilde{Z}_i}| m_{f'}, \quad (G.480)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(3)} = [(-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^u \beta_{\tilde{f}_1}^{\tilde{W}_j}] m_f |m_{\tilde{W}_j}|, \quad (G.481)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(4)} = (\omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} - (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^u \beta_{\tilde{f}_1}^{\tilde{W}_j}) |m_{\tilde{Z}_i}| m_f, \quad (G.482)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(5)} = -[(-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^u \alpha_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j}] m_{f'} |m_{\tilde{W}_j}|, \quad (G.483)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(6)} = -[(-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} - \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^u \beta_{\tilde{f}_1}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}|, \quad (G.484)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(7)} = -(\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^u \alpha_{\tilde{f}_1}^{\tilde{W}_j} - (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j}) m_{f'} m_f, \quad (G.485)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(8)} = -2[(-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^u \alpha_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_1\tilde{Z}_i}^d C_{Gff'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_{f'} m_f. \quad (G.486)$$

On the other hand, if it's instead a chargino decaying to neutralino:

$$\mathcal{V}_{G\tilde{f}_1}^{(1)} = \frac{1}{2}[(-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^u \alpha_{\tilde{f}_1\tilde{Z}_i}^d + \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^d \beta_{\tilde{f}_1\tilde{Z}_i}^d], \quad (G.487)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(2)} = [(-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^d \alpha_{\tilde{f}_1\tilde{Z}_i}^d + \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^u \beta_{\tilde{f}_1\tilde{Z}_i}^d] |m_{\tilde{Z}_i}| m_f, \quad (G.488)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(3)} = -(\omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^u \alpha_{\tilde{f}_1\tilde{Z}_i}^d + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^d \beta_{\tilde{f}_1\tilde{Z}_i}^d) m_{f'} |m_{\tilde{W}_j}|, \quad (G.489)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(4)} = -[(-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^u \alpha_{\tilde{f}_1\tilde{Z}_i}^d + \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^d \beta_{\tilde{f}_1\tilde{Z}_i}^d] |m_{\tilde{Z}_i}| m_{f'}, \quad (G.490)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(5)} = (\omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^d + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^u \beta_{\tilde{f}_1\tilde{Z}_i}^d) m_f |m_{\tilde{W}_j}|, \quad (G.491)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(6)} = (\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^u \alpha_{\tilde{f}_1\tilde{Z}_i}^d + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^d \beta_{\tilde{f}_1\tilde{Z}_i}^d) |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|, \quad (G.492)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(7)} = -[(-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^d \alpha_{\tilde{f}_1\tilde{Z}_i}^d + \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^u \beta_{\tilde{f}_1\tilde{Z}_i}^d] m_f m_{f'}, \quad (G.493)$$

$$\mathcal{V}_{G\tilde{f}_1}^{(8)} = -2[\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^d \alpha_{\tilde{f}_1\tilde{Z}_i}^d + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{Gff'}^u \beta_{\tilde{f}_1\tilde{Z}_i}^d] m_f m_{f'} |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|. \quad (G.494)$$

The integrals are exactly as for  $G\tilde{f}'_1$  except you must swap  $m_{f'} \leftrightarrow m_f$  and  $m_{\tilde{f}'_1} \leftrightarrow m_{\tilde{f}_1}$ . As for  $G\tilde{f}'_1$ , for the case of a chargino decaying to a neutralino you must relabel integrals such that integrals 2 and 4 are swapped as are integrals 3 and 5. The  $\Gamma_{G\tilde{f}_1}$  is then given analogously to  $\Gamma_{G\tilde{f}'_1}$  as the sum of the products of the coupling combinations with corresponding integrals.

$\Gamma_{G\tilde{f}_2}$

Here the coupling combinations are, if a neutralino is decaying:

$$\mathcal{V}_{G\tilde{f}_2}^{(1)} = -\frac{1}{2}[\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^d \alpha_{\tilde{f}_2}^{\tilde{W}_j} + (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^u \beta_{\tilde{f}_2}^{\tilde{W}_j}], \quad (G.495)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(2)} = [\omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^u \alpha_{\tilde{f}_2}^{\tilde{W}_j} + (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^d \beta_{\tilde{f}_2}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| m_{f'}, \quad (G.496)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(3)} = -((-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^d \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^u \beta_{\tilde{f}_2}^{\tilde{W}_j}) m_f |m_{\tilde{W}_j}|, \quad (G.497)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(4)} = -[\omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^d \alpha_{\tilde{f}_2}^{\tilde{W}_j} + (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^u \beta_{\tilde{f}_2}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| m_f, \quad (G.498)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(5)} = [(-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^u \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^d \beta_{\tilde{f}_2}^{\tilde{W}_j}] m_{f'} |m_{\tilde{W}_j}|, \quad (G.499)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(6)} = -((-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^d \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^u \beta_{\tilde{f}_2}^{\tilde{W}_j}) |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_j}, \quad (G.500)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(7)} = [\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^u \alpha_{\tilde{f}_2}^{\tilde{W}_j} + (-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^d \beta_{\tilde{f}_2}^{\tilde{W}_j}] m_{f'} m_f, \quad (G.501)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(8)} = 2[(-1)^{\theta_i} \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^u \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_2\tilde{Z}_i}^d C_{Gff'}^d \beta_{\tilde{f}_2}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| m_{f'} m_f |m_{\tilde{W}_j}| (-1)^{\theta_j}. \quad (G.502)$$

If it's a chargino decaying:

$$\mathcal{V}_{G\tilde{f}_2}^{(1)} = -\frac{1}{2}(-1)^{\theta_j} [\omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^u \alpha_{\tilde{f}_2\tilde{Z}_i}^d - \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^d \beta_{\tilde{f}_2\tilde{Z}_i}^d], \quad (G.503)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(2)} = -[\omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^d \alpha_{\tilde{f}_2\tilde{Z}_i}^d - (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^u \beta_{\tilde{f}_2\tilde{Z}_i}^d] |m_{\tilde{Z}_i}| m_f, \quad (G.504)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(3)} = [(-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^u \alpha_{\tilde{f}_2\tilde{Z}_i}^d - \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^d \beta_{\tilde{f}_2\tilde{Z}_i}^d] m_{f'} |m_{\tilde{W}_j}|, \quad (G.505)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(4)} = (\omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^u \alpha_{\tilde{f}_2\tilde{Z}_i}^d + (-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^d \beta_{\tilde{f}_2\tilde{Z}_i}^d) |m_{\tilde{Z}_i}| m_{f'}, \quad (G.506)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(5)} = -[(-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^d \alpha_{\tilde{f}_2\tilde{Z}_i}^d + \omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^u \beta_{\tilde{f}_2\tilde{Z}_i}^d] m_f |m_{\tilde{W}_j}|, \quad (G.507)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(6)} = [\omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^u \alpha_{\tilde{f}_2\tilde{Z}_i}^d + \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^d \beta_{\tilde{f}_2\tilde{Z}_i}^d] |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|, \quad (G.508)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(7)} = -(-1)^{\theta_j} [\omega_{G\tilde{W}\tilde{Z}}^L \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^d \alpha_{\tilde{f}_2\tilde{Z}_i}^d + \omega_{G\tilde{W}\tilde{Z}}^R \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^u \beta_{\tilde{f}_2\tilde{Z}_i}^d] m_f m_{f'}, \quad (G.509)$$

$$\mathcal{V}_{G\tilde{f}_2}^{(8)} = 2[(-1)^{\theta_j} \omega_{G\tilde{W}\tilde{Z}}^R \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^d \alpha_{\tilde{f}_2\tilde{Z}_i}^d + \omega_{G\tilde{W}\tilde{Z}}^L \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{Gff'}^u \beta_{\tilde{f}_2\tilde{Z}_i}^d] m_f m_{f'} |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|. \quad (G.510)$$

The integrals are exactly as for  $G\tilde{f}'_2$  except you must swap  $m_{f'} \leftrightarrow m_f$  and  $m_{\tilde{f}'_2} \leftrightarrow m_{\tilde{f}_2}$ . As for  $G\tilde{f}'_2$ , for the case of a chargino decaying to a neutralino you must relabel integrals such that integrals 2 and 4 are swapped as are integrals 3 and 5. The  $\Gamma_{G\tilde{f}_2}$  is then given analogously to  $\Gamma_{G\tilde{f}'_2}$  as the sum of the products of the coupling combinations with corresponding integrals.

$$\underline{\Gamma_{H^\pm \tilde{f}_1}}$$

Again couplings depend upon which direction the decay occurs, if it is a neutralino decaying the couplings are:

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(1)} = \frac{1}{2}[\omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^u \beta_{\tilde{f}_1}^{\tilde{W}_j}], \quad (\text{G.511})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(2)} = -[\omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^u \alpha_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| m_{f'}, \quad (\text{G.512})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(3)} = (\omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^u \beta_{\tilde{f}_1}^{\tilde{W}_j}) m_f |m_{\tilde{W}_j}| (-1)^{\theta_i}, \quad (\text{G.513})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(4)} = (\omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^u \beta_{\tilde{f}_1}^{\tilde{W}_j}) |m_{\tilde{Z}_i}| m_f, \quad (\text{G.514})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(5)} = -[(-1)^{\theta_i} \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^u \alpha_{\tilde{f}_1}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j}] m_{f'} |m_{\tilde{W}_j}|, \quad (\text{G.515})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(6)} = [(-1)^{\theta_i} \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^d \alpha_{\tilde{f}_1}^{\tilde{W}_j} - \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^u \beta_{\tilde{f}_1}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}|, \quad (\text{G.516})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(7)} = -[\omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^u \alpha_{\tilde{f}_1}^{\tilde{W}_j} - (-1)^{\theta_i} \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j}] m_f m_{f'}, \quad (\text{G.517})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(8)} = -2[(-1)^{\theta_i} \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^u \alpha_{\tilde{f}_1}^{\tilde{W}_j} - \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{Z}_i \tilde{f}_1}^d C_{H^\pm f f'}^d \beta_{\tilde{f}_1}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_f m_{f'}. \quad (\text{G.518})$$

Whilst if it's a chargino decaying:

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(1)} = \frac{1}{2}[(-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d + \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^d \beta_{\tilde{Z}_i \tilde{f}_1}^d], \quad (\text{G.519})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(2)} = [(-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^d \alpha_{\tilde{Z}_i \tilde{f}_1}^d + \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d] |m_{\tilde{Z}_i}| m_f, \quad (\text{G.520})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(3)} = -[\omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d + (-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^d \beta_{\tilde{Z}_i \tilde{f}_1}^d] m_{f'} |m_{\tilde{W}_j}|, \quad (\text{G.521})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(4)} = -[(-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d + \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^d \beta_{\tilde{Z}_i \tilde{f}_1}^d] |m_{\tilde{Z}_i}| m_{f'}, \quad (\text{G.522})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(5)} = (\omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^d \alpha_{\tilde{Z}_i \tilde{f}_1}^d + (-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d) m_f |m_{\tilde{W}_j}|, \quad (\text{G.523})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(6)} = (\omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^u \alpha_{\tilde{Z}_i \tilde{f}_1}^d + (-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^d \beta_{\tilde{Z}_i \tilde{f}_1}^d) |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|, \quad (\text{G.524})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(7)} = -[(-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^d \alpha_{\tilde{Z}_i \tilde{f}_1}^d + \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d] m_f m_{f'}, \quad (\text{G.525})$$

$$\mathcal{V}_{H^\pm \tilde{f}_1}^{(8)} = -2[\omega_{H^\pm \tilde{W}^+ \tilde{Z}}^R \beta_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^d \alpha_{\tilde{Z}_i \tilde{f}_1}^d + (-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ \tilde{Z}}^L \alpha_{\tilde{f}_1}^{\tilde{W}_j} C_{H^\pm f f'}^u \beta_{\tilde{Z}_i \tilde{f}_1}^d] m_f m_{f'} |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|. \quad (\text{G.526})$$

The integrals and overall contribution are given exactly as for  $H^\pm \tilde{f}'_1$  but  $m_{\tilde{f}'_1} \rightarrow m_{\tilde{f}_1}$ . Remember the integrals' labelling depends on whether it's a neutralino decaying (to a chargino) or a chargino decaying (to a neutralino).

$\Gamma_{H^\pm \tilde{f}_2}$

The coupling combinations again depend upon which way around the decay is occurring, for a neutralino decaying



the coupling combinations are:

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(1)} = -\frac{1}{2}[\omega_{H^\pm \tilde{W}^+ Z}^R \beta_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^d \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ Z}^L \alpha_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^u \beta_{\tilde{f}_2}^{\tilde{W}_j}], \quad (G.527)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(2)} = (\omega_{H^\pm \tilde{W}^+ Z}^R \alpha_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^u \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ Z}^L \beta_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^d \beta_{\tilde{f}_2}^{\tilde{W}_j}) |m_{\tilde{Z}_i}| m_{f'}, \quad (G.528)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(3)} = -[\omega_{H^\pm \tilde{W}^+ Z}^L \alpha_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^d \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ Z}^R \beta_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^u \beta_{\tilde{f}_2}^{\tilde{W}_j}] m_f |m_{\tilde{W}_j}| (-1)^{\theta_i}, \quad (G.529)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(4)} = -[\omega_{H^\pm \tilde{W}^+ Z}^R \alpha_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^d \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ Z}^L \beta_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^u \beta_{\tilde{f}_2}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| m_f, \quad (G.530)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(5)} = [(-1)^{\theta_i} \omega_{H^\pm \tilde{W}^+ Z}^L \alpha_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^u \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ Z}^R \beta_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^d \beta_{\tilde{f}_2}^{\tilde{W}_j}] m_{f'} |m_{\tilde{W}_j}|, \quad (G.531)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(6)} = -[\omega_{H^\pm \tilde{W}^+ Z}^L \beta_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^d \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ Z}^R \alpha_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^u \beta_{\tilde{f}_2}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| (-1)^{\theta_i} (-1)^{\theta_j}, \quad (G.532)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(7)} = [\omega_{H^\pm \tilde{W}^+ Z}^R \beta_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^u \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ Z}^L \alpha_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^d \beta_{\tilde{f}_2}^{\tilde{W}_j}] m_{f'} m_f, \quad (G.533)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(8)} = 2[(-1)^{\theta_i} \omega_{H^\pm \tilde{W}^+ Z}^L \beta_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^u \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \omega_{H^\pm \tilde{W}^+ Z}^R \alpha_{\tilde{f}_2 \tilde{Z}_i}^d C_{H^\pm f f'}^d \beta_{\tilde{f}_2}^{\tilde{W}_j}] |m_{\tilde{Z}_i}| m_{f'} m_f |m_{\tilde{W}_j}| (-1)^{\theta_j}. \quad (G.534)$$

If it's a chargino decaying:

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(1)} = \frac{1}{2}[\omega_{H^\pm \tilde{W}^+ Z}^L \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^u \alpha_{\tilde{f}_2 \tilde{Z}_i}^d + (-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ Z}^R \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^d \beta_{\tilde{f}_2 \tilde{Z}_i}^d], \quad (G.535)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(2)} = -[\omega_{H^\pm \tilde{W}^+ Z}^L \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^d \alpha_{\tilde{f}_2 \tilde{Z}_i}^d - (-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ Z}^R \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^u \beta_{\tilde{f}_2 \tilde{Z}_i}^d] |m_{\tilde{Z}_i}| m_f, \quad (G.536)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(3)} = [(-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ Z}^R \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^u \alpha_{\tilde{f}_2 \tilde{Z}_i}^d - \omega_{H^\pm \tilde{W}^+ Z}^L \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^d \beta_{\tilde{f}_2 \tilde{Z}_i}^d] m_{f'} |m_{\tilde{W}_j}|, \quad (G.537)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(4)} = (\omega_{H^\pm \tilde{W}^+ Z}^L \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^u \alpha_{\tilde{f}_2 \tilde{Z}_i}^d - (-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ Z}^R \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^d \beta_{\tilde{f}_2 \tilde{Z}_i}^d) |m_{\tilde{Z}_i}| m_{f'}, \quad (G.538)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(5)} = -[(-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ Z}^R \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^d \alpha_{\tilde{f}_2 \tilde{Z}_i}^d + \omega_{H^\pm \tilde{W}^+ Z}^L \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^u \beta_{\tilde{f}_2 \tilde{Z}_i}^d] m_f |m_{\tilde{W}_j}|, \quad (G.539)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(6)} = (\omega_{H^\pm \tilde{W}^+ Z}^R \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^u \alpha_{\tilde{f}_2 \tilde{Z}_i}^d + \omega_{H^\pm \tilde{W}^+ Z}^L \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^d \beta_{\tilde{f}_2 \tilde{Z}_i}^d) |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|, \quad (G.540)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(7)} = -(-1)^{\theta_j} [\omega_{H^\pm \tilde{W}^+ Z}^L \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^d \alpha_{\tilde{f}_2 \tilde{Z}_i}^d + \omega_{H^\pm \tilde{W}^+ Z}^R \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^u \beta_{\tilde{f}_2 \tilde{Z}_i}^d] m_f m_{f'}, \quad (G.541)$$

$$\mathcal{V}_{H^\pm \tilde{f}_2}^{(8)} = 2[(-1)^{\theta_j} \omega_{H^\pm \tilde{W}^+ Z}^R \beta_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^d \alpha_{\tilde{f}_2 \tilde{Z}_i}^d - \omega_{H^\pm \tilde{W}^+ Z}^L \alpha_{\tilde{f}_2}^{\tilde{W}_j} C_{H^\pm f f'}^u \beta_{\tilde{f}_2 \tilde{Z}_i}^d] m_f m_{f'} |m_{\tilde{W}_j}| |m_{\tilde{Z}_i}|. \quad (G.542)$$

The integrals and overall contribution are given exactly as for  $H^\pm \tilde{f}_2$  but  $m_{\tilde{f}_2} \rightarrow m_{\tilde{f}_2'}$ . Remember the integrals labelling depends on whether it's a neutralino decaying (to a chargino) or a chargino decaying (to a neutralino).

$\Gamma_{\tilde{f}_1 \tilde{f}_2'}$

Here the coupling combinations for the interference of the two positively charged sfermions depend upon whether the decay is neutralino to chargino or chargino to neutralino. For neutralino to chargino:

$$\mathcal{V}_{\tilde{f}_1 \tilde{f}_2'}^{(1)} = (\beta_{\tilde{f}_1 \tilde{Z}_i}^u \beta_{\tilde{f}_2 \tilde{Z}_i}^u + \alpha_{\tilde{f}_1 \tilde{Z}_i}^u \alpha_{\tilde{f}_2 \tilde{Z}_i}^u) (-1)^{\theta_i}, \quad (G.543)$$

$$\mathcal{V}_{\tilde{f}_1 \tilde{f}_2'}^{(2)} = (\alpha_{\tilde{f}_1 \tilde{Z}_i}^u \beta_{\tilde{f}_2 \tilde{Z}_i}^u + \beta_{\tilde{f}_1 \tilde{Z}_i}^u \alpha_{\tilde{f}_2 \tilde{Z}_i}^u) (-1)^{\theta_i}, \quad (G.544)$$

$$\mathcal{V}_{\tilde{f}_1 \tilde{f}_2'}^{(3)} = (-\alpha_{\tilde{f}_1}^{\tilde{W}_j} \alpha_{\tilde{f}_2}^{\tilde{W}_j} + \beta_{\tilde{f}_1}^{\tilde{W}_j} \beta_{\tilde{f}_2}^{\tilde{W}_j}) (-1)^{\theta_i}, \quad (G.545)$$

$$\mathcal{V}_{\tilde{f}_1 \tilde{f}_2'}^{(4)} = (\beta_{\tilde{f}_1}^{\tilde{W}_j} \alpha_{\tilde{f}_2}^{\tilde{W}_j} - \alpha_{\tilde{f}_1}^{\tilde{W}_j} \beta_{\tilde{f}_2}^{\tilde{W}_j}) (-1)^{\theta_i} (-1)^{\theta_j}, \quad (G.546)$$

whilst for chargino to neutralino:

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}'_2}^{(1)} = -\beta_{\tilde{f}'_1}^{\tilde{W}_j} \beta_{\tilde{f}'_2}^{\tilde{W}_j} + \alpha_{\tilde{f}'_1}^{\tilde{W}_j} \alpha_{\tilde{f}'_2}^{\tilde{W}_j} (-1)^{\theta_j}, \quad (\text{G.547})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}'_2}^{(2)} = (-1)^{\theta_j} \alpha_{\tilde{f}'_1}^{\tilde{W}_j} \beta_{\tilde{f}'_2}^{\tilde{W}_j} - \alpha_{\tilde{f}'_2}^{\tilde{W}_j} \beta_{\tilde{f}'_1}^{\tilde{W}_j}, \quad (\text{G.548})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}'_2}^{(3)} = (-\alpha_{\tilde{f}_1 \tilde{Z}_i}^u \alpha_{\tilde{f}_2 \tilde{Z}_i}^u + (-1)^{\theta_j} \beta_{\tilde{f}_1 \tilde{Z}_i}^u \beta_{\tilde{f}_2 \tilde{Z}_i}^u), \quad (\text{G.549})$$

$$\mathcal{V}_{\tilde{f}'_2 \tilde{f}'_2}^{(4)} = (-\alpha_{\tilde{f}_1 \tilde{Z}_i}^u \beta_{\tilde{f}_2 \tilde{Z}_i}^u + (-1)^{\theta_j} \alpha_{\tilde{f}_2 \tilde{Z}_i}^u \beta_{\tilde{f}_1 \tilde{Z}_i}^u). \quad (\text{G.550})$$

The integrals are as follows with  $s = m_{\tilde{Z}_i}^2 + m_{f'}^2 - 2|m_{\tilde{Z}_i}|E$  and  $\lambda = \sqrt{(s - (m_f + m_{\tilde{W}_j})^2)(s - (m_f - m_{\tilde{W}_j})^2)}$ :

$$I_{\tilde{f}'_1 \tilde{f}'_2}^1 = 2|m_{\tilde{Z}_i}| \int_{m_{f'}}^{E_{upper}} 2|m_{\tilde{Z}_i}| \sqrt{E^2 - m_{f'}^2} \frac{\lambda}{s(s - m_{\tilde{f}'_1}^2)(s - m_{\tilde{f}'_2}^2)}, \quad (\text{G.551})$$

$$I_{\tilde{f}'_1 \tilde{f}'_2}^2 = 2|m_{\tilde{Z}_i}| \int_{m_{f'}}^{E_{upper}} 2|m_{\tilde{Z}_i}| \sqrt{E^2 - m_{f'}^2} \frac{(s - m_f^2 - m_{\tilde{W}_j}^2)\lambda}{s(s - m_{\tilde{f}'_1}^2)(s - m_{\tilde{f}'_2}^2)}, \quad (\text{G.552})$$

$$I_{\tilde{f}'_1 \tilde{f}'_2}^3 = 2|m_{\tilde{Z}_i}| \int_{m_{f'}}^{E_{upper}} 2|m_{\tilde{Z}_i}| \sqrt{E^2 - m_{f'}^2} \frac{2|m_{\tilde{Z}_i}|E\lambda}{s(s - m_{\tilde{f}'_1}^2)(s - m_{\tilde{f}'_2}^2)}, \quad (\text{G.553})$$

$$I_{\tilde{f}'_1 \tilde{f}'_2}^4 = 2|m_{\tilde{Z}_i}| \int_{m_{f'}}^{E_{upper}} 2|m_{\tilde{Z}_i}| \sqrt{E^2 - m_{f'}^2} \frac{(s - m_f^2 - m_{\tilde{W}_j}^2)2|m_{\tilde{Z}_i}|E\lambda}{s(s - m_{\tilde{f}'_1}^2)(s - m_{\tilde{f}'_2}^2)}. \quad (\text{G.554})$$

Now if it's instead a chargino decaying, as described before, swap the chargino and neutralino masses throughout, but also here you must relabel the integrals  $I_{\tilde{f}'_1 \tilde{f}'_2}^2 \leftrightarrow I_{\tilde{f}'_1 \tilde{f}'_2}^3$ . Also for a chargino decaying one must interchange  $m_f$  and  $m_{f'}$ . For a neutralino decaying:

$$\begin{aligned} \Gamma_{\tilde{f}'_1 \tilde{f}'_2} = & -4\mathcal{V}_{\tilde{f}'_2 \tilde{f}'_2}^{(2)} \mathcal{V}_{\tilde{f}'_2 \tilde{f}'_2}^{(4)} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_{f'} m_f I_{\tilde{f}'_1 \tilde{f}'_2}^1 + 2\mathcal{V}_{\tilde{f}'_2 \tilde{f}'_2}^{(2)} \mathcal{V}_{\tilde{f}'_2 \tilde{f}'_2}^{(3)} |m_{\tilde{Z}_i}| m_{f'} I_{\tilde{f}'_1 \tilde{f}'_2}^2 \\ & - 2\mathcal{V}_{\tilde{f}'_2 \tilde{f}'_2}^{(1)} \mathcal{V}_{\tilde{f}'_2 \tilde{f}'_2}^{(4)} m_f |m_{\tilde{W}_j}| I_{\tilde{f}'_1 \tilde{f}'_2}^3 + \mathcal{V}_{\tilde{f}'_2 \tilde{f}'_2}^{(1)} \mathcal{V}_{\tilde{f}'_2 \tilde{f}'_2}^{(3)} I_{\tilde{f}'_1 \tilde{f}'_2}^4. \end{aligned} \quad (\text{G.555})$$

$\Gamma_{\tilde{f}_1 \tilde{f}_2}$

For this interference contribution the coupling combinations used are as follows:

$$\mathcal{V}_{\tilde{f}_1 \tilde{f}_2}^{(1)} = -[(-1)^{\theta_c} \alpha_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{Z}_i \tilde{f}_2}^d + \beta_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{Z}_i \tilde{f}_2}^d] (-1)^{\theta_j}, \quad (\text{G.556})$$

$$\mathcal{V}_{\tilde{f}_1 \tilde{f}_2}^{(2)} = [(-1)^{\theta_i} \beta_{\tilde{Z}_i \tilde{f}_1}^d \alpha_{\tilde{Z}_i \tilde{f}_2}^d - \alpha_{\tilde{Z}_i \tilde{f}_1}^d \beta_{\tilde{Z}_i \tilde{f}_2}^d] (-1)^{\theta_j} (-1)^{\theta_c}, \quad (\text{G.557})$$

$$\mathcal{V}_{\tilde{f}_1 \tilde{f}_2}^{(3)} = [\beta_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} - \alpha_{\tilde{f}_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}}] (-1)^{\theta_i}, \quad (\text{G.558})$$

$$\mathcal{V}_{\tilde{f}_1 \tilde{f}_2}^{(4)} = (\alpha_{\tilde{f}_1}^{\tilde{W}} \beta_{\tilde{f}_2}^{\tilde{W}} - \beta_{\tilde{f}_1}^{\tilde{W}} \alpha_{\tilde{f}_2}^{\tilde{W}}) (-1)^{\theta_c}. \quad (\text{G.559})$$

The integrals are as follows, now with  $s = m_{\tilde{Z}_i}^2 + m_f^2 - 2|m_{\tilde{Z}_i}|E$  and  $\lambda = \sqrt{(s - (m_{f'} - m_{\tilde{W}_j})^2)(s - (m_{f'} + m_{\tilde{W}_j})^2)}$ :

$$I_{\tilde{f}_1 \tilde{f}_2}^1 = 2|m_{\tilde{Z}_i}| \int_{m_f}^{E_{upper2}} dE \frac{2|m_{\tilde{Z}_i}|}{s} \lambda \sqrt{E^2 - m_f^2} \frac{1}{(s - m_{\tilde{f}_1}^2)(s - m_{\tilde{f}_2}^2)}, \quad (\text{G.560})$$

$$I_{\tilde{f}_1 \tilde{f}_2}^2 = 2|m_{\tilde{Z}_i}| \int_{m_f}^{E_{upper2}} dE \frac{2|m_{\tilde{Z}_i}|}{s} \lambda \sqrt{E^2 - m_f^2} \frac{s - m_{\tilde{W}_j}^2 - m_{f'}^2}{(s - m_{\tilde{f}_1}^2)(s - m_{\tilde{f}_2}^2)}, \quad (\text{G.561})$$

$$I_{\tilde{f}_1 \tilde{f}_2}^3 = 2|m_{\tilde{Z}_i}| \int_{m_f}^{E_{upper2}} dE \frac{2|m_{\tilde{Z}_i}|}{s} \lambda \sqrt{E^2 - m_f^2} \frac{2|m_{\tilde{Z}_i}|E}{(s - m_{\tilde{f}_1}^2)(s - m_{\tilde{f}_2}^2)}, \quad (\text{G.562})$$

$$I_{\tilde{f}_1 \tilde{f}_2}^4 = 2|m_{\tilde{Z}_i}| \int_{m_f}^{E_{upper2}} dE \frac{2|m_{\tilde{Z}_i}|}{s} \lambda \sqrt{E^2 - m_f^2} \frac{2|m_{\tilde{Z}_i}|E(s - m_{\tilde{W}_j}^2 - m_{f'}^2)}{(s - m_{\tilde{f}_1}^2)(s - m_{\tilde{f}_2}^2)}, \quad (\text{G.563})$$

The expression for the overall contribution as a product of these couplings and integrals is:

$$\Gamma_{\tilde{f}_1 \tilde{f}_2} = - \left[ (-1)^{\theta_i} (-1)^{\theta_c} \mathcal{V}_{\tilde{f}_1 \tilde{f}_2}^{(1)} \mathcal{V}_{\tilde{f}_1 \tilde{f}_2}^{(3)} I_{\tilde{f}_1 \tilde{f}_2}^4 + 2(-1)^{\theta_i} \mathcal{V}_{\tilde{f}_1 \tilde{f}_2}^{(1)} \mathcal{V}_{\tilde{f}_1 \tilde{f}_2}^{(4)} m_f |m_{\tilde{W}_j}| I_{\tilde{f}_1 \tilde{f}_2}^2 + 2(-1)^{\theta_c} \mathcal{V}_{\tilde{f}_1 \tilde{f}_2}^{(2)} \mathcal{V}_{\tilde{f}_1 \tilde{f}_2}^{(3)} |m_{\tilde{Z}_i}| m_{f'} I_{\tilde{f}_1 \tilde{f}_2}^3 + 4 \mathcal{V}_{\tilde{f}_1 \tilde{f}_2}^{(2)} \mathcal{V}_{\tilde{f}_1 \tilde{f}_2}^{(4)} |m_{\tilde{Z}_i}| |m_{\tilde{W}_j}| m_f m_{f'} I_{\tilde{f}_1 \tilde{f}_2}^1 \right]. \quad (\text{G.564})$$

It should be noted the partial widths of the decays  $\tilde{Z}_i \rightarrow \tilde{W}_j f' \bar{f}$  and the “reverse” decay  $\tilde{W}_j \rightarrow \tilde{Z}_i f' \bar{f}$  may show strong dependence on the quark masses taken for kinematic masses and for the running masses (which is used to set Yukawa couplings), of course depending on the details of the exact spectrum considered. These mass choice effects, along with the fact that sPHENO allows (small) mixing in the first two generations of sfermions whereas it is neglected in SOFTSUSY, can cause larger differences between SOFTSUSY and sPHENO of around 25%. If the same mass choices are made and sPHENO’s small mixing angles inserted by hand into the SOFTSUSY code then these differences are reduced to around 10%.

### Appendix G.3. Chargino $1 \rightarrow 3$ Decays

$$\tilde{W}_j \rightarrow \tilde{Z}_i f' \bar{f}$$

It is detailed above in the formulae for  $\tilde{Z}_i \rightarrow \tilde{W}_j f' \bar{f}$  how to adapt the formula for the chargino decaying into the neutralino rather than the neutralino decaying into the chargino.

## Appendix H. Decays to Gravitinos

In certain SUSY-breaking scenarios, particularly gauge-mediated SUSY breaking (GMSB), the gravitino can be very light and therefore may be the Lightest Supersymmetric Particle (LSP). Moreover, the gravitino has longitudinal components from the goldstino which couple much more strongly than gravitational strength, this therefore provides interactions relevant to collider phenomenology, resulting in gravitino-SUSY-SM couplings that affect collider signatures, when the gravitino is the LSP. Consequently Next-to-Lightest Supersymmetric Particle (NLSP) decays to gravitino LSPs may be of interest and are included within our decay calculator program SOFTSUSY. The following decay modes are relevant when the initial SUSY particle is the NLSP:

1.  $\tilde{g} \rightarrow g \tilde{G}$
2.  $\tilde{q}_i \rightarrow q \tilde{G}$
3.  $\tilde{l} \rightarrow l \tilde{G}$
4.  $\tilde{Z}_i \rightarrow \gamma \tilde{G}$
5.  $\tilde{Z}_i \rightarrow Z \tilde{G}$
6.  $\tilde{Z}_i \rightarrow \phi \tilde{G}$

Decays of Higgs bosons to the gravitino are not relevant as there are always other decays available to the Higgs (whether  $h$  or  $A$ ) which dominate its branching ratio, for example even a Higgs boson were the NLSP then decays to  $\gamma\gamma$  would be available and occur much more quickly than the Planck suppressed decays to gravitinos. The formulae used were rederived but nonetheless are as provided in [39]:

$$\Gamma(\tilde{g} \rightarrow g \tilde{G}) = \frac{m_{\tilde{g}}^5}{48\pi m_{\tilde{G}}^2 M_P^{red2}}. \quad (\text{H.1})$$

$$\Gamma(\tilde{q} \rightarrow q \tilde{G}) = \frac{(m_{\tilde{q}}^2 - m_q^2)^4}{48\pi m_{\tilde{q}}^3 m_{\tilde{G}}^2 M_P^{red2}}. \quad (\text{H.2})$$

$$\Gamma(\tilde{Z}_i \rightarrow \gamma \tilde{G}) = \frac{|m_{\tilde{Z}_i}|^5}{48\pi m_{\tilde{G}}^2 M_P^{red2}} [N_{1i} \cos \theta_W + N_{2i} \sin \theta_W]^2. \quad (\text{H.3})$$

$$\Gamma(\tilde{Z}_i \rightarrow Z \tilde{G}) = \frac{(m_{\tilde{Z}_i}^2 - m_Z^2)^4}{96\pi m_{\tilde{G}}^2 M_P^{red2} |m_{\tilde{Z}_i}|^3} \left[ 2(N_{1i} \sin \theta_W - N_{2i} \cos \theta_W)^2 + (N_{4i} \sin \beta - N_{3i} \cos \beta)^2 \right]. \quad (\text{H.4})$$

$$\Gamma(\tilde{Z}_i \rightarrow \phi \tilde{G}) = \frac{(m_{\tilde{Z}_i}^2 - m_\phi^2)^4}{96\pi m_{\tilde{G}}^2 M_P^{red2} |m_{\tilde{Z}_i}|^3} C_{h/H/A}^2, \quad (\text{H.5})$$

where

$$C_{h/H/A} = \begin{cases} N_{4i} \cos \alpha - N_{3i} \sin \alpha, & \text{for h,} \\ N_{4i} \sin \alpha + N_{3i} \cos \alpha, & \text{for H,} \\ N_{4i} \cos \beta + N_{3i} \sin \beta, & \text{for A.} \end{cases} \quad (\text{H.6})$$

Note that  $M_P^{red} = \frac{M_P}{\sqrt{8\pi}} \approx 2.4 \times 10^{18} \text{ GeV}$ .

## Appendix I. NMSSM Decays

Only 2 body decays have been included in the NMSSM, this does however include the important loop decays of the neutral Higgs bosons to  $\gamma\gamma$ ,  $Z\gamma$  and  $gg$ , as well as QCD corrections to the neutral Higgs decays to  $q\bar{q}$  and  $gg$ . We do however include the decays  $\phi \rightarrow VV^* \rightarrow V f \bar{f}$ .<sup>4</sup> Note that throughout  $S(A, B)$  is the now  $3 \times 3$  CP even Higgs mixing matrix and  $P(A, B)$  is the  $2 \times 3$  CP odd Higgs mixing matrix (with goldstone excluded). The additional NMSSM variables include  $\lambda$  (distinct from the  $\tilde{\lambda}(A, B, C)$  used for kinematic part of decay widths given above),  $\kappa$  and  $\mu_{eff} = \frac{\lambda \langle S \rangle}{\sqrt{2}}$ . The conventions used for the NMSSM decay formulae were detailed earlier in section 3.2, with differences with respect to the SOFTSUSY NMSSM manual [3] noted. The conventions are those of NMSSMTools [12, 13, 24, 25], against which the formulae were checked for consistency and which provided a useful guide.

The NMSSM simply involves the addition of a gauge singlet chiral superfield to the MSSM, therefore the NMSSM has an additional neutralino, additional CP even neutral Higgs and additional CP odd neutral Higgs, therefore any decays not involving the extended neutralino or extended Higgs sectors are as in the MSSM.

### Appendix I.1. CP Even Higgs Decays

First the decay to a fermion and antifermion, with no QCD corrections (QCD corrected formulae given later in Appendix J).

$$\Gamma(h_i \rightarrow f \bar{f}) = \frac{N_c G_F m_{h_i}}{\sqrt{2} 4\pi} m_q^2 \left( 1 - 4 \frac{m_q^2}{m_{h_i}^2} \right)^{\frac{3}{2}} \mathcal{A}_{h_i f \bar{f}}^{NMSSM}, \quad (\text{I.1})$$

where

$$\mathcal{A}_{h_i f \bar{f}}^{NMSSM} = \begin{cases} \frac{S(i,1)}{\sin \beta}, & \text{for 'u'-type fermions,} \\ \frac{S(i,2)}{\cos \beta}, & \text{for 'd'-type fermions.} \end{cases} \quad (\text{I.2})$$

For squarks of the same handedness of the first two generations (so no mixing and negligible quark masses) the decay widths for the CP even Higgs  $i$  ( $i=1,2,3$  are mass ordered CP even neutral Higgs bosons) are

$$\Gamma(h_i \rightarrow \tilde{q}_{L/R} \tilde{q}_{L/R}) = \frac{N_c}{16\pi m_{h_i}} \tilde{\lambda}^{\frac{1}{2}}(m_{h_i}, m_{\tilde{q}_{L/R}}, m_{\tilde{q}_{L/R}}) C^2, \quad (\text{I.3})$$

<sup>4</sup>The decays  $\phi \rightarrow VV^* \rightarrow V f \bar{f}$ , whilst strictly being 3 body, are classified as having NDA (Number of Daughters) of 2 according to SLHA conventions.

where

$$C = \begin{cases} g\left(m_W\left(\frac{1}{2} - \frac{\tan^2 \theta_W}{6}\right)[\sin \beta S(i, 1) - \cos \beta S(i, 2)] - \frac{m_q^2 S(i, 1)}{m_W \sin \beta}\right), & \text{for up-type LH squarks,} \\ g\left(m_W\left(-\frac{1}{2} - \frac{\tan^2 \theta_W}{6}\right)[\sin \beta S(i, 1) - \cos \beta S(i, 2)] - \frac{m_q^2 S(i, 2)}{m_W \cos \beta}\right), & \text{for down-type LH squarks,} \\ g\left(\frac{2}{3}m_W \tan^2 \theta_W [\sin \beta S(i, 1) - \cos \beta S(i, 2)] - \frac{m_q^2 S(i, 1)}{m_W \sin \beta}\right), & \text{for up-type RH squarks,} \\ g\left(\frac{-1}{3}m_W \tan^2 \theta_W [\sin \beta S(i, 1) - \cos \beta S(i, 2)] - \frac{m_q^2 S(i, 2)}{m_W \cos \beta}\right), & \text{for down-type RH squarks.} \end{cases} \quad (\text{I.4})$$

For sleptons the same formulae apply but without the factor of 3 in the prefactor from  $N_c$  and with the coupling  $C$  now (note “down type” here means the charged sleptons (i.e. sleptons of the electron, muon, tau)):

$$C = \begin{cases} g\left(m_W\left(\frac{1}{2} + \frac{\tan^2 \theta_W}{2}\right)[\sin \beta S(i, 1) - \cos \beta S(i, 2)]\right), & \text{for sneutrinos (i.e. equivalent of up-type LH),} \\ g\left(m_W\left(\frac{1}{2} - \frac{\tan^2 \theta_W}{6}\right)[\sin \beta S(i, 1) - \cos \beta S(i, 2)] - \frac{m_q^2 S(i, 2)}{m_W \cos \beta}\right), & \text{for down-type LH sleptons,} \\ g\left(m_W(\tan^2 \theta_W [\sin \beta S(i, 1) - \cos \beta S(i, 2)] - \frac{m_q^2 S(i, 1)}{m_W \cos \beta})\right), & \text{for down-type RH sleptons.} \end{cases} \quad (\text{I.5})$$

For squarks of opposite handedness:

$$\Gamma(h_i \rightarrow \tilde{q}_{L/R} \tilde{q}_{R/L}) = \frac{N_c}{16\pi m_{h_i}} \tilde{\lambda}^{\frac{1}{2}}(m_{h_i}, m_{\tilde{q}_L}, m_{\tilde{q}_R}) \mathcal{D}^2, \quad (\text{I.6})$$

where

$$\mathcal{D} = \begin{cases} \frac{gm_q}{2m_W \sin \beta} [A_q S(i, 1) - \mu_{eff} S(i, 2) - \lambda \frac{\sqrt{2}m_W \cos \beta}{gS(i, 3)}], & \text{for up-type squarks of different handedness,} \\ \frac{gm_q}{2m_W \cos \beta} [A_q S(i, 2) - \mu_{eff} S(i, 1) - \lambda \frac{\sqrt{2}m_W \sin \beta}{gS(i, 3)}], & \text{for down-type squarks of different handedness.} \end{cases} \quad (\text{I.7})$$

For the decay to two sleptons with different handedness then we can use the same formulae as for two squarks of opposite handedness above but dividing by 3 to account for the fact sleptons aren't coloured (i.e. no factor  $N_c$ ). Note as only LH sneutrinos exist, only decays to charged sleptons of opposite handedness are possible, i.e. only “down-type sleptons” of different handedness. For third generation squarks and sleptons the formulae are more complicated ( $j = 1, 2$  indicates  $\tilde{t}_1$  and  $\tilde{t}_2$ ):

$$\Gamma(h_i \rightarrow \tilde{t}_j \tilde{t}_j) = \frac{3}{16\pi m_{h_i}} \tilde{\lambda}^{\frac{1}{2}}(m_{h_i}, m_{\tilde{t}_j}, m_{\tilde{t}_j}) C_{t_j t_j}^2, \quad (\text{I.8})$$

where

$$\begin{aligned} C_{t_1 t_1} = & \cos^2 \theta_t \sqrt{2} \left[ h_u^2 \langle h_1 \rangle S(i, 1) + \left( \frac{g'^2}{12} - \frac{g^2}{4} \right) \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right] \\ & + \sin^2 \theta_t \sqrt{2} \left[ h_u^2 \langle h_1 \rangle S(i, 1) - \frac{g'^2}{3} \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right] \\ & + 2 \cos \theta_t \sin \theta_t \frac{h_u}{\sqrt{2}} \left( A_t S(i, 1) - \mu_{eff} S(i, 2) - \lambda \langle h_2 \rangle S(i, 3) \right). \end{aligned} \quad (\text{I.9})$$

This was for  $j = 1$  i.e. for  $\tilde{t}_1 \tilde{t}_1$ . For  $j = 2$ , make the replacements  $\cos \theta_t \rightarrow -\sin \theta_t$  and  $\sin \theta_t \rightarrow \cos \theta_t$ .

For different stops:

$$\Gamma(h_i \rightarrow \tilde{t}_1 \tilde{t}_2) = \frac{3}{16\pi m_{h_i}} \tilde{\lambda}^{\frac{1}{2}}(m_{h_i}, m_{\tilde{t}_j}, m_{\tilde{t}_j}) C_{t_1 t_2}^2, \quad (\text{I.10})$$

where

$$\begin{aligned} C_{t_1 t_2} = & \cos \theta_t \sin \theta_t \left[ \sqrt{2} \left( h_u^2 \langle h_1 \rangle S(i, 1) - \frac{g'^2}{3} \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right) \right. \\ & \left. - \sqrt{2} \left( h_u^2 \langle h_1 \rangle S(i, 1) + \left( \frac{g'^2}{12} - \frac{g^2}{4} \right) \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right) \right] \\ & + (\cos^2 \theta_t - \sin^2 \theta_t) \frac{h_u}{\sqrt{2}} \left( A_t S(i, 1) - \mu_{eff} S(i, 2) - \lambda \langle h_2 \rangle S(i, 3) \right). \end{aligned} \quad (\text{I.11})$$

Note  $\langle h_1 \rangle = \frac{\sqrt{2}m_W \sin \beta}{g}$  and  $\langle h_2 \rangle = \frac{\sqrt{2}m_W \cos \beta}{g}$ , whilst  $h_u = \frac{m_t^{run}}{\langle h_1 \rangle}$ . For decays to sbottoms the decay formulae are the same (with the expected mass changes) except:

$$\begin{aligned} C_{b_1 b_1} = & \cos^2 \theta_b \sqrt{2} \left[ h_d^2 \langle h_2 \rangle S(i, 2) + \left( \frac{g'^2}{12} + \frac{g^2}{4} \right) \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right] \\ & + \sin^2 \theta_b \sqrt{2} \left[ h_d^2 \langle h_2 \rangle S(i, 2) + \frac{g'^2}{6} \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right] \\ & + 2 \cos \theta_b \sin \theta_b \frac{h_d}{\sqrt{2}} \left( A_b S(i, 2) - \mu_{eff} S(i, 1) - \lambda \langle h_1 \rangle S(i, 3) \right), \end{aligned} \quad (\text{I.12})$$

and

$$\begin{aligned} C_{b_1 b_2} = & \cos \theta_b \sin \theta_b \left[ \sqrt{2} \left( h_d^2 \langle h_2 \rangle S(i, 2) + \frac{g'^2}{6} \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right) \right. \\ & \left. - \sqrt{2} \left( h_d^2 \langle h_2 \rangle S(i, 2) + \left( \frac{g'^2}{12} + \frac{g^2}{4} \right) \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right) \right] \\ & + (\cos^2 \theta_b - \sin^2 \theta_b) \frac{h_d}{\sqrt{2}} \left( A_b S(i, 2) - \mu_{eff} S(i, 1) - \lambda \langle h_1 \rangle S(i, 3) \right). \end{aligned} \quad (\text{I.13})$$

Note  $h_d = \frac{m_b^{run}}{\langle h_2 \rangle}$ . Again to get the  $\tilde{b}_2 \tilde{b}_2$  coupling from the  $\tilde{b}_1 \tilde{b}_1$ , make the changes  $\cos \theta_b \rightarrow -\sin \theta_b$  and  $\sin \theta_b \rightarrow \cos \theta_b$ . For staus:

$$\Gamma(h_i \rightarrow \tilde{\tau}_j \tilde{\tau}_k) = \frac{1}{16\pi m_{h_i}} \tilde{\lambda}^{\frac{1}{2}}(m_{h_i}, m_{\tilde{\tau}_j}, m_{\tilde{\tau}_k}) C_{\tau_j \tau_k}^2. \quad (\text{I.14})$$

$$\begin{aligned} C_{\tau_1 \tau_1} = & \sin^2 \theta_\tau \sqrt{2} \left[ h_\tau^2 \langle h_2 \rangle S(i, 2) + \left( -\frac{g'^2}{4} + \frac{g^2}{4} \right) \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right] \\ & + \cos^2 \theta_\tau \sqrt{2} \left[ h_\tau^2 \langle h_2 \rangle S(i, 2) + \frac{g'^2}{2} \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right] \\ & + 2 \cos \theta_\tau \sin \theta_\tau \frac{h_\tau}{\sqrt{2}} \left( A_\tau S(i, 2) - \mu_{eff} S(i, 1) - \lambda \langle h_1 \rangle S(i, 3) \right), \end{aligned} \quad (\text{I.15})$$

$$\begin{aligned} C_{\tau_2 \tau_2} = & \cos^2 \theta_\tau \sqrt{2} \left[ h_\tau^2 \langle h_2 \rangle S(i, 2) + \left( -\frac{g'^2}{4} + \frac{g^2}{4} \right) \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right] \\ & + \sin^2 \theta_\tau \sqrt{2} \left[ h_\tau^2 \langle h_2 \rangle S(i, 2) + \frac{g'^2}{2} \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right] \\ & - 2 \cos \theta_\tau \sin \theta_\tau \frac{h_\tau}{\sqrt{2}} \left( A_\tau S(i, 2) - \mu_{eff} S(i, 1) - \lambda \langle h_1 \rangle S(i, 3) \right), \end{aligned} \quad (\text{I.16})$$

and

$$\begin{aligned} C_{\tau_1 \tau_2} = & -\cos \theta_\tau \sin \theta_\tau \left[ \sqrt{2} \left( h_\tau^2 \langle h_2 \rangle S(i, 2) + \frac{g'^2}{2} \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right) \right. \\ & \left. - \sqrt{2} \left( h_\tau^2 \langle h_2 \rangle S(i, 2) + \left( -\frac{g'^2}{4} + \frac{g^2}{4} \right) \{ \langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2) \} \right) \right] \\ & + (\sin^2 \theta_\tau - \cos^2 \theta_\tau) \frac{h_\tau}{\sqrt{2}} \left( A_\tau S(i, 2) - \mu_{eff} S(i, 1) - \lambda \langle h_1 \rangle S(i, 3) \right). \end{aligned} \quad (\text{I.17})$$

Now the decays to charginos, first of all decays to the same chargino:

$$\Gamma(h_i \rightarrow \tilde{W}_j \tilde{W}_j) = \frac{m_{h_i}}{8\pi} \tilde{\lambda}^{\frac{3}{2}}(m_{h_i}, m_{\tilde{W}_j}, m_{\tilde{W}_j}) \mathcal{F}_{jj}^2, \quad (\text{I.18})$$

where

$$\mathcal{F}_{jj} = \begin{cases} \frac{\lambda}{\sqrt{2}} S(i, 3) \cos \theta_L \cos \theta_R + \frac{g}{\sqrt{2}} [S(i, 1) \sin \theta_L \cos \theta_R + S(i, 2) \cos \theta_L \sin \theta_R], & \text{for } j = 1, \\ \frac{\lambda}{\sqrt{2}} S(i, 3) \sin \theta_L \sin \theta_R - \frac{g}{\sqrt{2}} [S(i, 1) \cos \theta_L \sin \theta_R + S(i, 2) \sin \theta_L \cos \theta_R], & \text{for } j = 2. \end{cases} \quad (\text{I.19})$$

For decays to different charginos:

$$\Gamma(h_i \rightarrow \tilde{W}_1 \tilde{W}_2) = \frac{m_{h_i}}{16\pi} \tilde{\lambda}^{\frac{1}{2}}(m_{h_i}, m_{\tilde{W}_1}, m_{\tilde{W}_2}) \left[ (c_1^2 + c_2^2) \frac{1}{m_{h_i}^2} (m_{h_i}^2 - m_{\tilde{W}_1}^2 - m_{\tilde{W}_2}^2) - 4c_1 c_2 \frac{m_{\tilde{W}_1} m_{\tilde{W}_2}}{m_{h_i}^2} \right], \quad (\text{I.20})$$

where

$$c_1 = \frac{\lambda}{\sqrt{2}} S(i, 3) \cos \theta_L \sin \theta_R + \frac{g}{\sqrt{2}} (S(i, 1) \sin \theta_L \sin \theta_R - S(i, 2) \cos \theta_L \cos \theta_R), \quad (\text{I.21})$$

$$c_2 = \frac{\lambda}{\sqrt{2}} S(i, 3) \sin \theta_L \cos \theta_R - \frac{g}{\sqrt{2}} (S(i, 1) \cos \theta_L \cos \theta_R - S(i, 2) \sin \theta_L \sin \theta_R). \quad (\text{I.22})$$

Now the decays to neutralinos:

$$\Gamma(h_i \rightarrow \tilde{Z}_j \tilde{Z}_k) = \frac{\alpha_{jk} m_{h_i}}{16\pi} \left( 1 - \left( \frac{m_{\tilde{Z}_j} + m_{\tilde{Z}_k}}{m_{h_i}} \right)^2 \right) \tilde{\lambda}^{\frac{1}{2}}(m_{h_i}, m_{\tilde{Z}_j}, m_{\tilde{Z}_k}) \mathcal{G}_{ijk}^2, \quad (\text{I.23})$$

where

$$\begin{aligned} \mathcal{G}_{ijk} = & \frac{\lambda}{\sqrt{2}} \left[ S(i, 1)(N_{3j}N_{5k} + N_{5j}N_{3k}) + S(i, 2)(N_{4j}N_{5k} + N_{5j}N_{4k}) \right. \\ & + S(i, 3)(N_{3j}N_{4k} + N_{4j}N_{3k}) \left. \right] - \sqrt{2} \kappa S(i, 3) N_{5j} N_{5k} \\ & + \frac{g'}{2} \left[ -S(i, 1)(N_{1j}N_{4k} + N_{1k}N_{4j}) \right. \\ & + S(i, 2)(N_{1j}N_{3k} + N_{3j}N_{1k}) \left. \right] + \frac{g}{2} \left[ S(i, 1)(N_{2j}N_{4k} + N_{4j}N_{2k}) \right. \\ & \left. - S(i, 2)(N_{2j}N_{3k} + N_{3j}N_{2k}) \right]. \end{aligned} \quad (\text{I.24})$$

Here  $N_{ab}$  is the neutralino mixing matrix which is now  $5 \times 5$  as the singlino mixes with the four original neutralinos. The neutralinos here are in order of increasing mass. The conventions for the NMSSM were detailed previously in section 3.2. Note the  $\alpha_{jk}$  is 2 if  $j \neq k$  and 1 if  $j = k$  in order to account for indistinguishability of particles.

The neutral Higgs decays to CP odd neutral Higgs bosons are given by:

$$\Gamma(h_i \rightarrow A_j A_k) = \frac{1}{16\pi m_{h_i}} \tilde{\lambda}^{\frac{1}{2}}(m_{h_i}, m_{A_j}, m_{A_k}) Q_{jk}^2, \quad (\text{I.25})$$

where

$$\begin{aligned} Q_{jk} = & \frac{g^2 + g'^2}{4\sqrt{2}} \left[ \langle h_1 \rangle \{C(i, j, k, 1, 1, 1) - C(i, j, k, 1, 2, 2)\} + \langle h_2 \rangle \{C(i, j, k, 2, 2, 2) - C(i, j, k, 2, 1, 1)\} \right] \\ & + \frac{\lambda A_\lambda}{\sqrt{2}} \{C(i, j, k, 1, 2, 3) + C(i, j, k, 2, 1, 3) + C(i, j, k, 3, 1, 2)\} - \frac{\kappa A_\kappa}{\sqrt{2}} C(i, j, k, 3, 3, 3) \\ & + \frac{\lambda^2}{\sqrt{2}} \left[ \langle h_1 \rangle \{C(i, j, k, 1, 2, 2) + C(i, j, k, 1, 3, 3)\} + \langle h_2 \rangle \{C(i, j, k, 2, 1, 1) + C(i, j, k, 2, 3, 3)\} \right] \\ & + \frac{\mu_{eff}}{\lambda} \{C(i, j, k, 3, 1, 1) + C(i, j, k, 3, 2, 2)\} + \frac{\kappa^2 \sqrt{2} \mu_{eff}}{\lambda} C(i, j, k, 3, 3, 3) \\ & + \frac{\lambda \kappa}{\sqrt{2}} \left[ \langle h_1 \rangle \{C(i, j, k, 2, 3, 3) - 2C(i, j, k, 3, 2, 3)\} + \langle h_2 \rangle \{C(i, j, k, 1, 3, 3) - 2C(i, j, k, 3, 1, 3)\} \right] \\ & + 2 \frac{\mu_{eff}}{\lambda} \{C(i, j, k, 3, 1, 2) - C(i, j, k, 1, 2, 3) - C(i, j, k, 2, 1, 3)\}. \end{aligned} \quad (\text{I.26})$$

The  $C(i, j, k, x, y, z)$  is the same coupling which appears later in (I.111).

$$\Gamma(h_i \rightarrow A_j Z) = \frac{(g^2 + g'^2) m_{h_i}^3}{64\pi m_Z^2} \tilde{\lambda}^{\frac{3}{2}}(m_{h_i}, m_{A_j}, m_Z) \mathcal{R}_{ij}^2, \quad (\text{I.27})$$

where

$$\mathcal{R}_{ij} = (S(i, 1) \cos \beta - S(i, 2) \sin \beta)(P(j, 1) \cos \beta - P(j, 2) \sin \beta). \quad (\text{I.28})$$

$$\Gamma(h_i \rightarrow H^+ H^-) = \frac{1}{16\pi m_{h_i}} \tilde{\lambda}^{\frac{1}{2}}(m_{h_i}, m_{H^\pm}, m_{H^\pm}) S_i^2, \quad (\text{I.29})$$

where

$$\begin{aligned} S_i = & \frac{\lambda \mu_{eff}}{\sqrt{2}} \left[ 2S(i, 3) \cos^2 \beta + 2S(i, 3) \sin^2 \beta \right] - \frac{\lambda^2 m_W \sin \beta}{g} (2S(i, 2) + 2S(i, 1)) \cos \beta \sin \beta \\ & + \mu_{eff} \kappa 2 \sqrt{2} S(i, 3) \cos \beta \sin \beta + \lambda A_\lambda \sqrt{2} S(i, 3) \cos \beta \sin \beta \\ & + \frac{g'^2}{4} \frac{m_W}{g} \left[ \sin \beta (2S(i, 1) \cos^2 \beta - 2S(i, 1) \sin^2 \beta) + \cos \beta (2S(i, 2) \sin^2 \beta - 2S(i, 2) \cos^2 \beta) \right] \\ & + \frac{g m_W}{4} \left[ \sin \beta (2S(i, 1) \sin^2 \beta - 2S(i, 2) \cos^2 \beta + 4S(i, 2) \sin \beta \cos \beta) \right. \\ & \left. + \cos \beta (2S(i, 2) \cos^2 \beta + 2S(i, 2) \sin^2 \beta + 4S(i, 1) \sin \beta \cos \beta) \right]. \end{aligned} \quad (\text{I.30})$$

$$\Gamma(h_i \rightarrow W^\pm H^\pm) = \frac{G_F m_{h_i}^3}{8\pi} [S(i, 1) \cos \beta - S(i, 2) \sin \beta]^2 \tilde{\lambda}^{\frac{3}{2}}(m_{h_i}, m_{W^\pm}, m_{H^\pm}). \quad (\text{I.31})$$

In this equation a factor of 2 has been included as it could be either  $W^+ H^-$  or  $W^- H^+$ .

$$\Gamma(h_i \rightarrow h_j h_k) = \frac{\alpha_{jk}}{32\pi m_{h_i}} \tilde{\lambda}^{\frac{1}{2}}(m_{h_i}, m_{h_j}, m_{h_k}) [C_{h_i h_j h_k}^{NMSSM}]^2, \quad (\text{I.32})$$

where again  $\alpha_{jk} = 1$  if  $j = k$  and 2 otherwise. Also remember that  $\mu_{eff} = \frac{\lambda \langle s \rangle}{\sqrt{2}}$ ,  $\langle h_1 \rangle = \frac{\sqrt{2} m_W \sin \beta}{g}$  and  $\langle h_2 \rangle = \frac{\sqrt{2} m_W \cos \beta}{g}$ , where  $\lambda$  is a parameter of the NMSSM which couples the singlino to higgsinos. Here  $C_{h_i h_j h_k}^{NMSSM}$  is given by:

$$\begin{aligned} C_{h_i h_j h_k}^{NMSSM} = & \frac{g^2 + g'^2}{4\sqrt{2}} \left[ \langle h_1 \rangle [S_{ijk}(1, 1, 1) - S_{ijk}(1, 2, 2)] + \langle h_2 \rangle [S_{ijk}(2, 2, 2) - S_{ijk}(2, 1, 1)] \right] - \frac{\lambda A_\lambda}{\sqrt{2}} S_{ijk}(1, 2, 3) + \frac{\kappa A_\kappa}{3\sqrt{2}} S_{ijk}(3, 3, 3) \\ & + \frac{\lambda^2}{\sqrt{2}} \left[ \langle h_1 \rangle [S_{ijk}(1, 2, 2) + S_{ijk}(1, 3, 3)] + \langle h_2 \rangle [S_{ijk}(2, 1, 1) + S_{ijk}(2, 3, 3)] + \frac{\mu_{eff}}{\lambda} [S_{ijk}(3, 1, 1) + S_{ijk}(3, 2, 2)] \right] \\ & + \kappa^2 \sqrt{2} \frac{\mu_{eff}}{\lambda} S_{ijk}(3, 3, 3) - \frac{\lambda \kappa}{\sqrt{2}} [\langle h_1 \rangle S_{ijk}(3, 2, 3) + \langle h_2 \rangle S_{ijk}(3, 1, 3) + 2 \frac{\mu_{eff}}{\lambda} S_{ijk}(1, 2, 3)], \end{aligned} \quad (\text{I.33})$$

where  $S_{ijk}(x, y, z)$  is just the symmetric combination of triples of  $S$  matrix elements with each of  $i, j, k$  with each of  $x, y, z$ , i.e.:

$$\begin{aligned} S_{ijk}(x, y, z) = & S(i, x) S(j, y) S(k, z) + S(i, x) S(k, y) S(j, z) + S(j, x) S(i, y) S(k, z) \\ & + S(j, x) S(i, z) S(k, y) + S(k, x) S(i, y) S(j, z) + S(k, x) S(j, y) S(i, z). \end{aligned} \quad (\text{I.34})$$

Decays to two vector bosons are complicated by the consideration of whether the Higgs boson mass is greater than twice the mass of the vector boson, just as they were complicated in the MSSM. Included in SOFTSUSY for the NMSSM are the cases both where the Higgs has mass  $m_{h/H/H3} > 2m_V$ , and so decays to two on-shell vector bosons, and also the case where the Higgs has mass  $m_V < m_{h/H/H3} < 2m_V$  so that it may only undergo a decay to one on-shell vector boson and one off-shell vector boson, which then decays into a fermion anti-fermion pair, i.e.  $h/H/H3 \rightarrow VV^* \rightarrow V f \bar{f}$ , exactly as were included for the MSSM.

As in the MSSM, first consider  $m_V < m_{h/H/H3} < 2m_V$ . The only difference compared with the MSSM formulae is in the couplings  $c_{h/H/H3VV}$ .

$$\Gamma(h/H/H3 \rightarrow ZZ^*) = \frac{g^4 m_{h/H/H3} c_{h/H/H3VV}^2}{2048\pi^3} \cos^4 \theta_W F(\epsilon_Z) \left[ 7 - \frac{40}{3} \sin^2 \theta_W + \frac{160}{9} \sin^4 \theta_W \right], \quad (\text{I.35})$$

$$\Gamma(h/H/H3 \rightarrow WW^*) = \frac{3g^4 m_{h/H/H3} c_{h/H/H3VV}^2}{512\pi^3} F(\epsilon_W), \quad (\text{I.36})$$



where here

$$\epsilon_V = \frac{m_V}{m_{h/H/H3}}, \quad (\text{I.37})$$

$$c_{hVV} = S(1, 1) \sin \beta + S(1, 2) \cos \beta, \quad (\text{I.38})$$

$$c_{HVV} = S(2, 1) \sin \beta + S(2, 2) \cos \beta, \quad (\text{I.39})$$

$$c_{H3VV} = S(3, 1) \sin \beta + S(3, 2) \cos \beta, \quad (\text{I.40})$$

and as before

$$F(\epsilon_V) = \frac{3(1 - 8\epsilon_V^2 + 20\epsilon_V^4)}{\sqrt{4\epsilon_V^2 - 1}} \cos^{-1} \left[ \frac{3\epsilon_V^2 - 1}{2\epsilon_V^3} \right] - (1 - \epsilon_V^2) \left( \frac{47}{2} \epsilon_V^2 - \frac{13}{2} + \frac{1}{\epsilon_V^2} \right) - 3(1 - 6\epsilon_V^2 + 4\epsilon_V^4) \log(\epsilon_V). \quad (\text{I.41})$$

If however  $m_{h/H/H3} > 2m_V$  then the decay to two on-shell vector bosons occurs instead and the formulae are then:

$$\Gamma(h/H/H3 \rightarrow WW) = \frac{g^2 m_{h/H/H3}^3}{64\pi m_W^2} \tilde{\lambda}^{\frac{1}{2}}(m_{h/H/H3}, m_W, m_W) (1 - r^2 + \frac{3}{4} r^4) c_{h/H/H3WW}^2, \quad (\text{I.42})$$

$$\Gamma(h/H/H3 \rightarrow ZZ) = \frac{g^2 m_{h/H/H3}^3}{128\pi m_Z^2} \tilde{\lambda}^{\frac{1}{2}}(m_{h/H/H3}, m_Z, m_Z) (1 - r^2 + \frac{3}{4} r^4) c_{h/H/H3ZZ}^2. \quad (\text{I.43})$$

Remember  $r = 2 \frac{m_V}{m_{h/H/H3}}$ .

Now onto the loop decays of the neutral Higgs bosons in the NMSSM:

$$\Gamma(h_i \rightarrow \gamma\gamma) = \frac{G_F}{\sqrt{2}} \frac{m_{h_i}^3 \alpha_{em}^2}{32\pi^2} |M_{\gamma\gamma}|^2, \quad (\text{I.44})$$

where

$$\begin{aligned} |M_{\gamma\gamma}|^2 = & \left[ I_t^r + I_b^r + I_c^r + I_\tau^r + I_{\tilde{W}_1}^r + I_{\tilde{W}_2}^r + I_W^r + I_{H^\pm}^r + I_{\tilde{c}_L}^r + I_{\tilde{c}_R}^r + I_{\tilde{s}_L}^r + I_{\tilde{s}_R}^r + I_{\tilde{\mu}_L}^r + I_{\tilde{\mu}_R}^r + I_{\tilde{t}_1}^r + I_{\tilde{t}_2}^r \right. \\ & + I_{\tilde{b}_1}^r + I_{\tilde{b}_2}^r + I_{\tilde{\tau}_1}^r + I_{\tilde{\tau}_2}^r \left. \right]^2 + \left[ I_t^i + I_b^i + I_c^i + I_\tau^i + I_{\tilde{W}_1}^i + I_{\tilde{W}_2}^i + I_W^i + I_{H^\pm}^i + I_{\tilde{c}_L}^i + I_{\tilde{c}_R}^i + I_{\tilde{s}_L}^i \right. \\ & + I_{\tilde{s}_R}^i + I_{\tilde{\mu}_L}^i + I_{\tilde{\mu}_R}^i + I_{\tilde{t}_1}^i + I_{\tilde{t}_2}^i + I_{\tilde{b}_1}^i + I_{\tilde{b}_2}^i + I_{\tilde{\tau}_1}^i + I_{\tilde{\tau}_2}^i \left. \right]^2. \end{aligned} \quad (\text{I.45})$$

The  $I_a^{r/i}$  are given below and are the real (r) and imaginary (i) parts.

$$I_a = c_a k_a, \quad (\text{I.46})$$

where the  $c_a$  is the coupling for particle  $a$  and the  $k_a$  is the kinetic part for particle  $a$ . The coupling is real whilst the kinetic part may be complex, it is from the kinetic part therefore that we get real and imaginary contributions. The couplings of the various loop particles are:

$$c_t = \frac{4}{3} \frac{S(i, 1)}{\sin \beta}, \quad (\text{I.47})$$

$$c_c = \frac{4}{3} \frac{S(i, 1)}{\sin \beta}, \quad (\text{I.48})$$

$$c_b = \frac{1}{3} \frac{S(i, 2)}{\cos \beta}, \quad (\text{I.49})$$

$$c_\tau = \frac{S(i, 2)}{\cos \beta}, \quad (\text{I.50})$$

$$c_W = S(i, 1) \sin \beta + S(i, 2) \cos \beta, \quad (\text{I.51})$$

$$c_{\tilde{W}_1} = \frac{1}{\sqrt{\sqrt{2}G_F m_{\tilde{W}_1}}} \frac{\lambda}{\sqrt{2}} S(i, 3) \cos \theta_L \cos \theta_R + \frac{g}{\sqrt{2}} [S(i, 1) \sin \theta_L \cos \theta_R + S(i, 2) \cos \theta_L \sin \theta_R], \quad (\text{I.52})$$

$$c_{\tilde{W}_2} = \frac{1}{\sqrt{\sqrt{2}G_F m_{\tilde{W}_2}}} \frac{\lambda}{\sqrt{2}} S(i, 3) \sin \theta_L \sin \theta_R - \frac{g}{\sqrt{2}} [S(i, 1) \cos \theta_L \sin \theta_R + S(i, 2) \sin \theta_L \cos \theta_R], \quad (\text{I.53})$$

$$c_{\tilde{c}_L} = \frac{4}{3} \frac{2m_W}{gm_{\tilde{c}_L}^2} \left[ \frac{g'^2}{12} + \frac{g^2}{4} \right] \frac{2m_W}{g} (\sin \beta S(i, 1) - \cos \beta S(i, 2)), \quad (\text{I.54})$$

$$c_{\tilde{c}_R} = \frac{4}{3} \frac{2m_W}{gm_{\tilde{c}_R}^2} \frac{g'^2}{6} \frac{2m_W}{g} (\sin \beta S(i, 1) - \cos \beta S(i, 2)), \quad (\text{I.55})$$

$$c_{\tilde{s}_L} = \frac{1}{3} \frac{2m_W}{gm_{\tilde{s}_L}^2} \left[ \frac{g'^2}{12} + \frac{g^2}{4} \right] \frac{2m_W}{g} (\sin \beta S(i, 1) - \cos \beta S(i, 2)), \quad (\text{I.56})$$

$$c_{\tilde{s}_R} = \frac{1}{3} \frac{2m_W}{gm_{\tilde{s}_R}^2} \frac{g'^2}{6} \frac{2m_W}{g} (\sin \beta S(i, 1) - \cos \beta S(i, 2)), \quad (\text{I.57})$$

$$c_{\tilde{\mu}_L} = \frac{2m_W}{gm_{\tilde{\mu}_L}^2} \left[ -\frac{g'^2}{4} + \frac{g^2}{4} \right] \frac{2m_W}{g} (\sin \beta S(i, 1) - \cos \beta S(i, 2)), \quad (\text{I.58})$$

$$c_{\tilde{\mu}_R} = \frac{2m_W}{gm_{\tilde{\mu}_R}^2} \frac{g'^2}{2} \frac{2m_W}{g} (\sin \beta S(i, 1) - \cos \beta S(i, 2)), \quad (\text{I.59})$$

$$\begin{aligned} c_{\tilde{t}_1} = & \frac{1}{2\sqrt{\sqrt{2}G_F m_{\tilde{t}_1}^2}} \left\{ \cos^2 \theta_t \sqrt{2} \left[ f_t^2 \sqrt{2} \frac{m_W \sin \beta}{g} S(i, 1) + \left( \frac{g'^2}{12} - \frac{g^2}{4} \right) \left\{ \frac{\sqrt{2}m_W \sin \beta}{g} S(i, 1) \right. \right. \right. \\ & - \left. \left. \frac{\sqrt{2}m_W \cos \beta}{g} S(i, 2) \right\} \right] + \sin^2 \theta_t \sqrt{2} \left[ f_t^2 \frac{\sqrt{2}m_W \sin \beta}{g} S(i, 1) \right. \\ & - \left. \frac{g'^2}{3} \left\{ \frac{\sqrt{2}m_W \sin \beta}{g} S(i, 1) - \frac{\sqrt{2}m_W \cos \beta}{g} S(i, 2) \right\} \right] \\ & \left. + 2 \sin \theta_t \cos \theta_t \frac{f_t}{\sqrt{2}} \left[ A_t S(i, 1) - \mu_{eff} S(i, 2) - \lambda \frac{\sqrt{2}m_W \cos \beta}{g} S(i, 3) \right] \right\}, \end{aligned} \quad (\text{I.60})$$

$$\begin{aligned} c_{\tilde{t}_2} = & \frac{1}{2\sqrt{\sqrt{2}G_F m_{\tilde{t}_2}^2}} \left\{ \sin^2 \theta_t \sqrt{2} \left[ f_t^2 \sqrt{2} \frac{m_W \sin \beta}{g} S(i, 1) + \left( \frac{g'^2}{12} - \frac{g^2}{4} \right) \left\{ \frac{\sqrt{2}m_W \sin \beta}{g} S(i, 1) \right. \right. \right. \\ & - \left. \left. \frac{\sqrt{2}m_W \cos \beta}{g} S(i, 2) \right\} \right] + \cos^2 \theta_t \sqrt{2} \left[ f_t^2 \frac{\sqrt{2}m_W \sin \beta}{g} S(i, 1) \right. \\ & - \left. \frac{g'^2}{3} \left\{ \frac{\sqrt{2}m_W \sin \beta}{g} S(i, 1) - \frac{\sqrt{2}m_W \cos \beta}{g} S(i, 2) \right\} \right] \\ & \left. - 2 \sin \theta_t \cos \theta_t \frac{f_t}{\sqrt{2}} \left[ A_t S(i, 1) - \mu_{eff} S(i, 2) - \lambda \frac{\sqrt{2}m_W \cos \beta}{g} S(i, 3) \right] \right\}, \end{aligned} \quad (\text{I.61})$$

$$\begin{aligned} c_{\tilde{b}_1} = & \frac{1}{2\sqrt{\sqrt{2}G_F m_{\tilde{b}_1}^2}} \left\{ \cos^2 \theta_b \sqrt{2} \left[ f_b^2 \sqrt{2} \frac{m_W \cos \beta}{g} S(i, 2) + \left( \frac{g'^2}{12} + \frac{g^2}{4} \right) \left\{ \frac{\sqrt{2}m_W \sin \beta}{g} S(i, 1) \right. \right. \right. \\ & - \left. \left. \frac{\sqrt{2}m_W \cos \beta}{g} S(i, 2) \right\} \right] + \sin^2 \theta_b \sqrt{2} \left[ f_b^2 \frac{\sqrt{2}m_W \cos \beta}{g} S(i, 2) + \frac{g'^2}{6} \left\{ \frac{\sqrt{2}m_W \sin \beta}{g} S(i, 1) \right. \right. \\ & - \left. \left. \frac{\sqrt{2}m_W \cos \beta}{g} S(i, 2) \right\} \right] + 2 \sin \theta_b \cos \theta_b \frac{f_b}{\sqrt{2}} \left[ A_b S(i, 2) - \mu_{eff} S(i, 1) - \lambda \frac{\sqrt{2}m_W \sin \beta}{g} S(i, 3) \right] \right\}, \end{aligned} \quad (\text{I.62})$$

$$c_{\tilde{b}_2} = \frac{1}{2\sqrt{\sqrt{2}G_F m_{\tilde{b}_2}^2}} \left\{ \sin^2 \theta_b \sqrt{2} \left[ f_b^2 \sqrt{2} \frac{m_W \cos \beta}{g} S(i, 2) + \left( \frac{g'^2}{12} + \frac{g^2}{4} \right) \left\{ \frac{\sqrt{2} m_W \sin \beta}{g} S(i, 1) - \frac{\sqrt{2} m_W \cos \beta}{g} S(i, 2) \right\} \right] + \cos^2 \theta_b \sqrt{2} \left[ f_b^2 \frac{\sqrt{2} m_W \cos \beta}{g} S(i, 2) + \frac{g'^2}{6} \left\{ \frac{\sqrt{2} m_W \sin \beta}{g} S(i, 1) - \frac{\sqrt{2} m_W \cos \beta}{g} S(i, 2) \right\} \right] - 2 \sin \theta_b \cos \theta_b \frac{f_b}{\sqrt{2}} \left[ A_b S(i, 2) - \mu_{eff} S(i, 1) - \lambda \frac{\sqrt{2} m_W \sin \beta}{g} S(i, 3) \right] \right\}, \quad (I.63)$$

$$c_{\tilde{\tau}_1} = \frac{1}{2\sqrt{\sqrt{2}G_F m_{\tilde{\tau}_1}^2}} \left\{ \cos^2 \theta_\tau \sqrt{2} \left[ f_\tau^2 \sqrt{2} \frac{m_W \cos \beta}{g} S(i, 2) + \left( -\frac{g'^2}{4} + \frac{g^2}{4} \right) \left\{ \frac{\sqrt{2} m_W \sin \beta}{g} S(i, 1) - \frac{\sqrt{2} m_W \cos \beta}{g} S(i, 2) \right\} \right] + \sin^2 \theta_\tau \sqrt{2} \left[ f_\tau^2 \frac{\sqrt{2} m_W \cos \beta}{g} S(i, 2) + \frac{g'^2}{2} \left\{ \frac{\sqrt{2} m_W \sin \beta}{g} S(i, 1) - \frac{\sqrt{2} m_W \cos \beta}{g} S(i, 2) \right\} \right] + 2 \sin \theta_\tau \cos \theta_\tau \frac{f_\tau}{\sqrt{2}} \left[ A_\tau S(i, 2) - \mu_{eff} S(i, 1) - \lambda \frac{\sqrt{2} m_W \sin \beta}{g} S(i, 3) \right] \right\}, \quad (I.64)$$

$$c_{\tilde{\tau}_2} = \frac{1}{2\sqrt{\sqrt{2}G_F m_{\tilde{\tau}_2}^2}} \left\{ \sin^2 \theta_\tau \sqrt{2} \left[ f_\tau^2 \sqrt{2} \frac{m_W \cos \beta}{g} S(i, 2) + \left( -\frac{g'^2}{4} + \frac{g^2}{4} \right) \left\{ \frac{\sqrt{2} m_W \sin \beta}{g} S(i, 1) - \frac{\sqrt{2} m_W \cos \beta}{g} S(i, 2) \right\} \right] + \cos^2 \theta_\tau \sqrt{2} \left[ f_\tau^2 \frac{\sqrt{2} m_W \cos \beta}{g} S(i, 2) + \frac{g'^2}{2} \left\{ \frac{\sqrt{2} m_W \sin \beta}{g} S(i, 1) - \frac{\sqrt{2} m_W \cos \beta}{g} S(i, 2) \right\} \right] - 2 \sin \theta_\tau \cos \theta_\tau \frac{f_\tau}{\sqrt{2}} \left[ A_\tau S(i, 2) - \mu_{eff} S(i, 1) - \lambda \frac{\sqrt{2} m_W \sin \beta}{g} S(i, 3) \right] \right\}, \quad (I.65)$$

$$c_{H^\pm} = \frac{\lambda \mu_{eff}}{\sqrt{2}} \left[ 2S(i, 3) \cos^2 \beta + 2S(i, 3) \sin^2 \beta \right] - \lambda^2 \frac{m_W \sin \beta}{g} 2S(i, 2) \cos \beta \sin \beta - \frac{m_W \sin \beta}{g} 2S(i, 1) \cos \beta \sin \beta + \mu_{eff} \kappa 2\sqrt{2} S(i, 3) \cos \beta \sin \beta + \frac{\lambda A_\lambda}{\sqrt{2}} 2S(i, 3) \cos \beta \sin \beta + \frac{g'^2}{4} \frac{m_W}{g} \left[ \sin \beta \{ 2S(i, 1) \cos^2 \beta - 2S(i, 1) \sin^2 \beta \} + \cos \beta \{ 2S(i, 2) \sin^2 \beta - 2S(i, 2) \cos^2 \beta \} \right] + \frac{gm_W}{4} \left[ \sin \beta \{ 2S(i, 1) \cos^2 \beta + 2S(i, 1) \sin^2 \beta + 4S(i, 2) \sin \beta \cos \beta \} + \cos \beta \{ 2S(i, 2) \cos^2 \beta + 2S(i, 2) \sin^2 \beta + 4S(i, 1) \sin \beta \cos \beta \} \right], \quad (I.66)$$

where we remember that

$$f_t = \frac{gm_t}{\sqrt{2}m_W \sin \beta}, \quad f_b = \frac{gm_b}{\sqrt{2}m_W \cos \beta}, \quad f_\tau = \frac{gm_\tau}{\sqrt{2}m_W \cos \beta}. \quad (I.67)$$

The kinetic parts meanwhile are as follows, they depend upon the  $f(\tau)$  given in eq. (E.2).

For fermions (spin  $\frac{1}{2}$ ), i.e. the quarks and the charginos:

$$k_a = 2\tau_a [1 - \tau_a f(\tau_a)]. \quad (I.68)$$

For scalars (e.g. sfermions and  $H^\pm$ ):

$$k_a = \tau_a (\tau_a f(\tau_a) - 1). \quad (I.69)$$

For spin 1 (i.e.  $W^\pm$  bosons):

$$k_a = -[2 + 3\tau_a + 3\tau_a(2 - \tau_a)f(\tau_a)]. \quad (I.70)$$

That's all the information needed for  $h_i \rightarrow \gamma\gamma$ .

$$\Gamma(h_i \rightarrow Z\gamma) = \frac{g^2 m_{h_i}^3 \alpha_{em}^2}{512\pi^3 m_W^2} \left(1 - \left(\frac{m_Z}{m_{h_i}}\right)^2\right)^3 |M_{Z\gamma}|^2, \quad (\text{I.71})$$

where

$$|M_{Z\gamma}|^2 = \left(\mathcal{I}_t^r + \mathcal{I}_b^r + \mathcal{I}_c^r + \mathcal{I}_{\tilde{W}_1}^r + \mathcal{I}_{\tilde{W}_2}^r + \mathcal{I}_W^r + \mathcal{I}_{H^\pm}^r\right)^2 + \left(\mathcal{I}_t^i + \mathcal{I}_b^i + \mathcal{I}_c^i + \mathcal{I}_{\tilde{W}_1}^i + \mathcal{I}_{\tilde{W}_2}^i + \mathcal{I}_W^i + \mathcal{I}_{H^\pm}^i\right)^2. \quad (\text{I.72})$$

Again  $\mathcal{I}_a^{r/i}$  are the real and imaginary parts of each contribution and  $\mathcal{I}_a = c_a k_a$ , where now the  $c_a$  and  $k_a$  are different to above as this is now for decays to  $Z\gamma$ . Now the couplings are:

$$c_t = -2\left(1 - \frac{8}{3} \sin^2 \theta_W\right) \frac{1}{\sin \theta_W \cos \theta_W} \frac{S(i, 1)}{\sin \beta}, \quad (\text{I.73})$$

$$c_c = -2\left(1 - \frac{8}{3} \sin^2 \theta_W\right) \frac{1}{\sin \theta_W \cos \theta_W} \frac{S(i, 1)}{\sin \beta}, \quad (\text{I.74})$$

$$c_b = \left(-1 + \frac{4}{3} \sin^2 \theta_W\right) \frac{1}{\sin \theta_W \cos \theta_W} \frac{S(i, 2)}{\cos \beta}, \quad (\text{I.75})$$

$$c_W = -\frac{g}{g'} [S(i, 1) \sin \beta + S(i, 2) \cos \beta], \quad (\text{I.76})$$

$$c_{\tilde{W}_1} = \frac{4m_W}{m_{\tilde{W}_1} g \sin \theta_W \cos \theta_W} \left[ \frac{\lambda}{\sqrt{2}} S(i, 3) \cos \theta_L \cos \theta_R + \frac{g}{\sqrt{2}} \{S(i, 1) \sin \theta_L \cos \theta_R + S(i, 2) \cos \theta_L \sin \theta_R\} \right] \\ \times \left[ -\sin^2 \theta_R - \frac{1}{2} \cos^2 \theta_R + 2 \sin^2 \theta_W - \sin^2 \theta_L - \frac{1}{2} \cos^2 \theta_L \right], \quad (\text{I.77})$$

$$c_{\tilde{W}_2} = \frac{4m_W}{m_{\tilde{W}_2} g \sin \theta_W \cos \theta_W} \left[ \frac{\lambda}{\sqrt{2}} S(i, 3) \sin \theta_L \sin \theta_R - \frac{g}{\sqrt{2}} \{S(i, 1) \cos \theta_L \sin \theta_R + S(i, 2) \sin \theta_L \cos \theta_R\} \right] \\ \times \left[ -\cos^2 \theta_R - \frac{1}{2} \sin^2 \theta_R + 2 \sin^2 \theta_W - \cos^2 \theta_L - \frac{1}{2} \sin^2 \theta_L \right], \quad (\text{I.78})$$

$$c_{H^\pm} = (1 - 2 \sin^2 \theta_W) \frac{1}{2 \sin \theta_W \cos \theta_W m_{H^\pm}^2} \frac{1}{\sqrt{\sqrt{2} G_F}} \left\{ \frac{\lambda \mu_{eff}}{\sqrt{2}} [2S(i, 3) \cos^2 \beta + 2S(i, 3) \sin^2 \beta] \right. \\ - \frac{\lambda^2 m_W \sin \beta}{g} 2S(i, 2) \cos \beta \sin \beta - \frac{\lambda^2 m_W \sin \beta}{g} 2S(i, 1) \cos \beta \sin \beta + \mu_{eff} \kappa 2 \sqrt{2} S(i, 3) \cos \beta \sin \beta \\ + \lambda A_\lambda \sqrt{2} S(i, 3) \cos \beta \sin \beta + \frac{g'^2 m_W}{4g} [\sin \beta (2S(i, 1) \cos^2 \beta - 2S(i, 1) \sin^2 \beta) + \cos \beta (2S(i, 2) \sin^2 \beta \\ - 2S(i, 2) \cos^2 \beta) + \frac{g m_W}{4} (\sin \beta [2S(i, 1) \cos^2 \beta + 2S(i, 1) \sin^2 \beta + 4S(i, 2) \sin \beta \cos \beta] \\ \left. + \cos \beta [2S(i, 2) \cos^2 \beta + 2S(i, 2) \sin^2 \beta + 4S(i, 1) \sin \beta \cos \beta])] \right\}. \quad (\text{I.79})$$

Now the kinetic parts depend upon both  $f(\tau_a)$  in eq. (E.2),  $g(\tau_a)$  in eq. (E.3) and  $f(\tau_{aZ})$ ,  $g(\tau_{aZ})$  where  $\tau_{aZ} = 4\left(\frac{m_a}{m_Z}\right)^2$  cf  $\tau_a = 4\left(\frac{m_a}{m_{h_i}}\right)^2$ .

For the spin  $\frac{1}{2}$  particles (quarks or charginos):

$$k_a = \frac{\tau_a \tau_{aZ}}{2(\tau_a - \tau_{aZ})} + \frac{(\tau_a \tau_{aZ})^2}{2(\tau_a - \tau_{aZ})^2} [f(\tau_a) - f(\tau_{aZ})] + \frac{\tau_a^2 \tau_{aZ}}{(\tau_a - \tau_{aZ})^2} [g(\tau_a) - g(\tau_{aZ})] + \frac{\tau_a \tau_{aZ}}{2(\tau_a - \tau_{aZ})} [f(\tau_a) - f(\tau_{aZ})]. \quad (\text{I.80})$$

For scalars (charged Higgs bosons):

$$k_a = \frac{\tau_a \tau_{aZ}}{2(\tau_a - \tau_{aZ})} + \frac{(\tau_a \tau_{aZ})^2}{2(\tau_a - \tau_{aZ})^2} [f(\tau_a) - f(\tau_{aZ})] + \frac{\tau_a^2 \tau_{aZ}}{(\tau_a - \tau_{aZ})^2} [g(\tau_a) - g(\tau_{aZ})]. \quad (\text{I.81})$$

For the spin 1 bosons (W bosons):

$$k_a = -4(3 - \tan^2 \theta_W) \frac{\tau_a \tau_{aZ}}{2(\tau_a - \tau_{aZ})} [f(\tau_a) - f(\tau_{aZ})] + \left\{ \left(1 + \frac{2}{\tau_a}\right) \tan^2 \theta_W - \left(5 + \frac{2}{\tau_a}\right) \right\} \left[ \frac{\tau_a \tau_{aZ}}{2(\tau_a - \tau_{aZ})} \right. \\ \left. + \frac{(\tau_a \tau_{aZ})^2}{2(\tau_a - \tau_{aZ})^2} [f(\tau_a) - f(\tau_{aZ})] + \frac{\tau_a^2 \tau_{aZ}}{(\tau_a - \tau_{aZ})^2} [g(\tau_a) - g(\tau_{aZ})] \right]. \quad (\text{I.82})$$

That's all the information required for  $h_i \rightarrow Z\gamma$ .

Next consider gluon gluon:

$$\Gamma(h_i \rightarrow gg) = \frac{G_F m_{h_i}^3 \alpha_s^2}{64 \sqrt{2} \pi^3} |M_{gg}|^2, \quad (\text{I.83})$$

where

$$|M_{gg}|^2 = [J_t^r + J_b^r + J_c^r + J_{\tilde{c}_L}^r + J_{\tilde{c}_R}^r + J_{\tilde{s}_L}^r + J_{\tilde{s}_R}^r + J_{\tilde{t}_1}^r + J_{\tilde{t}_2}^r + J_{\tilde{b}_1}^r + J_{\tilde{b}_2}^r]^2 \\ + [J_t^i + J_b^i + J_c^i + J_{\tilde{c}_L}^i + J_{\tilde{c}_R}^i + J_{\tilde{s}_L}^i + J_{\tilde{s}_R}^i + J_{\tilde{t}_1}^i + J_{\tilde{t}_2}^i + J_{\tilde{b}_1}^i + J_{\tilde{b}_2}^i]^2. \quad (\text{I.84})$$

As usual each of the  $J_a$  are given by  $J_a = c_a k_a$ , where the coupling parts  $c_a$  and kinetic parts  $k_a$  are given below:

$$c_t = \frac{S(i, 1)}{\sin \beta} \quad c_c = \frac{S(i, 1)}{\sin \beta} \quad c_b = \frac{S(i, 2)}{\cos \beta}, \quad (\text{I.85})$$

$$c_{\tilde{c}_L} = \frac{2m_W}{gm_{\tilde{c}_L}^2} \left( \frac{g'^2}{12} + \frac{g^2}{4} \right) \frac{2m_W}{g} [\sin \beta S(i, 1) - \cos \beta S(i, 2)], \quad (\text{I.86})$$

$$c_{\tilde{c}_R} = \frac{2m_W}{gm_{\tilde{c}_R}^2} \frac{g'^2}{6} \frac{2m_W}{g} [\sin \beta S(i, 1) - \cos \beta S(i, 2)], \quad (\text{I.87})$$

$$c_{\tilde{s}_L} = \frac{2m_W}{gm_{\tilde{s}_L}^2} \left( \frac{g'^2}{12} + \frac{g^2}{4} \right) \frac{2m_W}{g} [\sin \beta S(i, 1) - \cos \beta S(i, 2)], \quad (\text{I.88})$$

$$c_{\tilde{s}_R} = \frac{2m_W}{gm_{\tilde{s}_R}^2} \frac{g'^2}{6} \frac{2m_W}{g} [\sin \beta S(i, 1) - \cos \beta S(i, 2)], \quad (\text{I.89})$$

$$(\text{I.90})$$

$$c_{\tilde{t}_1} = \frac{m_W}{gm_{\tilde{t}_1}^2} \left[ \cos^2 \theta_t \sqrt{2} \left( f_t^2 \sqrt{2} \frac{m_W \sin \beta}{g} S(i, 1) + \left( \frac{g'^2}{12} - \frac{g^2}{4} \right) \left\{ \sqrt{2} \frac{m_W}{g} (S(i, 1) \sin \beta - \cos \beta S(i, 2)) \right\} \right) \right. \\ \left. + \sin^2 \theta_t \sqrt{2} \left( f_t^2 \sqrt{2} \frac{m_W \sin \beta}{g} S(i, 1) - \frac{g'^2}{3} \left\{ \sqrt{2} \frac{m_W}{g} (\sin \beta S(i, 1) - \cos \beta S(i, 2)) \right\} \right) \right. \\ \left. + 2 \sin \theta_t \cos \theta_t \frac{f_t}{\sqrt{2}} \left( A_t S(i, 1) - \mu_{eff} S(i, 2) - \lambda \sqrt{2} \frac{m_W \cos \beta}{g} S(i, 3) \right) \right], \quad (\text{I.91})$$

$$c_{\tilde{t}_2} = \frac{m_W}{gm_{\tilde{t}_2}^2} \left[ \sin^2 \theta_t \sqrt{2} \left( f_t^2 \sqrt{2} \frac{m_W \sin \beta}{g} S(i, 1) + \left( \frac{g'^2}{12} - \frac{g^2}{4} \right) \left\{ \sqrt{2} \frac{m_W}{g} (S(i, 1) \sin \beta - \cos \beta S(i, 2)) \right\} \right) \right. \\ \left. + \cos^2 \theta_t \sqrt{2} \left( f_t^2 \sqrt{2} \frac{m_W \sin \beta}{g} S(i, 1) - \frac{g'^2}{3} \left\{ \sqrt{2} \frac{m_W}{g} (\sin \beta S(i, 1) - \cos \beta S(i, 2)) \right\} \right) \right. \\ \left. - 2 \sin \theta_t \cos \theta_t \frac{f_t}{\sqrt{2}} \left( A_t S(i, 1) - \mu_{eff} S(i, 2) - \lambda \sqrt{2} \frac{m_W \cos \beta}{g} S(i, 3) \right) \right], \quad (\text{I.92})$$

$$c_{\tilde{b}_1} = \frac{m_W}{gm_{\tilde{b}_1}^2} \left[ \cos^2 \theta_b \sqrt{2} \left( f_b^2 \sqrt{2} \frac{m_W \cos \beta}{g} S(i, 2) + \left( \frac{g'^2}{12} + \frac{g^2}{4} \right) \left\{ \sqrt{2} \frac{m_W}{g} (S(i, 1) \sin \beta - \cos \beta S(i, 2)) \right\} \right) \right. \\ \left. + \sin^2 \theta_b \sqrt{2} \left( f_b^2 \sqrt{2} \frac{m_W \cos \beta}{g} S(i, 2) + \frac{g'^2}{6} \left\{ \sqrt{2} \frac{m_W}{g} (\sin \beta S(i, 1) - \cos \beta S(i, 2)) \right\} \right) \right. \\ \left. + 2 \sin \theta_b \cos \theta_b \frac{f_b}{\sqrt{2}} \left( A_b S(i, 2) - \mu_{eff} S(i, 1) - \lambda \sqrt{2} \frac{m_W \sin \beta}{g} S(i, 3) \right) \right], \quad (\text{I.93})$$

$$\begin{aligned}
c_{\tilde{b}_2} = & \frac{m_W}{gm_{\tilde{b}_2}^2} \left[ \sin^2 \theta_B \sqrt{2} \left( f_b^2 \sqrt{2} \frac{m_W \cos \beta}{g} S(i, 2) + \left( \frac{g'^2}{12} + \frac{g^2}{4} \right) \left\{ \sqrt{2} \frac{m_W}{g} (S(i, 1) \sin \beta - \cos \beta S(i, 2)) \right\} \right) \right. \\
& + \cos^2 \theta_b \sqrt{2} \left( f_b^2 \sqrt{2} \frac{m_W \cos \beta}{g} S(i, 2) + \frac{g'^2}{6} \left\{ \sqrt{2} \frac{m_W}{g} (\sin \beta S(i, 1) - \cos \beta S(i, 2)) \right\} \right) \\
& \left. - 2 \sin \theta_b \cos \theta_b \frac{f_b}{\sqrt{2}} \left( A_b S(i, 2) - \mu_{eff} S(i, 1) - \lambda \sqrt{2} \frac{m_W \sin \beta}{g} S(i, 3) \right) \right].
\end{aligned} \tag{I.94}$$

The kinetic parts are then exactly as in the  $h_i \rightarrow \gamma\gamma$  case.

### Appendix I.2. CP Odd Higgs Decays

First of all decays to a quark and an antiquark:

$$\Gamma(A_i \rightarrow q\bar{q}) = \frac{3G_F}{4\pi\sqrt{2}} m_q^2 m_{A_i} \sqrt{1 - \frac{4m_q^2}{m_{A_i}^2}} \mathcal{A}, \tag{I.95}$$

where

$$\mathcal{A} = \begin{cases} \left[ \frac{S(i,1)}{\sin \beta} \right]^2, & \text{for up-type quarks (u,c,t),} \\ \left[ \frac{S(i,2)}{\cos \beta} \right]^2, & \text{for down-type quarks (d,s,b).} \end{cases} \tag{I.96}$$

Use the same formulae for decays to leptons but divide by 3 as the 3 in the prefactor is  $N_c$ . Now decays to sfermions, because of CP conservation, decays can only go to sfermions of different handedness.

$$\Gamma(A_i \rightarrow \tilde{q}_L \tilde{q}_R^*) = \frac{1}{16\pi m_{A_i}} \tilde{\lambda}^{\frac{1}{2}}(m_{A_i}, m_{\tilde{q}_L}, m_{\tilde{q}_R}) C_{A_i \tilde{q} \tilde{q}}^2, \tag{I.97}$$

where

$$C_{A_i \tilde{q} \tilde{q}} = \begin{cases} \frac{f_q}{\sqrt{2}} [A_{\tilde{q}} P(i, 1) + \mu_{eff} P(i, 2) + \lambda \sqrt{2} \frac{m_W \cos \beta}{g} P(i, 3)], & \text{for } u\text{-type squarks,} \\ \frac{f_q}{\sqrt{2}} [A_{\tilde{q}} P(i, 2) + \mu_{eff} P(i, 1) + \lambda \sqrt{2} \frac{m_W \sin \beta}{g} P(i, 3)], & \text{for } d\text{-type squarks.} \end{cases} \tag{I.98}$$

Where you must remember that the expression for  $f_q$  differs for up type and down type quarks, for example see (F.8).

Note, (I.97) holds even for third generation squarks; as we see in the MSSM, in the NMSSM the decays of CP odd Higgs bosons to squarks are independent of the sfermion mixing angles. The formulae for the decays to squarks also hold for decays to sleptons, but again one must divide by 3.

Decays to neutralinos:

$$\Gamma(A_i \rightarrow \tilde{Z}_j \tilde{Z}_k) = \frac{m_{A_i}}{16\pi} \left[ 1 - \left( \frac{m_{\tilde{Z}_j} - m_{\tilde{Z}_k}}{m_{A_i}} \right)^2 \right] \tilde{\lambda}^{\frac{1}{2}}(m_{A_i}, m_{\tilde{Z}_j}, m_{\tilde{Z}_k}) \alpha_{ij} C_{A_i \tilde{Z}_j \tilde{Z}_k}^2, \tag{I.99}$$

where, as for the CP even decays to neutralinos, the  $\alpha_{jk}$  factor accounts for indistinguishability and so is 1 if  $j = k$  (i.e. decay to two of the same neutralinos) and 2 if  $j \neq k$  (i.e. decays to two different neutralino mass eigenstates). The coupling is given by:

$$\begin{aligned}
C_{A_i \tilde{Z}_j \tilde{Z}_k} = & \frac{\lambda}{\sqrt{2}} \left[ P(i, 1)(N_{3j} N_{5k} + N_{5j} N_{3k}) + P(i, 2)(N_{4j} N_{5k} + N_{5j} N_{4k}) \right. \\
& + P(i, 3)(N_{3j} N_{4k} + N_{4j} N_{3k}) \left. \right] - \sqrt{2} \kappa P(i, 3) N_{5j} N_{5k} \\
& - \tan \theta_W \frac{g}{2} \left[ -P(i, 1)(N_{1j} N_{4k} + N_{4j} N_{1k}) + P(i, 2)(N_{1j} N_{3k} \right. \\
& + N_{3j} N_{1k}) \left. \right] - \frac{g}{2} \left[ P(i, 1)(N_{2j} N_{4k} + N_{4j} N_{2k}) - P(i, 2)(N_{2j} N_{3k} \right. \\
& + N_{3j} N_{2k}) \left. \right].
\end{aligned} \tag{I.100}$$

Decays to charginos, first consider decays to the two of the same chargino:

$$\Gamma(A_i \rightarrow \tilde{W}_j \tilde{W}_j) = \frac{m_{A_i}}{8\pi} \tilde{\lambda}^{\frac{1}{2}}(m_{A_i}, m_{\tilde{W}_j}, m_{\tilde{W}_j}) S^2, \tag{I.101}$$

here

$$S = \begin{cases} \frac{\lambda}{\sqrt{2}} P(i, 3) \cos \theta_L \cos \theta_R - \frac{g}{\sqrt{2}} [P(i, 1) \sin \theta_L \cos \theta_R + P(i, 2) \cos \theta_L \sin \theta_R], & \text{for } j = 1, \\ \frac{\lambda}{\sqrt{2}} P(i, 3) \sin \theta_L \sin \theta_R + \frac{g}{\sqrt{2}} [P(i, 1) \cos \theta_L \sin \theta_R + P(i, 2) \sin \theta_L \cos \theta_R], & \text{for } j = 2. \end{cases} \quad (\text{I.102})$$

Meanwhile for decays to different charginos:

$$\Gamma(A_i \rightarrow \tilde{W}_1 \tilde{W}_2) = \frac{m_{A_i}}{8\pi} \tilde{\lambda}^{\frac{1}{2}}(m_{A_i}, m_{\tilde{W}_1}, m_{\tilde{W}_2}) \left[ (c_1^2 + c_2^2) \frac{1}{m_{A_i}^2} (m_{A_i}^2 - m_{\tilde{W}_1}^2 - m_{\tilde{W}_2}^2) + 4c_1 c_2 \frac{m_{\tilde{W}_1} m_{\tilde{W}_2}}{m_{A_i}^2} \right], \quad (\text{I.103})$$

Here,

$$c_1 = \frac{\lambda}{\sqrt{2}} P(i, 3) \cos \theta_L \sin \theta_R - \frac{g}{\sqrt{2}} [P(i, 1) \sin \theta_L \sin \theta_R - P(i, 2) \cos \theta_L \cos \theta_R], \quad (\text{I.104})$$

$$c_2 = \frac{\lambda}{\sqrt{2}} P(i, 3) \sin \theta_L \cos \theta_R + \frac{g}{\sqrt{2}} [P(i, 1) \cos \theta_L \cos \theta_R - P(i, 2) \sin \theta_L \sin \theta_R]. \quad (\text{I.105})$$

Decays to CP even neutral Higgs bosons and a Z boson:

$$\Gamma(A_i \rightarrow h_j Z) = \frac{G_F m_{A_i}^3}{8\pi \sqrt{2}} \tilde{\lambda}^{\frac{3}{2}}(m_{A_i}, m_{h_j}, m_Z) C_{A_i h_j Z}^2, \quad (\text{I.106})$$

where

$$C_{A_i h_j Z} = \begin{cases} [S(1, 1) \cos \beta - S(1, 2) \sin \beta] \cos \theta_A, & \text{for } i = j = 1, \\ [S(1, 1) \cos \beta - S(1, 2) \sin \beta] \sin \theta_A, & \text{for } i = 2, j = 1, \\ [S(2, 1) \cos \beta - S(2, 2) \sin \beta] \cos \theta_A, & \text{for } i = 1, j = 2, \\ [S(2, 1) \cos \beta - S(2, 2) \sin \beta] \sin \theta_A, & \text{for } i = j = 2, \\ [S(3, 1) \cos \beta - S(3, 2) \sin \beta] \cos \theta_A, & \text{for } i = 1, j = 3, \\ [S(3, 1) \cos \beta - S(3, 2) \sin \beta] \sin \theta_A, & \text{for } i = 2, j = 3. \end{cases} \quad (\text{I.107})$$

The decay of a CP odd neutral Higgs to a charged Higgs and a W boson in the NMSSM is given by:

$$\Gamma(A \rightarrow H^\pm W^\pm) = \frac{G_F m_A^3}{8\pi \sqrt{2}} \tilde{\lambda}^{\frac{3}{2}}(m_A, m_{H^\pm}, m_W) \cos^2 \theta_A, \quad (\text{I.108})$$

for  $A_2$  undergoing the same decay transform  $\cos \theta_A \rightarrow \sin \theta_A$  and  $m_A \rightarrow m_{A_2}$ .

$$\Gamma(A_2 \rightarrow A h_i) = \frac{1}{16\pi m_{h_i}} \tilde{\lambda}^{\frac{1}{2}}(m_{A_2}, m_A, m_{h_i}) [C_{AA_2 h_i}^{NMSSM}]^2, \quad (\text{I.109})$$

where the coupling  $C_{AA_2 h_i}^{NMSSM}$  is:

$$\begin{aligned} C_{AA_2 h_i}^{NMSSM} = & \frac{g^2 + g'^2}{4\sqrt{2}} \left[ \langle h_1 \rangle [C(i, 1, 2, 1, 1, 1) - C(i, 1, 2, 1, 2, 2)] + \langle h_2 \rangle [C(i, 1, 2, 2, 2, 2) - C(i, 1, 2, 2, 1, 1)] \right] \\ & + \frac{\lambda A_1}{\sqrt{2}} [C(i, 1, 2, 1, 2, 3) + C(i, 1, 2, 2, 1, 3) + C(i, 1, 2, 3, 1, 2)] - \frac{\kappa A_\kappa}{\sqrt{2}} C(i, 1, 2, 3, 3, 3) \\ & + \frac{\lambda^2}{\sqrt{2}} \left[ \langle h_1 \rangle [C(i, 1, 2, 1, 2, 2) + C(i, 1, 2, 1, 3, 3)] + \langle h_2 \rangle [C(i, 1, 2, 2, 1, 1) + C(i, 1, 2, 2, 3, 3)] \right] \\ & + \mu_{eff} \lambda [C(i, 1, 2, 3, 1, 1) + C(i, 1, 2, 3, 2, 2)] + \frac{\kappa^2 \sqrt{2} \mu_{eff}}{\lambda} C(i, 1, 2, 3, 3, 3) \\ & + \frac{\lambda \kappa}{\sqrt{2}} \left[ \langle h_1 \rangle [C(i, 1, 2, 2, 3, 3) - 2C(i, 1, 2, 3, 2, 3)] + \langle h_2 \rangle [C(i, 1, 2, 1, 3, 3) - 2C(i, 1, 2, 3, 1, 3)] \right] \\ & + 2 \frac{\mu_{eff}}{\lambda} [C(i, 1, 2, 3, 1, 2) - C(i, 1, 2, 1, 2, 3) - C(i, 1, 2, 2, 1, 3)], \end{aligned} \quad (\text{I.110})$$

where here  $C(i, 1, 2, x, y, z)$  is notation for

$$C(i, 1, 2, x, y, z) = S(i, x)[P(1, y)P(2, z) + P(1, z)P(2, y)]. \quad (\text{I.111})$$

Now the loop decays of the CP odd Higgs bosons. First consider decays to  $\gamma\gamma$ :

$$\Gamma(A_i \rightarrow \gamma\gamma) = \frac{G_F \alpha_{em}^2 m_{A_i}^3}{32\pi^3 \sqrt{2}} |M_{A_i\gamma\gamma}|^2, \quad (\text{I.112})$$

where

$$|M_{A_i\gamma\gamma}|^2 = (\mathcal{J}_t^r + \mathcal{J}_b^r + \mathcal{J}_c^r + \mathcal{J}_\tau^r + \mathcal{J}_{\tilde{W}_1}^r + \mathcal{J}_{\tilde{W}_2}^r)^2 + (\mathcal{J}_t^i + \mathcal{J}_b^i + \mathcal{J}_c^i + \mathcal{J}_\tau^i + \mathcal{J}_{\tilde{W}_1}^i + \mathcal{J}_{\tilde{W}_2}^i)^2. \quad (\text{I.113})$$

The  $\mathcal{J}_a^{r/i}$  here are the real and imaginary parts respectively of  $c_a k_a$  where the  $c_a$  and  $k_a$  for this decay mode are given below:

$$c_t = \frac{4}{3} \frac{P(i, 1)}{\sin\beta}, \quad c_b = \frac{1}{3} \frac{P(i, 2)}{\cos\beta}, \quad c_c = \frac{4}{3} \frac{P(i, 1)}{\sin\beta}, \quad c_\tau = \frac{P(i, 2)}{\cos\beta}, \quad (\text{I.114})$$

$$c_{\tilde{W}_1} = \frac{2m_W}{gm_{\tilde{W}_1}} \left[ \frac{\lambda}{\sqrt{2}} P(i, 3) \cos\theta_L \cos\theta_R - \frac{g}{\sqrt{2}} (P(i, 1) \sin\theta_L \cos\theta_R + P(i, 2) \cos\theta_L \sin\theta_R) \right], \quad (\text{I.115})$$

$$c_{\tilde{W}_2} = \frac{2m_W}{gm_{\tilde{W}_2}} \left[ \frac{\lambda}{\sqrt{2}} P(i, 3) \sin\theta_L \sin\theta_R + \frac{g}{\sqrt{2}} (P(i, 1) \cos\theta_L \sin\theta_R + P(i, 2) \sin\theta_L \cos\theta_R) \right]. \quad (\text{I.116})$$

Meanwhile the kinetic parts are, for the quarks or the charginos (as both are spin  $\frac{1}{2}$ ):

$$k_a = \tau_a f(\tau_a). \quad (\text{I.117})$$

The next loop decay is to  $Z\gamma$ :

$$\Gamma(A_i \rightarrow Z\gamma) = \frac{g^2 m_{A_i}^3 \alpha_{em}^2}{512\pi^3 m_W^2} \left( 1 - \left( \frac{m_Z}{m_{A_i}} \right)^2 \right) |M_{A_i Z\gamma}|^2, \quad (\text{I.118})$$

where

$$|M_{A_i Z\gamma}|^2 = (\mathcal{K}_t^r + \mathcal{K}_b^r + \mathcal{K}_c^r + \mathcal{K}_{\tilde{W}_1}^r + \mathcal{K}_{\tilde{W}_2}^r)^2 + (\mathcal{K}_t^i + \mathcal{K}_b^i + \mathcal{K}_c^i + \mathcal{K}_{\tilde{W}_1}^i + \mathcal{K}_{\tilde{W}_2}^i)^2. \quad (\text{I.119})$$

As usual each  $\mathcal{K}_a^{r/i}$  is the real/imaginary part of  $c_a k_a$  where the  $c_a$  and  $k_a$  for this mode are given below:

$$c_t = -2(1 - \frac{8}{3} \sin^2 \theta_W) \frac{1}{\sin\theta_W \cos\theta_W} \frac{P(i, 1)}{\sin\beta}, \quad c_c = -2(1 - \frac{8}{3} \sin^2 \theta_W) \frac{1}{\sin\theta_W \cos\theta_W} \frac{P(i, 1)}{\sin\beta}, \quad (\text{I.120})$$

$$c_b = (-1 + \frac{4}{3} \sin^2 \theta_W) \frac{1}{\sin\theta_W \cos\theta_W} \frac{P(i, 2)}{\cos\beta}, \quad (\text{I.121})$$

$$c_{\tilde{W}_1} = 4 \frac{m_W}{m_{\tilde{W}_1} g \sin\theta_W \cos\theta_W} \left[ -\sin^2 \theta_R - \frac{1}{2} \cos^2 \theta_R - \sin^2 \theta_L - \frac{1}{2} \cos^2 \theta_L + 2 \sin^2 \theta_W \right] \\ \times \left( \frac{\lambda}{\sqrt{2}} P(i, 3) \cos\theta_L \cos\theta_R - \frac{g}{\sqrt{2}} [P(i, 1) \sin\theta_L \cos\theta_R + P(i, 2) \cos\theta_L \sin\theta_R] \right), \quad (\text{I.122})$$

$$c_{\tilde{W}_2} = 4 \frac{m_W}{m_{\tilde{W}_2} g \sin\theta_W \cos\theta_W} \left[ -\cos^2 \theta_R - \frac{1}{2} \sin^2 \theta_R - \cos^2 \theta_L - \frac{1}{2} \sin^2 \theta_L + 2 \sin^2 \theta_W \right] \\ \times \left( \frac{\lambda}{\sqrt{2}} P(i, 3) \sin\theta_L \sin\theta_R + \frac{g}{\sqrt{2}} [P(i, 1) \cos\theta_L \sin\theta_R + P(i, 2) \sin\theta_L \cos\theta_R] \right), \quad (\text{I.123})$$

The kinetic parts are all of the following form, where remember  $\tau_{aZ} = 4(\frac{m_a}{m_Z})^2$  cf  $\tau_a = 4(\frac{m_a}{m_\phi})^2$ :

$$k_a = \frac{\tau_a \tau_{aZ}}{2(\tau_a - \tau_{aZ})} [f(\tau_a) - f(\tau_{aZ})]. \quad (\text{I.124})$$



Finally, for the loop decay to  $gg$ :

$$\Gamma(A_i \rightarrow gg) = \frac{g^2 \alpha_s^2 m_{A_i}^3}{128 \pi^3 m_W^2} |M_{A_i gg}|^2, \quad (\text{I.125})$$

where

$$|M_{A_i gg}|^2 = (\mathcal{R}_t^r + \mathcal{R}_b^r + \mathcal{R}_c^r)^2 + (\mathcal{R}_t^i + \mathcal{R}_b^i + \mathcal{R}_c^i)^2. \quad (\text{I.126})$$

Again the  $\mathcal{R}_a^{r/i}$  are the real and imaginary parts of  $c_a k_a$ , where for this mode they are:

$$c_t = \frac{P(i, 1)}{\sin \beta}, \quad c_b = \frac{P(i, 2)}{\cos \beta}, \quad c_c = \frac{P(i, 1)}{\sin \beta}. \quad (\text{I.127})$$

The kinetic parts are just:

$$k_a = \tau_a f(\tau_a). \quad (\text{I.128})$$

### Appendix I.3. Decays into Higgs Bosons

$$\Gamma(\tilde{b}_2 \rightarrow \tilde{b}_1 h_i) = \frac{1}{16 \pi m_{\tilde{b}_2}} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{b}_2}, m_{\tilde{b}_1}, m_{h_i}) \left[ \cos \theta_b \sin \theta_b (c_R - c_L) + (\cos^2 \theta_b - \sin^2 \theta_b) c_{LR} \right]^2, \quad (\text{I.129})$$

where

$$c_L = -\sqrt{2} \left[ f_b^2 \langle h_2 \rangle S(i, 2) + \left( \frac{g'^2}{12} + \frac{g^2}{4} \right) [\langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2)] \right], \quad (\text{I.130})$$

$$c_R = -\sqrt{2} \left[ f_b^2 \langle h_2 \rangle S(i, 2) + \left( \frac{g'^2}{6} \right) [\langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2)] \right], \quad (\text{I.131})$$

$$c_{LR} = -\frac{f_b}{\sqrt{2}} \left[ A_b S(i, 2) - \mu_{eff} S(i, 1) - \lambda \langle h_1 \rangle S(i, 3) \right]. \quad (\text{I.132})$$

$$\Gamma(\tilde{t}_2 \rightarrow \tilde{t}_1 h_i) = \frac{1}{16 \pi m_{\tilde{t}_2}} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{t}_2}, m_{\tilde{t}_1}, m_{h_i}) \left[ \cos \theta_t \sin \theta_t (c_R - c_L) + (\cos^2 \theta_t - \sin^2 \theta_t) c_{LR} \right]^2, \quad (\text{I.133})$$

where

$$c_L = -\sqrt{2} \left[ f_t^2 \langle h_1 \rangle S(i, 1) + \left( \frac{g'^2}{12} - \frac{g^2}{4} \right) [\langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2)] \right], \quad (\text{I.134})$$

$$c_R = -\sqrt{2} \left[ f_t^2 \langle h_1 \rangle S(i, 1) - \left( \frac{g'^2}{3} \right) [\langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2)] \right], \quad (\text{I.135})$$

$$c_{LR} = -\frac{f_t}{\sqrt{2}} \left[ A_t S(i, 1) - \mu_{eff} S(i, 2) - \lambda \langle h_2 \rangle S(i, 3) \right]. \quad (\text{I.136})$$

$$\Gamma(\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 h_i) = \frac{1}{16 \pi m_{\tilde{\tau}_2}} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{\tau}_2}, m_{\tilde{\tau}_1}, m_{h_i}) \left[ \cos \theta_\tau \sin \theta_\tau (c_R - c_L) + (\cos^2 \theta_\tau - \sin^2 \theta_\tau) c_{LR} \right]^2, \quad (\text{I.137})$$

where

$$c_L = -\sqrt{2} \left[ f_\tau^2 \langle h_2 \rangle S(i, 2) + \left( \frac{-g'^2}{4} + \frac{g^2}{4} \right) [\langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2)] \right], \quad (\text{I.138})$$

$$c_R = -\sqrt{2} \left[ f_\tau^2 \langle h_2 \rangle S(i, 2) + \left( \frac{g'^2}{2} \right) [\langle h_1 \rangle S(i, 1) - \langle h_2 \rangle S(i, 2)] \right], \quad (\text{I.139})$$

$$c_{LR} = -\frac{f_\tau}{\sqrt{2}} \left[ A_\tau S(i, 2) - \mu_{eff} S(i, 1) - \lambda \langle h_1 \rangle S(i, 3) \right]. \quad (\text{I.140})$$

$$\Gamma(\tilde{b}_2 \rightarrow \tilde{b}_1 A_i) = \frac{1}{16 \pi m_{\tilde{b}_2}} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{b}_2}, m_{\tilde{b}_1}, m_{A_i}) [\cos^2 \theta_b - \sin^2 \theta_b]^2 A_{LR}^2, \quad (\text{I.141})$$

where

$$A_{LR} = \frac{f_b}{\sqrt{2}} \left[ A_b P(i, 2) + \mu_{eff} P(i, 1) + \lambda \langle h_1 \rangle P(i, 3) \right]. \quad (\text{I.142})$$

For  $\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 A_i$  the formulae are the same except the changes  $\theta_b \rightarrow \theta_\tau$ ,  $m_b \rightarrow m_\tau$  and now  $A_{LR}$  is given by:

$$A_{LR} = \frac{f_\tau}{\sqrt{2}} \left[ A_\tau P(i, 2) + \mu_{eff} P(i, 1) + \lambda \langle h_1 \rangle P(i, 3) \right]. \quad (\text{I.143})$$

For  $\tilde{t}_2 \rightarrow \tilde{t}_1 A_i$  the formulae are the same except the changes  $\theta_b \rightarrow \theta_t$ ,  $m_b \rightarrow m_t$  and now  $A_{LR}$  is given by:

$$A_{LR} = \frac{f_t}{\sqrt{2}} \left[ A_t P(i, 1) + \mu_{eff} P(i, 2) + \lambda \langle h_2 \rangle P(i, 3) \right]. \quad (\text{I.144})$$

The chargino decays to lighter charginos and a CP even neutral Higgs:

$$\Gamma(\tilde{W}_2 \rightarrow \tilde{W}_1 h_i) = \frac{1}{32\pi |m_{\tilde{W}_2}|} \tilde{\lambda}^{1/2}(m_{\tilde{W}_2}, m_{\tilde{W}_1}, m_{h_i}) \left[ (c_1^2 + c_2^2)(m_{\tilde{W}_1}^2 + m_{\tilde{W}_2}^2 - m_{h_i}^2) + 4c_1 c_2 m_{\tilde{W}_1} m_{\tilde{W}_2} \right]^2, \quad (\text{I.145})$$

here the  $c_1$  and  $c_2$  are:

$$c_1 = \frac{\lambda}{\sqrt{2}} S(i, 3) \cos \theta_L \sin \theta_R + \frac{g}{\sqrt{2}} [S(i, 1) \sin \theta_L \sin \theta_R - S(i, 2) \cos \theta_L \cos \theta_R], \quad (\text{I.146})$$

$$c_2 = \frac{\lambda}{\sqrt{2}} S(i, 3) \sin \theta_L \cos \theta_R - \frac{g}{\sqrt{2}} [S(i, 1) \cos \theta_L \cos \theta_R - S(i, 2) \sin \theta_L \sin \theta_R]. \quad (\text{I.147})$$

The chargino decays to lighter charginos and a CP odd neutral Higgs:

$$\Gamma(\tilde{W}_2 \rightarrow \tilde{W}_1 A_i) = \frac{1}{32\pi |m_{\tilde{W}_2}|} \tilde{\lambda}^{1/2}(m_{\tilde{W}_2}, m_{\tilde{W}_1}, m_{A_i}) \left[ (C_1^2 + C_2^2)(m_{\tilde{W}_1}^2 + m_{\tilde{W}_2}^2 - m_{A_i}^2) + 4C_1 C_2 m_{\tilde{W}_1} m_{\tilde{W}_2} \right]^2, \quad (\text{I.148})$$

here the  $C_1$  and  $C_2$  are:

$$C_1 = \frac{\lambda}{\sqrt{2}} P(i, 3) \cos \theta_L \sin \theta_R - \frac{g}{\sqrt{2}} [P(i, 1) \sin \theta_L \sin \theta_R - P(i, 2) \cos \theta_L \cos \theta_R], \quad (\text{I.149})$$

$$C_2 = \frac{\lambda}{\sqrt{2}} P(i, 3) \sin \theta_L \cos \theta_R + \frac{g}{\sqrt{2}} [P(i, 1) \cos \theta_L \cos \theta_R - P(i, 2) \sin \theta_L \sin \theta_R]. \quad (\text{I.150})$$

The formulae for  $\Gamma(H^\pm \rightarrow Wh_i)$  are just as above for  $\Gamma(h_i \rightarrow WH^\pm)$  in (I.31) but with  $m_{h_i} \leftrightarrow m_{H^\pm}$ . Similarly the formulae for  $\Gamma(H^\pm \rightarrow WA_i)$  are just as above for  $\Gamma(A_i \rightarrow WH^\pm)$  in (I.108) but with the replacement  $m_{A_i} \leftrightarrow m_{H^\pm}$ .

#### Appendix I.4. Neutralino Decays

$$\Gamma(\tilde{Z}_i \rightarrow \tilde{q}_{L/R} \bar{q}) = \frac{N_c g^2}{32\pi |m_{\tilde{Z}_i}|} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{Z}_i}, m_{\tilde{q}_{L/R}}, m_q) \mathcal{C}_{\mathcal{L}/\mathcal{R}}^2 (m_{\tilde{Z}_i}^2 - m_{\tilde{q}_{L/R}}^2 + m_q^2), \quad (\text{I.151})$$

where

$$C_L = \begin{cases} -\sqrt{2} \left[ \frac{2}{3} c(1) \sin \theta_W + \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \frac{c(2)}{\cos \theta_W} \right], & \text{for } \tilde{u}_L, \\ \sqrt{2} \left[ \frac{1}{3} c(1) \sin \theta_W + \left( \frac{1}{2} - \frac{1}{3} \sin^2 \theta_W \right) \frac{c(2)}{\cos \theta_W} \right], & \text{for } \tilde{d}_L, \\ \sqrt{2} \left[ -c(1) \sin \theta_W + \left( \frac{1}{2} - \sin^2 \theta_W \right) \frac{c(2)}{\cos \theta_W} \right], & \text{for } \tilde{l}_L, \\ \frac{-c(2)}{\sqrt{2}} \cos \theta_W, & \text{for } \tilde{\nu}_L, \end{cases} \quad (\text{I.152})$$

$$C_R = \begin{cases} -\sqrt{2} \frac{2}{3} \sin \theta_W [c(2) \tan \theta_W - c(1)], & \text{for } \tilde{u}_R, \\ \sqrt{2} \frac{1}{3} \sin \theta_W [c(2) \tan \theta_W - c(1)], & \text{for } \tilde{d}_R, \\ \sqrt{2} \sin \theta_W [c(2) \tan \theta_W - c(1)], & \text{for } \tilde{l}_R, \\ 0, & \text{for } \tilde{\nu}_R. \end{cases} \quad (\text{I.153})$$

Here  $N_c = 3$  for quarks and 1 for leptons and  $c(1)$  and  $c(2)$  are given by:

$$c(1) = N_{1i} \cos \theta_W + N_{2i} \sin \theta_W, \quad c(2) = -N_{1i} \sin \theta_W + N_{2i} \cos \theta_W. \quad (\text{I.154})$$

For the third generation, the generalisation is as expected but with extra Yukawa interactions:

$$\Gamma(\tilde{Z}_i \rightarrow \tilde{t}_{1/2} \bar{t}) = \frac{3g^2}{32\pi|m_{\tilde{Z}_i}|} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{Z}_i}, m_{\tilde{t}_{1/2}}, m_t) \left[ (c_1^2 + c_2^2)(m_{\tilde{Z}_i}^2 - m_{\tilde{t}_{1/2}}^2 + m_t^2) + 4m_t m_{\tilde{Z}_i} c_1 c_2 \right], \quad (\text{I.155})$$

where:

$$c_1 = \begin{cases} \sqrt{2} \cos \theta_t [-\frac{2}{3} c(1) \sin \theta_W + (-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W) \frac{c(2)}{\cos \theta_W}] - \sin \theta_t \frac{f_t}{g} N_{4i}, & \text{for } \tilde{t}_1, \\ -\sqrt{2} \sin \theta_t [-\frac{2}{3} c(1) \sin \theta_W + (-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W) \frac{c(2)}{\cos \theta_W}] - \cos \theta_t \frac{f_t}{g} N_{4i}, & \text{for } \tilde{t}_2, \end{cases} \quad (\text{I.156})$$

$$c_2 = \begin{cases} -\sqrt{2} \sin \theta_t \frac{2}{3} \sin \theta_W [c(2) \tan \theta_W - c(1)] - \cos \theta_t \frac{f_t}{g} N_{4i}, & \text{for } \tilde{t}_1, \\ -\sqrt{2} \cos \theta_t \frac{2}{3} \sin \theta_W [c(2) \tan \theta_W - c(1)] + \sin \theta_t \frac{f_t}{g} N_{4i}, & \text{for } \tilde{t}_2, \end{cases} \quad (\text{I.157})$$

$$\Gamma(\tilde{Z}_i \rightarrow \tilde{b}_{1/2} \bar{b}) = \frac{3g^2}{32\pi|m_{\tilde{Z}_i}|} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{Z}_i}, m_{\tilde{b}_{1/2}}, m_b) \left[ (c_1^2 + c_2^2)(m_{\tilde{Z}_i}^2 - m_{\tilde{b}_{1/2}}^2 + m_b^2) + 4m_b m_{\tilde{Z}_i} c_1 c_2 \right], \quad (\text{I.158})$$

where:

$$c_1 = \begin{cases} \sqrt{2} \cos \theta_b [\frac{1}{3} c(1) \sin \theta_W + (\frac{1}{2} - \frac{1}{3} \sin^2 \theta_W) \frac{c(2)}{\cos \theta_W}] - \sin \theta_b \frac{f_b}{g} N_{3i}, & \text{for } \tilde{b}_1, \\ -\sqrt{2} \sin \theta_b [\frac{1}{3} c(1) \sin \theta_W + (\frac{1}{2} - \frac{1}{3} \sin^2 \theta_W) \frac{c(2)}{\cos \theta_W}] - \cos \theta_b \frac{f_b}{g} N_{3i}, & \text{for } \tilde{b}_2, \end{cases} \quad (\text{I.159})$$

$$c_2 = \begin{cases} \sqrt{2} \sin \theta_b \frac{1}{3} \sin \theta_W [c(2) \tan \theta_W - c(1)] - \cos \theta_b \frac{f_b}{g} N_{3i}, & \text{for } \tilde{b}_1, \\ \sqrt{2} \cos \theta_b \frac{1}{3} \sin \theta_W [c(2) \tan \theta_W - c(1)] + \sin \theta_b \frac{f_b}{g} N_{3i}, & \text{for } \tilde{b}_2. \end{cases} \quad (\text{I.160})$$

$$\Gamma(\tilde{Z}_i \rightarrow \tilde{\tau}_{1/2} \bar{\tau}) = \frac{g^2}{32\pi|m_{\tilde{Z}_i}|} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{Z}_i}, m_{\tilde{\tau}_{1/2}}, m_\tau) \left[ (c_1^2 + c_2^2)(m_{\tilde{Z}_i}^2 - m_{\tilde{\tau}_{1/2}}^2 + m_\tau^2) + 4m_\tau m_{\tilde{Z}_i} c_1 c_2 \right], \quad (\text{I.161})$$

where:

$$c_1 = \begin{cases} \sqrt{2} \cos \theta_\tau [c(1) \sin \theta_W + (\frac{1}{2} - \sin^2 \theta_W) \frac{c(2)}{\cos \theta_W}] - \sin \theta_\tau \frac{f_\tau}{g} N_{3i}, & \text{for } \tilde{\tau}_1, \\ -\sqrt{2} \sin \theta_\tau [c(1) \sin \theta_W + (\frac{1}{2} - \sin^2 \theta_W) \frac{c(2)}{\cos \theta_W}] - \cos \theta_\tau \frac{f_\tau}{g} N_{3i}, & \text{for } \tilde{\tau}_2, \end{cases} \quad (\text{I.162})$$

$$c_2 = \begin{cases} \sqrt{2} \sin \theta_\tau \sin \theta_W [c(2) \tan \theta_W - c(1)] - \cos \theta_\tau \frac{f_\tau}{g} N_{3i}, & \text{for } \tilde{\tau}_1, \\ \sqrt{2} \cos \theta_\tau \sin \theta_W [c(2) \tan \theta_W - c(1)] + \sin \theta_\tau \frac{f_\tau}{g} N_{3i}, & \text{for } \tilde{\tau}_2. \end{cases} \quad (\text{I.163})$$

Neutralino decays to a chargino and W boson:

$$\Gamma(\tilde{Z}_i \rightarrow W \tilde{W}_1) = \frac{g^2}{32\pi|m_{\tilde{Z}_i}|} \tilde{\lambda}^{1/2}(m_{\tilde{Z}_i}, m_{\tilde{W}_1}, m_W) \left[ -12m_{\tilde{Z}_i} m_{\tilde{W}_1} c_L c_R + (c_L^2 + c_R^2) \{ (m_{\tilde{W}_1}^2 + m_{\tilde{Z}_i}^2 - m_W^2) + (m_{\tilde{Z}_i}^2 + m_W^2 - m_{\tilde{W}_1}^2) (m_{\tilde{Z}_i}^2 - m_W^2 - m_{\tilde{W}_1}^2) \frac{1}{m_{\tilde{W}_1}^2} \} \right], \quad (\text{I.164})$$

where:

$$c_L = \frac{-1}{\sqrt{2}} N_{4i} \cos \theta_R + N_{2i} \sin \theta_R, \quad (\text{I.165})$$

$$c_R = \frac{1}{\sqrt{2}} N_{3i} \cos \theta_L + N_{2i} \sin \theta_L. \quad (\text{I.166})$$

For  $\tilde{W}_2$  just take  $m_{\tilde{W}_1} \rightarrow m_{\tilde{W}_2}$ ,  $\cos \theta_R \rightarrow \sin \theta_R$ ,  $\cos \theta_L \rightarrow \sin \theta_L$ ,  $\sin \theta_R \rightarrow -\cos \theta_R$  and  $\sin \theta_L \rightarrow -\cos \theta_L$ .

Neutralino decays to a chargino and charged Higgs boson:

$$\Gamma(\tilde{Z}_i \rightarrow H^\pm \tilde{W}_1) = \frac{1}{32\pi|m_{\tilde{Z}_i}|} \tilde{\lambda}^{1/2}(m_{\tilde{Z}_i}, m_{\tilde{W}_1}, m_{H^\pm}) \left[ (C_L^2 + C_R^2) (m_{\tilde{W}_1}^2 + m_{\tilde{Z}_i}^2 - m_{H^\pm}^2) + 4C_L C_R m_{\tilde{Z}_i} m_{\tilde{W}_1} \right], \quad (\text{I.167})$$

where:

$$C_L = \lambda \cos \beta N_{5i} \cos \theta_L - \frac{\sin \beta}{\sqrt{2}} [g' N_{1i} + g N_{2i}] \cos \theta_L + g \sin \beta N_{3i} \sin \theta_L, \quad (\text{I.168})$$

$$C_R = \lambda \sin \beta N_{5i} \cos \theta_R + \frac{\cos \beta}{\sqrt{2}} [g' N_{1i} + g N_{2i}] \cos \theta_R + g \cos \beta N_{4i} \sin \theta_R. \quad (\text{I.169})$$

Again for  $\tilde{W}_2$  just take  $m_{\tilde{W}_1} \rightarrow m_{\tilde{W}_2}$ ,  $\cos \theta_R \rightarrow \sin \theta_R$ ,  $\cos \theta_L \rightarrow \sin \theta_L$ ,  $\sin \theta_R \rightarrow -\cos \theta_R$  and  $\sin \theta_L \rightarrow -\cos \theta_L$ .

Neutralino decays to a lighter neutralino and Z boson:

$$\Gamma(\tilde{Z}_i \rightarrow Z \tilde{Z}_j) = \frac{g^2}{32\pi|m_{\tilde{Z}_i}|} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{Z}_i}, m_{\tilde{Z}_j}, m_Z) \left[ -12m_{\tilde{Z}_i} m_{\tilde{Z}_j} c_{LZ} c_{RZ} + (c_{LZ}^2 + c_{RZ}^2) (m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2 - m_Z^2) + (m_{\tilde{Z}_i}^2 - m_{\tilde{Z}_j}^2 + m_Z^2) (m_{\tilde{Z}_i}^2 - m_{\tilde{Z}_j}^2 - m_Z^2) \frac{1}{m_Z^2} \right], \quad (\text{I.170})$$

here we have:

$$c_{LZ} = -c_{RZ} = \frac{1}{2 \cos \theta_W} [N_{3i} N_{3j} - N_{4i} N_{4j}]. \quad (\text{I.171})$$

Neutralino decays to a lighter neutralino and CP even neutral Higgs boson:

$$\Gamma(\tilde{Z}_i \rightarrow h_k \tilde{Z}_j) = \frac{1}{4\pi|m_{\tilde{Z}_i}|} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{Z}_i}, m_{h_k}, m_{\tilde{Z}_j}) C_{\tilde{Z}_i \tilde{Z}_j h_k}^2 \left[ m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2 - m_{h_k}^2 + 2m_{\tilde{Z}_i} m_{\tilde{Z}_j} \right], \quad (\text{I.172})$$

where

$$\begin{aligned} C_{\tilde{Z}_i \tilde{Z}_j h_k} = & \frac{\lambda}{2\sqrt{2}} \left[ S(k, 1)(N_{3i} N_{5j} + N_{3j} N_{5i}) + S(k, 2)(N_{4i} N_{5j} + N_{4j} N_{5i}) \right. \\ & + S(k, 3)(N_{3i} N_{4j} + N_{4i} N_{3j}) \left. \right] - \sqrt{2} \kappa S(k, 3) N_{5i} N_{5j} \\ & + \frac{g'}{2} \left[ -S(k, 1)(N_{1i} N_{4j} + N_{1j} N_{4i}) + S(k, 2)(N_{1i} N_{3j} + N_{1j} N_{3i}) \right] \\ & + \frac{g}{2} \left[ S(k, 1)(N_{2i} N_{4j} + N_{2j} N_{4i}) - S(k, 2)(N_{2i} N_{3j} + N_{2j} N_{3i}) \right]. \end{aligned} \quad (\text{I.173})$$

Neutralino decays to a lighter neutralino and CP odd neutral Higgs boson:

$$\Gamma(\tilde{Z}_i \rightarrow A_k \tilde{Z}_j) = \frac{1}{4\pi|m_{\tilde{Z}_i}|} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{Z}_i}, m_{A_k}, m_{\tilde{Z}_j}) [\mathcal{G}_{\tilde{Z}_i \tilde{Z}_j A_k}^2] [m_{\tilde{Z}_i}^2 + m_{\tilde{Z}_j}^2 - m_{A_k}^2 - 2m_{\tilde{Z}_i} m_{\tilde{Z}_j}], \quad (\text{I.174})$$

where

$$\begin{aligned} \mathcal{G}_{\tilde{Z}_i \tilde{Z}_j A_k} = & \frac{-\lambda}{2\sqrt{2}} \left[ P(k, 1)(N_{3i} N_{5j} + N_{3j} N_{5i}) + P(k, 2)(N_{4i} N_{5j} + N_{4j} N_{5i}) \right. \\ & + P(k, 3)(N_{3i} N_{4j} + N_{4i} N_{3j}) \left. \right] - \sqrt{2} \kappa P(k, 3) N_{5i} N_{5j} \\ & - \frac{g'}{2} \left[ -P(k, 1)(N_{1i} N_{4j} + N_{1j} N_{4i}) + P(k, 2)(N_{1i} N_{3j} + N_{1j} N_{3i}) \right] \\ & - \frac{g}{2} \left[ P(k, 1)(N_{2i} N_{4j} + N_{2j} N_{4i}) - P(k, 2)(N_{2i} N_{3j} + N_{2j} N_{3i}) \right]. \end{aligned} \quad (\text{I.175})$$

Note the  $C$  and  $\mathcal{G}$  couplings here are similar to those given for the reverse decays of Higgs bosons to neutralinos earlier, with  $i, j, k$  permuted accordingly.

### Appendix I.5. Decays into Neutralinos

For the first two generations of quarks and squarks:

$$\Gamma(\tilde{q}_{L/R} \rightarrow q\tilde{Z}_i) = \frac{g^2}{16\pi m_{\tilde{q}_{L/R}}} B_{\tilde{q}_{L/R}}^2 \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{q}_{L/R}}, m_{\tilde{Z}_i}, m_q) [m_{\tilde{q}_{L/R}}^2 - m_{\tilde{Z}_i}^2 - m_q^2], \quad (\text{I.176})$$

where:

$$B_{\tilde{q}_{L/R}} = \begin{cases} -\sqrt{2}[\frac{2}{3}c(1)\sin\theta_W + (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)\frac{c(2)}{\cos\theta_W}], & \text{for } \tilde{u}_L \text{ type,} \\ -\sqrt{2}\frac{2}{3}\sin\theta_W[c(2)\tan\theta_W - c(1)], & \text{for } \tilde{u}_R \text{ type,} \\ \sqrt{2}[c(1)\frac{1}{3}\sin\theta_W + (\frac{1}{2} - \frac{1}{3}\sin^2\theta_W)\frac{c(2)}{\cos\theta_W}], & \text{for } \tilde{d}_L \text{ type,} \\ \sqrt{2}\frac{1}{3}\sin\theta_W[c(2)\tan\theta_W - c(1)], & \text{for } \tilde{d}_R \text{ type.} \end{cases} \quad (\text{I.177})$$

The decay formulae for decays of the first two generations of sleptons to leptons and neutralinos are the same as for the squarks here but with the coupling change  $B_{\tilde{q}_{L/R}} \rightarrow B_{\tilde{l}_{L/R}}$  and squark masses exchanged for slepton masses and quark masses for lepton masses:

$$B_{\tilde{l}_{L/R}} = \begin{cases} -\frac{c(2)}{\sqrt{2}\cos\theta_W}, & \text{for } \tilde{\nu}_L \text{ type,} \\ 0, & \text{for } \tilde{\nu}_R \text{ type (as no RH sneutrinos),} \\ \sqrt{2}[c(1)\sin\theta_W + (\frac{1}{2} - \sin^2\theta_W)\frac{c(2)}{\cos\theta_W}], & \text{for } \tilde{l}_L \text{ type,} \\ \sqrt{2}\sin\theta_W[c(2)\tan\theta_W - c(1)], & \text{for } \tilde{l}_R \text{ type.} \end{cases} \quad (\text{I.178})$$

For the third generation:

$$\Gamma(\tilde{t}_{1/2} \rightarrow t\tilde{Z}_i) = \frac{g^2}{16\pi m_{\tilde{t}_{1/2}}} [(d_1^2 + d_2^2)(m_{\tilde{t}_{1/2}}^2 - m_t^2 - m_{\tilde{Z}_i}^2) - 4d_1d_2m_tm_{\tilde{Z}_i}] \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{t}_{1/2}}, m_{\tilde{Z}_i}, m_t), \quad (\text{I.179})$$

where

$$d_1 = \begin{cases} \cos\theta_t \sqrt{2}[-\frac{2}{3}c(1)\sin\theta_W + (-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W)\frac{c(2)}{\cos\theta_W}] - \sin\theta_t \frac{f_t}{g} N_{4i}, & \text{for } \tilde{t}_1, \\ -\sin\theta_t \sqrt{2}[-\frac{2}{3}c(1)\sin\theta_W + (-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W)\frac{c(2)}{\cos\theta_W}] - \cos\theta_t \frac{f_t}{g} N_{4i}, & \text{for } \tilde{t}_2, \end{cases} \quad (\text{I.180})$$

$$d_2 = \begin{cases} -\frac{2}{3}\sin\theta_t \sqrt{2}\sin\theta_W[c(2)\tan\theta_W - c(1)] - \cos\theta_t \frac{f_t}{g} N_{4i}, & \text{for } \tilde{t}_1, \\ -\frac{2}{3}\cos\theta_t \sqrt{2}\sin\theta_W[c(2)\tan\theta_W - c(1)] + \sin\theta_t \frac{f_t}{g} N_{4i}, & \text{for } \tilde{t}_2. \end{cases} \quad (\text{I.181})$$

$$\Gamma(\tilde{b}_{1/2} \rightarrow b\tilde{Z}_i) = \frac{g^2}{16\pi m_{\tilde{b}_{1/2}}} [(f_1^2 + f_2^2)(m_{\tilde{b}_{1/2}}^2 - m_b^2 - m_{\tilde{Z}_i}^2) - 4f_1f_2m_bm_{\tilde{Z}_i}] \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{b}_{1/2}}, m_{\tilde{Z}_i}, m_b), \quad (\text{I.182})$$

where

$$f_1 = \begin{cases} \cos\theta_b \sqrt{2}[\frac{1}{3}c(1)\sin\theta_W + (\frac{1}{2} - \frac{1}{3}\sin^2\theta_W)\frac{c(2)}{\cos\theta_W}] - \sin\theta_b \frac{f_b}{g} N_{3i}, & \text{for } \tilde{b}_1, \\ -\sin\theta_b \sqrt{2}[\frac{1}{3}c(1)\sin\theta_W + (\frac{1}{2} - \frac{1}{3}\sin^2\theta_W)\frac{c(2)}{\cos\theta_W}] - \cos\theta_b \frac{f_b}{g} N_{3i}, & \text{for } \tilde{b}_2, \end{cases} \quad (\text{I.183})$$

$$f_2 = \begin{cases} \frac{1}{3}\sin\theta_b \sqrt{2}\sin\theta_W[c(2)\tan\theta_W - c(1)] - \cos\theta_b \frac{f_b}{g} N_{3i}, & \text{for } \tilde{b}_1, \\ \frac{1}{3}\cos\theta_b \sqrt{2}\sin\theta_W[c(2)\tan\theta_W - c(1)] + \sin\theta_b \frac{f_b}{g} N_{3i}, & \text{for } \tilde{b}_2. \end{cases} \quad (\text{I.184})$$

$$\Gamma(\tilde{\tau}_{1/2} \rightarrow \tau\tilde{Z}_i) = \frac{g^2}{16\pi m_{\tilde{\tau}_{1/2}}} [(g_1^2 + g_2^2)(m_{\tilde{\tau}_{1/2}}^2 - m_\tau^2 - m_{\tilde{Z}_i}^2) - 4g_1g_2m_\tau m_{\tilde{Z}_i}] \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{\tau}_{1/2}}, m_{\tilde{Z}_i}, m_\tau), \quad (\text{I.185})$$

where

$$g_1 = \begin{cases} \cos\theta_\tau \sqrt{2}[c(1)\sin\theta_W + (\frac{1}{2} - \sin^2\theta_W)\frac{c(2)}{\cos\theta_W}] - \sin\theta_\tau \frac{f_\tau}{g} N_{3i}, & \text{for } \tilde{\tau}_1, \\ -\sin\theta_\tau \sqrt{2}[c(1)\sin\theta_W + (\frac{1}{2} - \sin^2\theta_W)\frac{c(2)}{\cos\theta_W}] - \cos\theta_\tau \frac{f_\tau}{g} N_{3i}, & \text{for } \tilde{\tau}_2, \end{cases} \quad (\text{I.186})$$

$$g_2 = \begin{cases} \sin\theta_\tau \sqrt{2}\sin\theta_W[c(2)\tan\theta_W - c(1)] - \cos\theta_\tau \frac{f_\tau}{g} N_{3i}, & \text{for } \tilde{\tau}_1, \\ \cos\theta_\tau \sqrt{2}\sin\theta_W[c(2)\tan\theta_W - c(1)] + \sin\theta_\tau \frac{f_\tau}{g} N_{3i}, & \text{for } \tilde{\tau}_2. \end{cases} \quad (\text{I.187})$$

Remember the  $c(1)$  and  $c(2)$  were given previously in (I.154).

Sneutrino decays into neutralinos are given by:

$$\Gamma(\tilde{\nu}_{\tau_{1/2}} \rightarrow \nu_{\tau} \tilde{Z}_i) = \frac{g^2}{16\pi m_{\tilde{\nu}_{\tau_{1/2}}}} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{\nu}_{\tau_{1/2}}}, 0, m_{\tilde{Z}_i})(m_{\tilde{\nu}_{\tau_{1/2}}}^2 - m_{\tilde{Z}_i}^2) \left( \frac{c(2)}{\sqrt{2} \cos \theta_W} \right)^2. \quad (\text{I.188})$$

For chargino decays into neutralinos and charged Higgs bosons the partial width is given by:

$$\Gamma(\tilde{W}_1 \rightarrow H^\pm \tilde{Z}_j) = \frac{g^2}{32\pi |m_{\tilde{W}_1}|} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{W}_1}, m_{\tilde{Z}_j}, m_{H^\pm}) \left[ (c1_{\tilde{W}_1 H^\pm \tilde{Z}_j}^2 + c2_{\tilde{W}_1 H^\pm \tilde{Z}_j}^2)(m_{\tilde{Z}_j}^2 + m_{\tilde{W}_1}^2 - m_{H^\pm}^2) + 4c1_{\tilde{W}_1 H^\pm \tilde{Z}_j}^2 c2_{\tilde{W}_1 H^\pm \tilde{Z}_j}^2 m_{\tilde{Z}_j} m_{\tilde{W}_1} \right], \quad (\text{I.189})$$

where:

$$c1_{\tilde{W}_1 H^\pm \tilde{Z}_j} = \frac{1}{g} [\lambda \sin \beta N_{5j} \cos \theta_R + \frac{\cos \beta}{\sqrt{2}} (g' N_{1j} + g N_{2j}) \cos \theta_R + g \cos \beta N_{4j} \sin \theta_R], \quad (\text{I.190})$$

$$c2_{\tilde{W}_1 H^\pm \tilde{Z}_j} = \frac{1}{g} [\lambda \cos \beta N_{5j} \cos \theta_L - \frac{\sin \beta}{\sqrt{2}} (g' N_{1j} + g N_{2j}) \cos \theta_L + g \sin \beta N_{3j} \sin \theta_L]. \quad (\text{I.191})$$

For  $\tilde{W}_2$  the formulae are the same, just make the replacements  $m_{\tilde{W}_1} \rightarrow m_{\tilde{W}_2}$ ,  $\cos \theta_{L/R} \rightarrow \sin \theta_{L/R}$  and  $\sin \theta_{L/R} \rightarrow -\cos \theta_{L/R}$ .

$$\Gamma(\tilde{W}_1 \rightarrow W \tilde{Z}_j) = \frac{g^2}{32\pi |m_{\tilde{W}_1}|} \tilde{\lambda}^{\frac{1}{2}}(m_{\tilde{W}_1}, m_{\tilde{Z}_j}, m_W) \left[ -12m_{\tilde{W}_1} m_{\tilde{Z}_j} cL_{\tilde{W}_1 W \tilde{Z}_j} cR_{\tilde{W}_1 W \tilde{Z}_j} + (cL_{\tilde{W}_1 W \tilde{Z}_j}^2 + cR_{\tilde{W}_1 W \tilde{Z}_j}^2) \{ (m_{\tilde{W}_1}^2 + m_{\tilde{Z}_j}^2 - m_W^2) + (m_{\tilde{W}_1}^2 + m_W^2 - m_{\tilde{Z}_j}^2)(m_{\tilde{W}_1}^2 - m_{\tilde{Z}_j}^2 - m_W^2) \frac{1}{m_W^2} \} \right], \quad (\text{I.192})$$

where:

$$cL_{\tilde{W}_1 W \tilde{Z}_j} = -\frac{1}{\sqrt{2}} N_{4j} \cos \theta_R + N_{2j} \sin \theta_R, \quad (\text{I.193})$$

$$cR_{\tilde{W}_1 W \tilde{Z}_j} = \frac{1}{\sqrt{2}} N_{3j} \cos \theta_L + N_{2j} \sin \theta_L. \quad (\text{I.194})$$

Again for  $\tilde{W}_2$  the formulae are the same, just make the replacements  $m_{\tilde{W}_1} \rightarrow m_{\tilde{W}_2}$ ,  $\cos \theta_{L/R} \rightarrow \sin \theta_{L/R}$  and  $\sin \theta_{L/R} \rightarrow -\cos \theta_{L/R}$ .

## Appendix J. QCD Corrections to Decays

Note, for the decays of neutral Higgs bosons to quarks or gluons, the possibility of including QCD corrections is included in the program, by default the QCD corrections are on. The formulae are those provided in HDECAY-3.4 in SUSYHIT [21, 22] and NMSSMTools-4.2.1 in NMHDECAY [12, 25]. With QCD corrections incorporated our formulae become as follows:

$$\Gamma(h \rightarrow qq)_{QCDcorr} = \Gamma(h \rightarrow qq)_{tree} \left( 1 + \frac{4\alpha_s(m_h)}{3\pi} \left[ \frac{A(\tilde{\beta})}{\tilde{\beta}} + \frac{3 + 34\tilde{\beta}^2 - 13\tilde{\beta}^4}{16\tilde{\beta}^3} \log \frac{1 + \tilde{\beta}}{1 - \tilde{\beta}} + \frac{3}{8\tilde{\beta}^2} (7\tilde{\beta}^2 - 1) \right] \right). \quad (\text{J.1})$$

This formula applies for all the CP even neutral Higgs bosons, whether in the MSSM or NMSSM, the difference between the MSSM and NMSSM comes in the tree-level formula. Note  $\alpha_s$  is evaluated at the mass of the decaying Higgs boson. Also note that  $\tilde{\beta}$  and  $A(\tilde{\beta})$  are given by:

$$\tilde{\beta} = \sqrt{1 - 4 \frac{m_q^2}{m_h^2}}, \quad (\text{J.2})$$

$$A(\tilde{\beta}) = (1 + \tilde{\beta}^2) \left[ 4Li_2\left(\frac{1 - \tilde{\beta}}{1 + \tilde{\beta}}\right) + 2Li_2\left(\frac{\tilde{\beta} - 1}{\tilde{\beta} + 1}\right) - 3 \log\left(\frac{1 + \tilde{\beta}}{1 - \tilde{\beta}}\right) \log \frac{2}{1 + \tilde{\beta}} - 2 \log\left(\frac{1 + \tilde{\beta}}{1 - \tilde{\beta}}\right) \log \tilde{\beta} \right] - 3\tilde{\beta} \log \frac{4}{1 - \tilde{\beta}^2} - 4\tilde{\beta} \log \tilde{\beta}. \quad (J.3)$$

This is exactly as given in equations (16) and (25) of [37].  $Li_2$  is the dilogarithm function, aka Spence's function.

For the CP odd Higgs bosons we have:

$$\Gamma(A \rightarrow qq)_{QCDcorr} = \Gamma(A \rightarrow qq)_{tree} \left( 1 + \frac{4\alpha_s(m_A)}{3\pi} \left[ \frac{A(\tilde{\beta})}{\tilde{\beta}} + \frac{19 + 2\tilde{\beta}^2 + 3\tilde{\beta}^4}{16\tilde{\beta}} \log \frac{1 + \tilde{\beta}}{1 - \tilde{\beta}} + \frac{3}{8}(7 - \tilde{\beta}^2) \right] \right). \quad (J.4)$$

$\tilde{\beta}$  and  $A(\tilde{\beta})$  are as given above but with the change  $m_h \rightarrow m_A$  as appropriate. This formula is as given in equations (25) and (26) of [37].

The QCD corrections for  $h \rightarrow gg$  are more complicated as they involve both standard QCD corrections due to gluons being radiated, gluons in the loop, tops, bottoms and other quarks, and additional SUSY-QCD corrections due to gluinos, stops, sbottoms and other squarks. This complicates matters as whilst the usual ‘‘fermionic’’ QCD (FQCD) corrections apply to all particles in the loop, the SUSY-QCD (SQCD) corrections only apply to the scalar squark contributions, therefore rather than multiply the whole width by a correction factor (as was the case for  $h \rightarrow qq$ ) we must now correct the SM and SUSY loop contributions separately. The usual MSSM equation for  $h \rightarrow gg$  with no corrections is (note that  $\phi$  is a CP even neutral Higgs here):

$$\Gamma(\phi \rightarrow gg)_{1-loop} = \frac{\alpha_s^2(m_\phi)}{128\pi^3} \frac{G_F}{\sqrt{2}} m_\phi^3 \frac{9}{8} \Sigma |I_{loop}^\phi|^2. \quad (J.5)$$

Here the  $\alpha_s$  is run to the mass of the decaying Higgs boson. The  $I_{loop}$  can be split into  $I_{quark}$  and  $I_{squark}$  loop contributions. So  $I_{looptot}^\phi = I_{qtot}^\phi + I_{sqtot}^\phi$ , where  $I_{qtot}^\phi = I_t^\phi + I_b^\phi + I_c^\phi$  and  $I_{sqtot}^\phi = I_{\tilde{t}_1}^\phi + I_{\tilde{t}_2}^\phi + I_{\tilde{b}_1}^\phi + I_{\tilde{b}_2}^\phi + I_{\tilde{c}_L}^\phi + I_{\tilde{c}_R}^\phi + I_{\tilde{s}_L}^\phi + I_{\tilde{s}_R}^\phi + I_{\tilde{u}_L}^\phi + I_{\tilde{u}_R}^\phi + I_{\tilde{d}_L}^\phi + I_{\tilde{d}_R}^\phi$ .

To account for the usual QCD corrections, i.e. ‘‘FQCD’’ corrections, as these affect all the loop contributions, the whole partial width is multiplied by  $\delta_{FQCD}$ :

$$\delta_{FQCD}^{CPevenHiggs} = 1 + \frac{\alpha_s(m_\phi)}{\pi} \left( \frac{95}{4} - \frac{7}{6} N_f \right), \quad (J.6)$$

or

$$\delta_{FQCD}^{CPoddHiggs} = 1 + \frac{\alpha_s(m_\phi)}{\pi} \left( \frac{97}{4} - \frac{7}{6} N_f \right). \quad (J.7)$$

$N_f$  is the number of active fermion flavours; 5 for the SM-like lightest CP even neutral Higgs boson and 6 for the heavier CP even neutral Higgs boson  $H$  and the CP odd Higgs boson  $A$  (and also for  $H3$  and  $A2$  in the NMSSM). The SUSY QCD corrections, i.e. ‘‘SQCD’’ corrections, apply only to the squark loop contributions. Therefore to incorporate these in the final partial width you must multiply both the squark loop squared contributions and the interference terms of the squark loops with the quark loops by the correction factor. Therefore the  $|I_{loop}|^2$  (which comes from the matrix element squared) above with both FQCD and SQCD corrections included becomes:

$$|I_{looptot}^\phi|^2 = \delta_{FQCD}^\phi |I_{looptot}^\phi|^2 + \text{Re}[(I_{looptot}^\phi)^* I_{sqtot}^\phi] \delta_{SQCD}. \quad (J.8)$$

To be clear the  $(I_{looptot}^\phi)^*$  here means the complex conjugate of  $I_{looptot}^\phi$ , given this is an interference term. The  $\delta_{SQCD}$  correction factor is the same for CP even and CP odd neutral Higgs bosons and is given by (J.9) below, note that  $\alpha_s$  is run to the mass of the decaying Higgs boson:

$$\delta_{SQCD} = \frac{17\alpha_s(m_\phi)}{6\pi}. \quad (J.9)$$

Consequently in the MSSM the overall formula for the QCD and SUSY-QCD corrected  $h \rightarrow gg$  decay (at 2-loop) is:

$$\Gamma(\phi \rightarrow gg)_{1-loop+QCDcorr} = \frac{\alpha_s^2(m_\phi)}{128\pi^3} \frac{G_F}{\sqrt{2}} m_\phi^3 \frac{9}{8} \left[ \delta_{FQCD}^\phi |I_{looptot}^\phi|^2 + \text{Re}[(I_{looptot}^\phi)^* I_{sqtot}^\phi] \delta_{SQCD} \right]. \quad (J.10)$$

$\phi$  is a CP even neutral Higgs here as CP odd Higgs bosons do not have squark loop contributions because of CP invariance of the decays. For the CP odd Higgs  $A$  in the MSSM we therefore only have the quark loops and FQCD corrections:

$$\Gamma(A \rightarrow gg)_{1-loop+QCDcorr} = \frac{\alpha_s^2(m_A)}{128\pi^3} \frac{G_F}{\sqrt{2}} m_A^3 \frac{9}{8} [\delta_{FQCD}^A |I_{qtot}^A|^2]. \quad (J.11)$$

The FQCD corrections for CP even and CP odd neutral Higgs bosons in the MSSM are as given in [38] in equations (7) and (15) but with the log terms dropped as the scale of alphas is run to the masses of the decaying Higgs. As the corrections are purely coloured and the NMSSM only alters the Higgs and neutralino sectors, the form of the QCD corrections is exactly the same in the NMSSM. The alterations to the Higgs sector in the NMSSM however result in the couplings of the neutral Higgs bosons to other particles, and therefore the leading order (i.e. 1-loop) formula for the loop contributions to  $h \rightarrow gg$ , being altered. The formula in the uncorrected NMSSM (i.e. at 1-loop) is (as detailed previously) as follows, with the  $\alpha_s$  evaluated at the scale of the decaying Higgs boson:

$$\Gamma(h_i \rightarrow gg)_{1-loop} = \frac{G_F m_{h_i}^3 \alpha_s^2(m_{h_i})}{64 \sqrt{2} \pi^3} |M_{gg}|^2, \quad (J.12)$$

where

$$|M_{gg}|^2 = [J_t^r + J_b^r + J_c^r + J_{\tilde{u}_L}^r + J_{\tilde{u}_R}^r + J_{\tilde{d}_L}^r + J_{\tilde{d}_R}^r + J_{\tilde{c}_L}^r + J_{\tilde{c}_R}^r + J_{\tilde{s}_L}^r + J_{\tilde{s}_R}^r + J_{\tilde{t}_1}^r + J_{\tilde{t}_2}^r + J_{\tilde{b}_1}^r + J_{\tilde{b}_2}^r]^2 \\ + [J_t^i + J_b^i + J_c^i + J_{\tilde{u}_L}^i + J_{\tilde{u}_R}^i + J_{\tilde{d}_L}^i + J_{\tilde{d}_R}^i + J_{\tilde{c}_L}^i + J_{\tilde{c}_R}^i + J_{\tilde{s}_L}^i + J_{\tilde{s}_R}^i + J_{\tilde{t}_1}^i + J_{\tilde{t}_2}^i + J_{\tilde{b}_1}^i + J_{\tilde{b}_2}^i]^2, \quad (J.13)$$

where the  $J_X$  contributions are different to those in the MSSM as the couplings are different. Here the  $r$  and  $i$  were used as shorthands for real and imaginary parts. The  $|M_{gg}|^2$  is therefore just the mod square of the sum of the complex loop contributions.

In order to incorporate the FQCD and SQCD corrections we again group the loop contributions into quark and squark so that  $J_{qtot}^\phi = J_t^\phi + J_b^\phi + J_c^\phi$ ,  $J_{sqtot}^\phi = J_{\tilde{u}_L}^\phi + J_{\tilde{u}_R}^\phi + J_{\tilde{d}_L}^\phi + J_{\tilde{d}_R}^\phi + J_{\tilde{c}_L}^\phi + J_{\tilde{c}_R}^\phi + J_{\tilde{s}_L}^\phi + J_{\tilde{s}_R}^\phi + J_{\tilde{t}_1}^\phi + J_{\tilde{t}_2}^\phi + J_{\tilde{b}_1}^\phi + J_{\tilde{b}_2}^\phi$  and  $J_{looptot}^\phi = J_{qtot}^\phi + J_{sqtot}^\phi$ . Then  $|M_{gg}|^2$  becomes:

$$|M_{gg}|^2 = [\delta_{FQCD}^\phi |J_{looptot}^\phi|^2 + \text{Re}[(J_{looptot}^\phi)^* J_{sqtot}^\phi] \delta_{SQCD}^\phi], \quad (J.14)$$

because the FQCD and SQCD corrections apply to the loop contributions exactly as in the MSSM, however the loop contributions themselves have changed between the MSSM and NMSSM. So overall in the NMSSM, the QCD corrected partial width for neutral Higgs decays to gluons is as follows, again note the  $\alpha_s$  is evaluated at the scale of the mass of the decaying Higgs boson:

$$\Gamma(h_i \rightarrow gg)_{1-loop+QCDcorr} = \frac{G_F m_{h_i}^3 \alpha_s^2(m_{h_i})}{64 \sqrt{2} \pi^3} [\delta_{FQCD}^\phi |J_{looptot}^\phi|^2 + \text{Re}[(J_{looptot}^\phi)^* J_{sqtot}^\phi] \delta_{SQCD}^\phi]. \quad (J.15)$$

Again  $\phi$  here is a CP even neutral Higgs boson as CP odd Higgs bosons do not have squark loop contributions because of CP invariance of the decays, as in the MSSM. Therefore CP odd Higgs bosons have only quark loop contributions and so only receive FQCD corrections, without corrections the formula was:

$$\Gamma(A_i \rightarrow gg)_{1-loop} = \frac{g^2 \alpha_s^2(m_{A_i}) m_{A_i}^3}{128 \pi^3 m_W^2} |M_{A_i gg}|^2, \quad (J.16)$$

remembering that the  $\alpha_s(m_{A_i})$  means  $\alpha_s$  evaluated at the mass of the decaying CP odd Higgs boson  $A_i$ . Here  $|M_{A_i gg}|^2$  is:

$$|M_{A_i gg}|^2 = (\mathcal{R}_t^r + \mathcal{R}_b^r + \mathcal{R}_c^r)^2 + (\mathcal{R}_t^i + \mathcal{R}_b^i + \mathcal{R}_c^i)^2. \quad (J.17)$$

The corrections are incorporated by multiplying by  $\delta_{FQCD}^A$ , so with the QCD corrections the CP odd Higgs decays in the NMSSM are given by:

$$\Gamma(A_i \rightarrow gg)_{1-loop+QCDcorr} = \frac{g^2 \alpha_s^2(m_{A_i}) m_{A_i}^3}{128 \pi^3 m_W^2} |M_{A_i gg}|^2 \delta_{FQCD}^A. \quad (J.18)$$

Throughout, the formulae used are those given in HDECAY-3.4 [22] (and hence SUSYHT [21]) and NMSSMTools-4.2.1 [2, 24, 25].



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